1 Appendix A: The Proof of Proposition 1

The projection of $p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})$ is expressed as

$$\mathcal{N}_c(\mathbf{z};\mathbf{m}_{\mathbf{z}}^{t+1,-},\mathbf{v}_{\mathbf{z}}^{t+1,-}) = \mathrm{Proj}_{\mathbf{z}}[p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})], \tag{53}$$

where the mean vector and the covariance matrix are given by

$$\mathbf{m}_{\mathbf{z}}^{t+1,-} = \frac{\int \mathbf{z} p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})\mathrm{d}\mathbf{z}}{\int p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})\mathrm{d}\mathbf{z}}, \tag{54}$$

$$\mathbf{v}_{\mathbf{z}}^{t+1,-} = \frac{\int ||\mathbf{z}-\mathbf{m}_{\mathbf{z}}^{t+1,-}||^2 p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})\mathrm{d}\mathbf{z}}{\int p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})\mathrm{d}\mathbf{z}}. \tag{55}$$

Based on the separable structure of $p(\mathbf{y}^{\mathrm{u},p}_{\Phi_c}|\mathbf{z})$ and $\mathcal{N}_c(\mathbf{z};\mathbf{m}_1^{t,+},\mathbf{v}_1^{t,+})$, we calculate one of element of $\mathbf{m}_{\mathbf{z}}^{t+1,-}$ and other elements of $\mathbf{m}_{\mathbf{z}}^{t+1,-}$ have similar expression. Moreover, we deal $\mathbf{v}_{\mathbf{z}}^{t+1,-}$ with the same method. For simplification, we abuse z to denote the real part of image part of any element of $\mathbf{z}$. The transition probability of (4) is given by

$$p(y^{\mathrm{u},p}_{\Phi_c}|z) = \Upsilon\left(y^{\mathrm{u},p}_{\Phi_c}; z, \frac{\sigma_p^2}{2}\right), \tag{56}$$

where $\Upsilon(y^{\mathrm{u},p}_{\Phi_c}; z, c^2) \triangleq \Psi\left(\frac{y^{\mathrm{up}}-z}{c}\right) - \Psi\left(\frac{y^{\mathrm{low}}-z}{c}\right)$ with $\Psi(x) = \int_{-\infty}^{x}\mathcal{N}(t;0,1)\mathrm{d}t$. With those notations, we then have

$$\int_z p(y^{\mathrm{u},p}_{\Phi_c}|z)\mathcal{N}\left(z;m_1^{t,+},\frac{v_1^{t,+}}{2}\right)\mathrm{d}z \overset{(a)}{=} \int_z\int_{y^{\mathrm{low}}}^{y^{\mathrm{up}}}\mathcal{N}\left(s;z,\frac{\sigma_p^2}{2}\right)\mathrm{d}s\mathcal{N}\left(z;m_1^{t,+},\frac{v_1^{t,+}}{2}\right)\mathrm{d}z \tag{57}$$

$$\overset{(b)}{=} \int_{y^{\mathrm{low}}}^{y^{\mathrm{up}}}\mathcal{N}\left(s;m_1^{t,+},\frac{\sigma_p^2}{2}\right)\mathrm{d}s \tag{58}$$

$$= \Psi(\eta_1(y^{\mathrm{u},p}_{\Phi_c})) - \Psi(\eta_2(y^{\mathrm{u},p}_{\Phi_c})), \tag{59}$$

where (a) holds with the fact [5] while (b) holds using Gaussian produce lemma. In addition, some definitions are as following for simplification

$$\eta_1(y^{\mathrm{u},p}_{\Phi_c}) \triangleq \frac{y^{\mathrm{up}}-m_1^{t,+}}{\sqrt{\frac{\sigma_p^2+v_1^{t,+}}{2}}}, \eta_2(y^{\mathrm{u},p}_{\Phi_c}) \triangleq \frac{y^{\mathrm{low}}-m_1^{t,+}}{\sqrt{\frac{\sigma_p^2+v_1^{t,+}}{2}}}. \tag{61}$$

Differentiating w.r.t. $m_1^{t,+}$ on the both of (59) yields

$$\int\left(\frac{z-m_1^{+}}{v_1^{t,+}/2}\right)p(y^{\mathrm{u},p}_{\Phi_c}|z)\mathcal{N}_c\left(z;m_1^{t,+},\frac{v_1^{t,+}}{2}\right)\mathrm{d}z = -\sqrt{\frac{2}{\sigma_p^2+v_1^{t,+}}}[\phi(\eta_1(y^{\mathrm{u},p}_{\Phi_c})) - \phi(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))], \tag{62}$$

where the fact $\frac{\partial\Psi(x)}{\partial m} = \Theta(x)\frac{\partial x}{\partial m}$ and $\Theta(x) = \mathcal{N}(x;0,1)$ are used.
Combining (59) and (62), we get

$$m_z^{t+1,-} = m_1^{t,+} - \frac{v_1^{t,+}}{\sqrt{2(\sigma_p^2+v_1^{t,+})}}\frac{\Theta(\eta_1(y^{\mathrm{u},p}_{\Phi_c})) - \Theta(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}{\Psi(\eta_1(y^{\mathrm{u},p}_{\Phi_c})) - \Psi(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}, \tag{63}$$

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5) For $\Psi\left(\frac{y^{\mathrm{up}}-z}{\sigma}\right) = \int_{-\infty}^{\frac{y^{\mathrm{up}}-z}{\sigma}}\mathcal{N}(t;0,1)\mathrm{d}t$, we define auxiliary variable $x = \sigma t + z$, then there are $\mathrm{d}x = \sigma\mathrm{d}t$ and

$$\int_{-\infty}^{\frac{y^{\mathrm{up}}-z}{\sigma}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2}\right)\mathrm{d}t = \int_{-\infty}^{y^{\mathrm{up}}}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-z)^2}{2\sigma^2}\right)\mathrm{d}x = \int_{-\infty}^{y^{\mathrm{up}}}\mathcal{N}(x;z,\sigma^2)\mathrm{d}x. \tag{60}$$

where the expectation is taken over $\frac{\int_z zp(y^{\mathrm{u},p}_{\Phi_c}|z)\mathcal{N}(z;m_1^{t,+},\frac{v_1^{t,+}}{2})\mathrm{d}z}{\int_z p(y^{\mathrm{u},p}_{\Phi_c}|z)\mathcal{N}(z;m_1^{t,+},\frac{v_1^{t,+}}{2})\mathrm{d}z}$.
Using the property of exponential family [6)] , clearly, we can obtain

$$v_z^{t+1,-}=\frac{v_1^{t,+}}{2}-\frac{(v_1^{t,+})^2}{2(\sigma_p^2+v_1^{t,+})}\left[\frac{\eta_1(y^{\mathrm{u},p}_{\Phi_c})\Theta(\eta_1(y))-\eta_2(y^{\mathrm{u},p}_{\Phi_c})\Theta(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}{\Psi(\eta_1(y^{\mathrm{u},p}_{\Phi_c}))-\Psi(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}+\left(\frac{\Theta(\eta_1(y^{\mathrm{u},p}_{\Phi_c}))-\Theta(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}{\Psi(\eta_1(y^{\mathrm{u},p}_{\Phi_c}))-\Psi(\eta_2(y^{\mathrm{u},p}_{\Phi_c}))}\right)^2\right]. \quad (64)$$

2 Appendix B: The Proof of Proposition 2

From (17), we reformulate $\mathbf{m}_{\mathbf{x}^p}^{t+1,+}$ in Algorithm 1 as

$$\mathbf{m}_{\mathbf{x}}^{t+1,+}=\frac{\int \mathbf{x}^p p(\mathbf{x}^p)\mathcal{N}_c\left(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-}\right)\mathrm{d}\mathbf{x}^p}{\int p(\mathbf{x}^p)\mathcal{N}_c\left(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-}\right)\mathrm{d}\mathbf{x}^p}. \quad (65)$$

With the prior probability of $\mathbf{x}$ in (18), we have

$$\begin{aligned}
&\int p(\mathbf{x}^p)\mathcal{N}_c\left(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-}\right)\mathrm{d}\mathbf{x}^p\\
&=\prod_{n=1}^N\int_{-\infty}^{+\infty}\left[(1-\epsilon)\delta(x_n^p)+\epsilon\mathcal{N}_c\left(x_n^p;0,\lambda_n\right)\right]\mathcal{N}_c\left(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}\right)\mathrm{d}x_n^p\\
&=\prod_{n=1}^N\left[\int_{-\infty}^{+\infty}(1-\epsilon)\delta(x_n)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p+\int_{-\infty}^{+\infty}\epsilon\mathcal{N}_c(x_n^p;0,\lambda_n)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\right]\\
&\overset{(a)}{=}\prod_{n=1}^N\left[(1-\epsilon)\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})+\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\right.\\
&\left.\int_{-\infty}^{+\infty}\mathcal{N}_c\left(x_n^p;\frac{m_{0,n}^{t+1,-}\lambda_n}{\lambda_n+v_{0,n}^{t+1,-}},\frac{\lambda_n v_{0,n}^{t+1,-}}{\lambda_n+v_{0,n}^{t+1,-}}\right)\mathrm{d}x_n^p\right]\\
&=\prod_{n=1}^N\left[(1-\epsilon)\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})+\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\right],
\end{aligned} \quad (66)$$

where the step (a) uses the fact that:
(1)$\int_{-\infty}^{\infty}\delta(x)\mathrm{d}x=1,\delta(x)=0,x\neq 0$;
(2)$\mathcal{N}_c(x;a,A)\mathcal{N}_c(x;b,B)=\mathcal{N}_c(0;a-b,A+B)\mathcal{N}_c(x;c,C)$, where $C=(\frac{1}{A}+\frac{1}{B})^{-1},c=C(\frac{a}{A}+\frac{b}{B})$.
Moreover,

$$\begin{aligned}
&\int \mathbf{x}^p p(\mathbf{x}^p)\mathcal{N}_c(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},v_0^{t+1,-})\mathrm{d}\mathbf{x}^p\\
&=\prod_{n=1}^N\int_{-\infty}^{+\infty}x_n^p[(1-\epsilon)\delta(x_n^p)+\epsilon\mathcal{N}_c(x_n^p;0,\lambda_n)]\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\\
&=\prod_{n=1}^N\left[\int_{-\infty}^{+\infty}(1-\epsilon)x_n^p\delta(x_n^p)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p+\int_{-\infty}^{+\infty}\epsilon x_n^p\mathcal{N}_c(x_n^p;0,\lambda_n)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\right]\\
&=\prod_{n=1}^N\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\int_{-\infty}^{+\infty}x_n^p\mathcal{N}_c\left(x_n^p;\frac{m_{0,n}^{t+1,-}\lambda_n}{\lambda_n+v_{0,n}^{t+1,-}},\frac{\lambda_n v_{0,n}^{t+1,-}}{\lambda_n+v_{0,n}^{t+1,-}}\right)\mathrm{d}x_n^p\\
&=\prod_{n=1}^N\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\cdot\frac{m_{0,n}^{t+1,-}\lambda_n}{\lambda_n+v_{0,n}^{t+1,-}}.
\end{aligned} \quad (67)$$

6) $\frac{\partial}{\partial m}\frac{\int xp(x)\mathcal{N}(x;m,v)\mathrm{d}x}{\int p(x)\mathcal{N}(x;m,v)\mathrm{d}x}=\frac{1}{v}\frac{\int(x-\hat{x})^2p(x)\mathcal{N}(x;m,v)\mathrm{d}x}{\int p(x)\mathcal{N}(x;m,v)\mathrm{d}x}$ where $p(x)$ can be any bounded function with non-negative value.

From (66) and (67), we have (22). On the other hand, the variance matrix $\mathbf{v}_{\mathbf{x}^p}^{t+1,+}$ can be written as

$$\mathbf{v}_{\mathbf{x}^p}^{t+1,+}=\frac{\int |\mathbf{x}^p|^2 p(\mathbf{x}^p)\mathcal{N}_c\left(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-}\right)\mathrm{d}\mathbf{x}^p}{\int p(\mathbf{x}^p)\mathcal{N}_c\left(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-}\right)\mathrm{d}\mathbf{x}^p}-|\mathbf{m}_{\mathbf{x}^p}^{t+1,+}|^2. \tag{68}$$

Since

$$\begin{aligned}
&\int |\mathbf{x}^p|^2 p(\mathbf{x}^p)\mathcal{N}_c(\mathbf{x}^p;\mathbf{m}_0^{t+1,-},\mathbf{v}_0^{t+1,-})\mathrm{d}\mathbf{x}^p\\
&=\prod_{n=1}^{N}\int_{-\infty}^{+\infty}|x_n^p|^2[(1-\epsilon)\delta(x_n^p)+\epsilon\mathcal{N}_c(x_n^p;0,\lambda_n)]\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\\
&=\prod_{n=1}^{N}\Big[\int_{-\infty}^{+\infty}(1-\epsilon)|x_n^p|^2\delta(x_n^p)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\\
&+\int_{-\infty}^{+\infty}\epsilon|x_n^p|^2\mathcal{N}_c(x_n^p;0,\lambda_n)\mathcal{N}_c(x_n^p;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-})\mathrm{d}x_n^p\Big]\\
&=\prod_{n=1}^{N}\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\int_{-\infty}^{+\infty}|x_n^p|^2\mathcal{N}_c\left(x_n^p;\frac{m_{0,n}^{t+1,-}\lambda_n}{\lambda_n+v_{0,n}^{t+1,-}},\frac{\lambda_n v_{0,n}^{t+1,-}}{\lambda_n+v_{0,n}^{t+1,-}}\right)\mathrm{d}x_n^p\\
&=\prod_{n=1}^{N}\epsilon\mathcal{N}_c(0;m_{0,n}^{t+1,-},v_{0,n}^{t+1,-}+\lambda_n)\cdot\left(\frac{\lambda_n v_{0,n}^{t+1,-}}{\lambda_n+v_{0,n}^{t+1,-}}+\left|\frac{m_{0,n}^{t+1,-}\lambda_n}{\lambda_n+v_{0,n}^{t+1,-}}\right|^2\right),
\end{aligned} \tag{69}$$

From (66), (68) and (69), we have (23).

3 Appendix C: The Proof of Lemma 1

From (32), the LS estimator of h_i^{d} is

$$\hat{h}_i^{\mathrm{d}}=\frac{1}{\alpha}\boldsymbol{\phi}_i^H\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p}. \tag{70}$$

Then, we have

$$\mathbb{E}\{\hat{h}_i(\hat{h}_i^{\mathrm{d}})^H\}=\mathbb{E}\left\{\frac{1}{\alpha}\boldsymbol{\phi}_i^H\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p}\frac{1}{\alpha}(\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p})^H\boldsymbol{\phi}_i\right\}=\frac{1}{\alpha^2}\boldsymbol{\phi}_i^H\mathbb{E}\left\{\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p}(\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p})^H\right\}\boldsymbol{\phi}_i. \tag{71}$$

Since

$$\begin{aligned}
\mathbb{E}\left\{\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p}(\mathbf{y}_{i,\Phi_c}^{\mathrm{d},p})^H\right\}&=\mathbb{E}\left\{(\alpha\boldsymbol{\phi}_i h_i^{\mathrm{d}}+\alpha\mathbf{w}^{\mathrm{d},p}+\mathbf{w}_q)(\alpha\boldsymbol{\phi}_i h_i^{\mathrm{d}}+\alpha\mathbf{w}^{\mathrm{d},p}+\mathbf{w}_q)^H\right\}\\
&=\alpha^2\lambda_i\boldsymbol{\phi}_i\boldsymbol{\phi}_i^H+\alpha^2\sigma_p^2\mathbf{I}_{L^q}+\alpha\varsigma(\lambda_i+\sigma_p^2)\mathbf{I}_{L^q}.
\end{aligned} \tag{72}$$

Submitting (72) into (71), we have

$$\mathbb{E}\left\{\hat{h}_i^{\mathrm{d}}(\hat{h}_i^{\mathrm{d}})^H\right\}=\lambda_i+\sigma_p^2+\frac{\varsigma}{\alpha}\left(\lambda_i+\sigma_p^2\right). \tag{73}$$

Note that, $\hat{h}_i^{\mathrm{d}}$ in (70) can be re-written as

$$\hat{h}_i^{\mathrm{d}}=\frac{1}{\alpha}\boldsymbol{\phi}_i^H(\alpha\boldsymbol{\phi}_i h_i^{\mathrm{d}}+\alpha\mathbf{w}^{\mathrm{d},p}+\mathbf{w}_q)=h_i^{\mathrm{d}}+\boldsymbol{\phi}_i^H\left(\mathbf{w}^{\mathrm{d},p}+\frac{1}{\alpha}\mathbf{w}_q\right)\triangleq h_i^{\mathrm{d}}+e_i, \tag{74}$$

where $e_i\triangleq\boldsymbol{\phi}_i^H\left(\mathbf{w}^{\mathrm{d},p}+\frac{1}{\alpha}\mathbf{w}_q\right)$ and

$$\mathbb{E}\{e_i e_i^H\}=\sigma_p^2+\frac{\varsigma}{\alpha}(\lambda_i+\sigma_p^2). \tag{75}$$

4 Appendix D: The Proof of Theorem 1

The symbols of device v at device u are always treated as interference, then $\bar{\mathcal{E}}_u = \bar{\mathcal{E}}_{u\to u}$. Let $|\hat{h}_u^{\mathrm{d}}|^2 = \tilde{\lambda}_u x$ with x following exponential distribution, the average BLER of decoding x_u^{d} is

$$\bar{\mathcal{E}}_{u\to u} \approx \int_0^{\infty} Q\left(\frac{\log_2\left(1+\frac{\alpha_u\tilde{\lambda}_u x}{\alpha_v\tilde{\lambda}_u x+\tilde{\sigma}_{eu}^2+1/\rho}\right)-\frac{B_u}{L^{\mathrm{d}}}}{\sqrt{1-\frac{1}{\left(1+\frac{\alpha_u\tilde{\lambda}_u x}{\alpha_v\tilde{\lambda}_u x+\tilde{\sigma}_{eu}^2+1/\rho}\right)^2}}}\cdot\frac{\sqrt{L^{\mathrm{d}}}}{\log_2 e}\right) e^{-\frac{x}{\tilde{\lambda}_u}}\mathrm{d}x, \tag{76}$$

where $e^{-\frac{x}{\tilde{\lambda}_u}}$ is the PDF of estimated channel gain, $|\hat{h}_u^{\mathrm{d}}|^2$. Defining $y = \frac{\alpha_u\tilde{\lambda}_u x}{\alpha_v\tilde{\lambda}_u x+\tilde{\sigma}_{eu}^2+1/\rho}$, we have

$$x = \frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)\tilde{\lambda}_u}, \mathrm{d}x = \frac{\alpha_u(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)^2\tilde{\lambda}_u}\mathrm{d}y, \tag{77}$$

where $y \leqslant \frac{\alpha_u}{\alpha_v}$. Base on (77), the average BLER of device u in (76) can be calculated as

$$\bar{\mathcal{E}}_{u\to u} \approx \int_0^{\frac{\alpha_u}{\alpha_v}} Q\left(\frac{\log_2(1+y)-\frac{B_u}{L^{\mathrm{d}}}}{\sqrt{1-\frac{1}{(1+y)^2}}}\cdot\frac{\sqrt{L^{\mathrm{d}}}}{\log_2 e}\right)\frac{\alpha_u(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)^2\tilde{\lambda}_u}\exp\left(-\frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)\tilde{\lambda}_u}\right)\mathrm{d}y. \tag{78}$$

By defining

$$Z(y) \triangleq Q\left(\frac{\log_2(1+y)-\frac{B_u}{L^{\mathrm{d}}}}{\sqrt{1-\frac{1}{(1+y)^2}}}\cdot\frac{\sqrt{L^{\mathrm{d}}}}{\log_2 e}\right), \tag{79}$$

similar to [1], $Z(y), y \in (-\infty, +\infty)$, can be approximated as follows

$$Z(y) = \begin{cases} 1, & y \leqslant \mu_{u,L^{\mathrm{d}}}, \\ \frac{1}{2} - \alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}(y-\beta_{u,L^{\mathrm{d}}}), & \mu_{u,L^{\mathrm{d}}} < y < \nu_{u,L^{\mathrm{d}}}, \\ 0, & y \geqslant \nu_{u,L^{\mathrm{d}}}, \end{cases} \tag{80}$$

where $\alpha_{u,L^{\mathrm{d}}} = \frac{1}{\sqrt{2\pi(2^{2B_u/L^{\mathrm{d}}}-1)}}, \beta_{u,L^{\mathrm{d}}} = 2^{\frac{B_u}{L^{\mathrm{d}}}}-1, \mu_{u,L^{\mathrm{d}}} = \beta_{u,L^{\mathrm{d}}} - \frac{1}{2\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}}$, and $\nu_{u,L^{\mathrm{d}}} = \beta_{u,L^{\mathrm{d}}} + \frac{1}{2\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}}$. Note that $\frac{1}{2\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}} = \sqrt{\frac{\pi(2^{2B_u/L^{\mathrm{d}}}-1)}{2L^{\mathrm{d}}}}$ is a small positive number when the coding rate is small [2] and the block-length is larger than 100 for device u. Therefore, we have $(\mu_{u,L^{\mathrm{d}}}, \nu_{u,L^{\mathrm{d}}}) \subset (0, \frac{\alpha_u}{\alpha_v})$, and

$$\frac{\partial Z(y)}{\partial y} = -\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}, \tag{81}$$

for $\mu_{u,L^{\mathrm{d}}} < y < \nu_{u,L^{\mathrm{d}}}$. From (78) and (81), the average BLER of decoding x_u^{d} at device u is

$$\begin{aligned}\bar{\mathcal{E}}_{u\to u} &\approx \int_0^{\frac{\alpha_u}{\alpha_v}} \frac{\partial Z(y)}{\partial y}\left[1-\exp\left(-\frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)\tilde{\lambda}_u}\right)\right]\mathrm{d}y \\ &= \int_{\mu_{u,L^{\mathrm{d}}}}^{\nu_{u,L^{\mathrm{d}}}} \alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\left[1-\exp\left(-\frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)\tilde{\lambda}_u}\right)\right]\mathrm{d}y \\ &= 1-\int_{\mu_{u,L^{\mathrm{d}}}}^{\nu_{u,L^{\mathrm{d}}}} \alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\exp\left(-\frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{(\alpha_u-\alpha_v y)\tilde{\lambda}_u}\right)\mathrm{d}y. \end{aligned} \tag{82}$$

$$\bar{\mathcal{E}}_{v\to u} \approx 1-\frac{\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\alpha_u(\tilde{\sigma}_{ev}^2+1/\rho)\exp(\frac{\tilde{\sigma}_{ev}^2+1/\rho}{\alpha_v\tilde{\lambda}_v})}{\alpha_v^2\tilde{\lambda}_v}\left[\mathrm{E}_1\left(\frac{\alpha_u(\tilde{\sigma}_{ev}^2+1/\rho)}{(\alpha_u-\alpha_v\mu_{u,L^{\mathrm{d}}})\alpha_v\tilde{\lambda}_v}\right)-\mathrm{E}_1\left(\frac{\alpha_u(\tilde{\sigma}_{ev}^2+1/\rho)}{(\alpha_u-\alpha_v\nu_{u,L^{\mathrm{d}}})\alpha_v\tilde{\lambda}_v}\right)\right]+ \frac{\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\exp(\frac{\tilde{\sigma}_{ev}^2+1/\rho}{\alpha_v\tilde{\lambda}_v})}{\alpha_v}\cdot(\alpha_u-\alpha_v\nu_{u,L^{\mathrm{d}}})\left[\exp\left(-\frac{\alpha_u(\tilde{\sigma}_{ev}^2+1/\rho)}{(\alpha_u-\alpha_v\nu_{u,L^{\mathrm{d}}})\alpha_v\tilde{\lambda}_v}\right)-\exp\left(-\frac{\alpha_u(\tilde{\sigma}_{ev}^2+1/\rho)}{(\alpha_u-\alpha_v\mu_{u,L^{\mathrm{d}}})\alpha_v\tilde{\lambda}_v}\right)\right]. \quad (86)$$

Let $z=\frac{y(\tilde{\sigma}_{eu}^2+1/\rho)}{\tilde{\lambda}_u(\alpha_u-\alpha_v y)}, \theta_1=\frac{\mu_{u,L^{\mathrm{d}}}}{\alpha_u-\alpha_v\mu_{u,L^{\mathrm{d}}}}, \theta_2=\frac{\nu_{u,L^{\mathrm{d}}}}{\alpha_u-\alpha_v\nu_{u,L^{\mathrm{d}}}}$, then (82) is expressed as

$$\bar{\mathcal{E}}_{u\to u} \approx 1-\frac{\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\alpha_u(\tilde{\sigma}_{eu}^2+1/\rho)}{\alpha_v^2\tilde{\lambda}_u}\int_{\frac{\theta_1(\tilde{\sigma}_{eu}^2+1/\rho)}{\tilde{\lambda}_u}}^{\frac{\theta_2(\tilde{\sigma}_{eu}^2+1/\rho)}{\tilde{\lambda}_u}}\frac{\exp(-z)}{\left(\frac{(\tilde{\sigma}_{eu}^2+1/\rho)}{\alpha_v\tilde{\lambda}_u}+z\right)^2}\mathrm{d}z. \quad (83)$$

By defining $t=\frac{\tilde{\sigma}_{eu}^2+1/\rho}{\alpha_v\tilde{\lambda}_u}+z$, (83) is further expressed as

$$\bar{\mathcal{E}}_{u\to u} \approx 1-\frac{\alpha_{u,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\alpha_u(\tilde{\sigma}_{eu}^2+1/\rho)\exp\left(\frac{(\tilde{\sigma}_{eu}^2+1/\rho)}{\alpha_v\tilde{\lambda}_u}\right)}{\alpha_v^2\tilde{\lambda}_u}\int_{\frac{\tilde{\sigma}_{eu}^2+1/\rho}{\tilde{\lambda}_u}\left[\theta_1+\frac{1}{\alpha_v}\right]}^{\frac{\tilde{\sigma}_{eu}^2+1/\rho}{\tilde{\lambda}_u}\left[\theta_2+\frac{1}{\alpha_v}\right]}\frac{\exp(-t)}{t^2}\mathrm{d}t. \quad (84)$$

From [3], we have

$$\int_{\mu}^{\infty}\frac{\exp(-x)}{x^2}\mathrm{d}x=\mathrm{E}_1(\mu)+\frac{\exp(-\mu)}{\mu}, \quad \mu>0. \quad (85)$$

Submitting (85) into (84)) yields (40).
Similarly, the average BLER of decoding x_u^{d} at device v is given by (86), shown at the top of this page. Since the cumulative distribution function (CDF) of $\gamma_{v\to v}$ is

$$F_{\gamma_{v\to v}}(x)=1-\exp\left(-\frac{x(\tilde{\sigma}_{ev}^2+1/\rho)}{\alpha_v\tilde{\lambda}_v}\right). \quad (1)$$

Then we have

$$\begin{aligned}\bar{\mathcal{E}}_{v\to v} &\approx \alpha_{v,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\int_{\mu_{v,L^{\mathrm{d}}}}^{\nu_{v,L^{\mathrm{d}}}}F_{\gamma_{v\to v}}(x)\mathrm{d}x\\ &=1-\frac{\alpha_{v,L^{\mathrm{d}}}\sqrt{L^{\mathrm{d}}}\alpha_v\tilde{\lambda}_v}{\tilde{\sigma}_{ev}^2+1/\rho}\left[\exp\left(-\frac{\mu_{v,L^{\mathrm{d}}}(\tilde{\sigma}_{ev}^2+1/\rho)}{\alpha_v\tilde{\lambda}_v}\right)-\exp\left(-\frac{\nu_{v,L^{\mathrm{d}}}(\tilde{\sigma}_{ev}^2+1/\rho)}{\alpha_v\tilde{\lambda}_v}\right)\right]. \quad (2)\end{aligned}$$

From (11), (86) and (2), we have the average BLER $\bar{\mathcal{E}}_v=\bar{\mathcal{E}}_{v\to u}+\bar{\mathcal{E}}_{v\to v}$ in (41).

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