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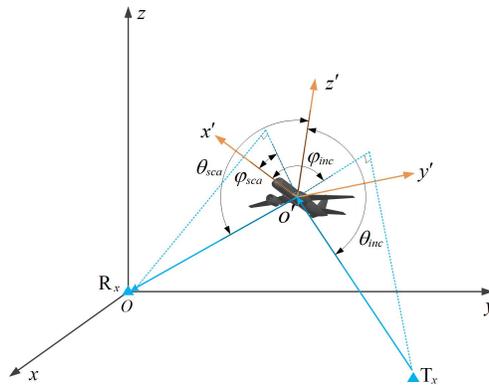
# Automatic Target Recognition Combining Angular Diversity and Time Diversity for Multistatic Passive Radar

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## Appendix A Sketch map of the angular subspace

Since each receiving station can only observe local information from a target in limited time, we divide the angular space consisting of four components of incident angle and scattering angle of electromagnetic wave into multiple subspaces, and then analyze target characteristics, build and train sub-recognizers, and do the preliminary recognition separately in these angular subspaces. Each angular subspace corresponds to an angle domain in the target coordinate system. We denote the two components of incident angle as  $\theta_{inc}$  and  $\phi_{inc}$ , the two components of scattering angle as  $\theta_{sca}$  and  $\phi_{sca}$ . Figure A1 shows the definitions of  $\theta_{inc}$ ,  $\phi_{inc}$ ,  $\theta_{sca}$ , and  $\phi_{sca}$ . The  $o - xyz$  is the radar coordinate system,  $o' - x'y'z'$  is the target coordinate system. The origin of  $o' - x'y'z'$  is defined at the center of mass of the target. The positive direction of the  $x'$ -axis points to the nose direction, and that of the  $y'$ -axis points to the left side of the fuselage. The  $z'$ -axis is defined by the right-hand rule. The incident angle and scattering angle are defined in the target coordinate system. The  $\theta_{inc}$  is defined as the angle between the line of transmitter-to-target and the positive semi-axis of the  $z'$ -axis. The  $\phi_{inc}$  is defined as the angle between the projection of the line of transmitter-to-target on the  $x'y'$  plane and the positive semi-axis of the  $x'$ -axis. The  $\theta_{sca}$  and  $\phi_{sca}$  are defined in the same way. The value ranges of  $\theta_{inc}$  and  $\theta_{sca}$  are set to  $[0^\circ, 180^\circ]$ , whereas value ranges of  $\phi_{inc}$  and  $\phi_{sca}$  are set to  $[0^\circ, 360^\circ]$ .



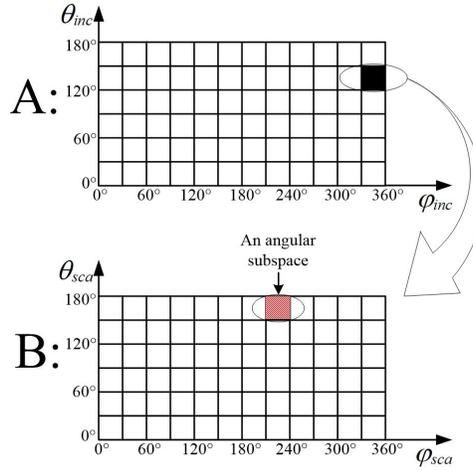
**Figure A1** Definitions of  $\theta_{inc}$ ,  $\phi_{inc}$ ,  $\theta_{sca}$ , and  $\phi_{sca}$ .  $R_x$  denotes the receiving station,  $T_x$  denotes the transmitting station.  $o - xyz$  is the radar coordinate system, whereas  $o' - x'y'z'$  is the target coordinate system. The incident angle, i.e.,  $\theta_{inc}$  and  $\phi_{inc}$ , and scattering angle, i.e.,  $\theta_{sca}$  and  $\phi_{sca}$ , are defined in the target coordinate system. The  $\theta_{inc}$  is defined as the angle between the line of transmitter-to-target and the positive semi-axis of the  $z'$ -axis. The  $\phi_{inc}$  is defined as the angle between the projection of the line of transmitter-to-target on the  $x'o'y'$  plane and the positive semi-axis of the  $x'$ -axis. The  $\theta_{sca}$  and  $\phi_{sca}$  are defined in the same way.

With a step of  $30^\circ$  for the four components, we divide the angular space into  $6 \times 12 \times 6 \times 12 = 5184$  subspaces. Figure A2 shows the sketch map of the angular subspace. Each small square of A denotes an angle range of incident angle corresponding to  $\Delta\theta_{inc}=30^\circ$ ,  $\Delta\phi_{inc}=30^\circ$ . And B denotes the division of scattering angle under the angle range of incident angle shown in black in A. Each small square in B denotes an angular subspace.

## Appendix B The time fusion module and decision module

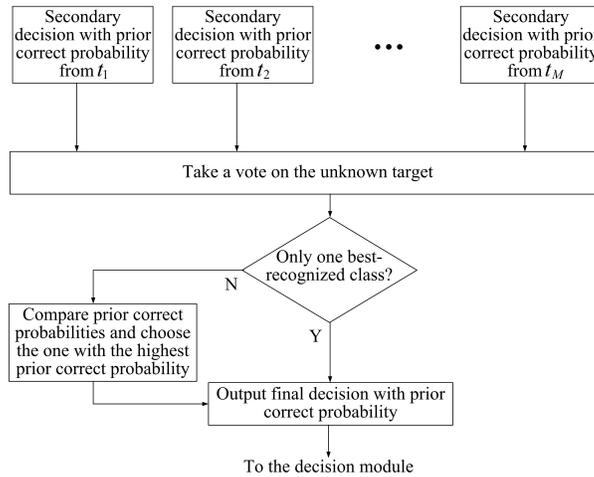
The details of the time fusion module are shown in Figure B1. We employ the voting strategy to make time fusion. That is, when the number of secondary decisions reaches the expected number, the time fusion module takes a vote on the unknown target. It counts the votes of each target class obtained from the secondary decisions and assigns the target class with the most votes to the unknown target. When there are more than one class with the highest number of votes, it goes back to compare the prior correct

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**Figure A2** Sketch map of the division of angular space. Each small square of A denotes an angle range of incident angle corresponding to  $\Delta\theta_{inc}=30^\circ$ ,  $\Delta\phi_{inc}=30^\circ$ . And B denotes the division of scattering angle under the angle range of incident angle shown in black in A. Each small square in B denotes an angular subspace.

probabilities of the secondary decisions voting for these classes, and chooses the one with the highest prior correct probability as the decision. It should be noted that the prior correct probability of the secondary decision is obtained from the highest prior correct probability of the preliminary decisions that vote for it, and the prior correct probability of the preliminary decision is obtained from the average correct recognition rate of the sub-recognizer that is called by the receiving station. We call the decision output by the time fusion module the final decision. In addition, we use the highest prior correct probability of those secondary decisions voting for the final decision to approximate the prior correct probability of the final decision. And then the prior correct probability is sent together with the final decision to the decision module.

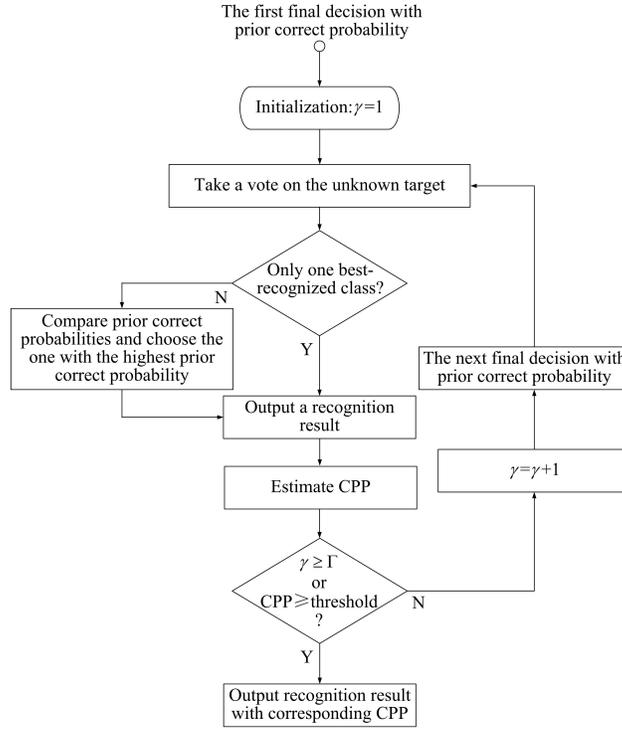


**Figure B1** Block diagram of the time fusion module. The voting strategy is used to make time fusion.

The block diagram of the decision module using SV strategy is shown in Figure B2, where  $\gamma$  is the current number of fused final decisions,  $\Gamma$  is the threshold of the number of final decisions to fuse. As we can see, the proposed SV strategy takes successive vote on the unknown target according to the final decisions. That is, each time a final decision enters the decision module, we take a vote on the unknown target according to current final decision and those old final decisions. In the voting stage, we use the same voting strategy mentioned in the time fusion module to decide the recognition result. Then, we estimate the CPP of the recognition result to see if it satisfies the CPP threshold. If the CPP satisfies the threshold or  $\gamma \geq \Gamma$ , the decision module outputs the recognition result and the corresponding CPP. Otherwise, it continues to fuse the final decisions.

### Appendix C Detailed derivation of CPP

The CPP is estimated with the Total Probability Theorem. The detailed estimation method of CPP is as follows. We assume that  $A_i, i = 1, 2, \dots, \gamma$  is the event that the recognizer correctly recognizes the target  $i$  times in  $\gamma$  final decisions,  $B$  the event that the recognizer correctly recognizes the unknown target with fusing  $\gamma$  final decisions. It is no doubt that all of  $A_i, i = 1, 2, \dots, \gamma$  are mutually exclusive events and they form the exhaustive events. According to the Total Probability Theorem, we know the probability of event  $B$  happening can be determined as follows,



**Figure B2** Block diagram of the decision module using the proposed SV strategy.  $\gamma$  is the counter used to count the successively fused final decisions. CPP is the probability that the outputted recognition result is correct recognition.

$$P(B) = \sum_{i=1}^{\gamma} P(A_i)P(B|A_i) \quad (C1)$$

where  $P(B)$  is the probability of event  $B$  happening, i.e., the CPP,  $P(A_i)$  is the probability of event  $A_i$  happening,  $P(B|A_i)$  is the conditional probability of event  $B$  happening conditioned on event  $A_i$ . The probability  $P(A_i)$  is expressed as follows,

$$P(A_i) = \sum_{s=1}^{C_{\gamma}^i} \xi_s \quad (C2)$$

where  $C_{\gamma}^i$  is the number of possible situations of event  $A_i$  happening, which is expressed as follows,

$$C_{\gamma}^i = \frac{\gamma!}{i!(\gamma-i)!} \quad (C3)$$

$\xi_s$  is the probability of the  $s$ th situation happening among  $C_{\gamma}^i$  situations corresponding to the event  $A_i$ , which can be determined as follows,

$$\xi_s = \prod_{d=1}^i p_d \prod_{g=1}^{\gamma-i} (1-p_g) \quad (C4)$$

where  $p_d$  is the correct probability of each final decision among  $i$  correct final decisions,  $(1-p_g)$  is the wrong probability of each final decision among  $(\gamma-i)$  wrong final decisions.  $p_d$  and  $p_g$  are the prior probabilities. To calculate  $P(A_i)$ , we first enumerate all the  $C_{\gamma}^i$  possible situations of event  $A_i$ . Then, using (C2), (C3), and (C4), we can obtain  $P(A_i)$ .

Next, we derive the  $P(B|A_i)$ . Assume there are  $Q$  classes of targets in the standard target database. Then we can infer the conditional probability  $P(B|A_i)$  from the principle of the voting strategy. As aforementioned,  $A_i$  denotes the event that the recognizer correctly recognizes the target  $i$  times in  $\gamma$  final decisions. Under this circumstance, it is easy to know that the probability of the recognizer correctly recognizing the unknown target is zero when  $i < \gamma/Q$  since the right recognition cannot get the highest number of votes under this condition, which means the right recognition is voted out. And when  $i > \gamma/2$ , the recognizer can definitely get the right recognition since the right recognition will win the highest number of votes for sure, regardless of the number of votes of other classes. In this situation,  $P(B|A_i) = 1$ . As for the situation of  $\gamma/Q \leq i \leq \gamma/2$ , the probability of the recognizer correctly recognizing the unknown target can be determined as follows. We can easily know that there are still  $(\gamma-i)$  votes left for  $(Q-i)$  targets under this condition. According to the Theory of Permutation and Combination, there could be  $C_{\gamma-i+Q-2}^{Q-2}$  possible voting results under this circumstance. It should be noted that these possible voting results are equal in probability. Although each final decision has a prior probability to be correct recognition, its possibility of belonging to each target class is equal since we do not know the class of the unknown target yet and either one of the standard target classes could be the correct one. That is, the chance of each final decision voting for either standard target class is equal and has nothing to do with its prior probability. As a result of this, all possible voting results are equal in probability, i.e., the probability of each possible voting result happening is  $1/C_{\gamma-i+Q-2}^{Q-2}$ . Assume that the right recognition probability for the  $j$ th voting result among  $C_{\gamma-i+Q-2}^{Q-2}$

possible voting results is  $p'_j$ , and the class with the highest number of votes in the remaining  $(Q - i)$  classes obtains  $V$  votes from  $(\gamma - i)$  votes. Then the  $p'_j$  can be determined as follows,

$$p'_j = \begin{cases} 0, & i < V \\ 0.5, & i = V \\ 1, & i > V \end{cases} \quad (C5)$$

There is no doubt about  $p'_j$  when  $i < V$  or  $i > V$ . As for  $i = V$ , we assign 0.5 to  $p'_j$  since there is no obvious evidence to decide which target class is to choose. According to the proposed SV strategy, it is the corresponding prior correct probability of the final decision that determines the recognition result under such circumstance, i.e., target class voted by the final decision with the highest prior correct probability is chosen. As we have mentioned in Appendix B that the prior correct probability of the final decision is related to that of the preliminary decision, and the preliminary decision depends on the observation angle of the radar to the target. Considering that the target attitude is random in the actual scene, the prior correct probability of each preliminary decision is also random. In other words, the probability for which target class to be chosen is equal. Thus, we assign 0.5 to  $p'_j$  under such circumstance. Thus, The corresponding  $P(B|A_i)$  can be expressed as follows,

$$P(B|A_i) = \sum_{j=1}^{C_{\gamma-i+Q-2}^{Q-2}} \frac{p'_j}{C_{\gamma-i+Q-2}^{Q-2}}, \gamma/Q \leq i \leq \gamma/2 \quad (C6)$$

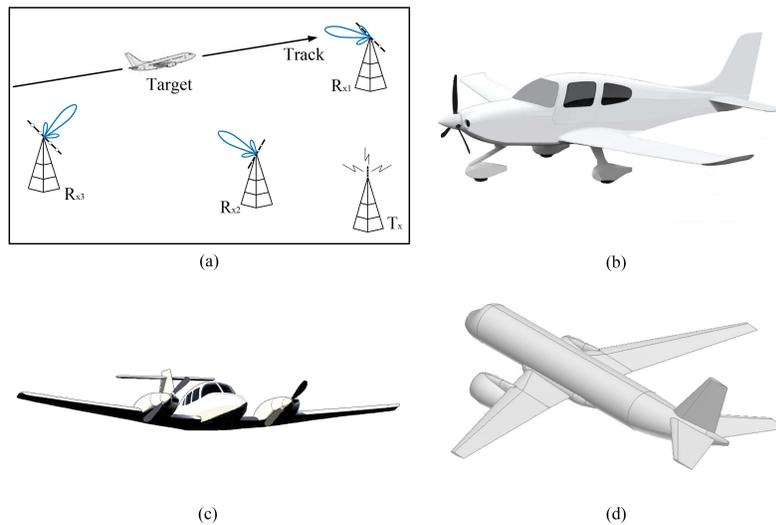
To sum up, we have the conditional probability as follows,

$$P(B|A_i) = \begin{cases} 0, & i < \frac{\gamma}{Q} \\ \sum_{j=1}^{C_{\gamma-i+Q-2}^{Q-2}} \frac{p'_j}{C_{\gamma-i+Q-2}^{Q-2}}, & \gamma/Q \leq i \leq \gamma/2 \\ 1, & i > \frac{\gamma}{2} \end{cases} \quad (C7)$$

With (C1), (C2), (C3), (C4), (C5), and (C7), we can obtain  $P(B)$ , i.e., the CPP of each recognition result.

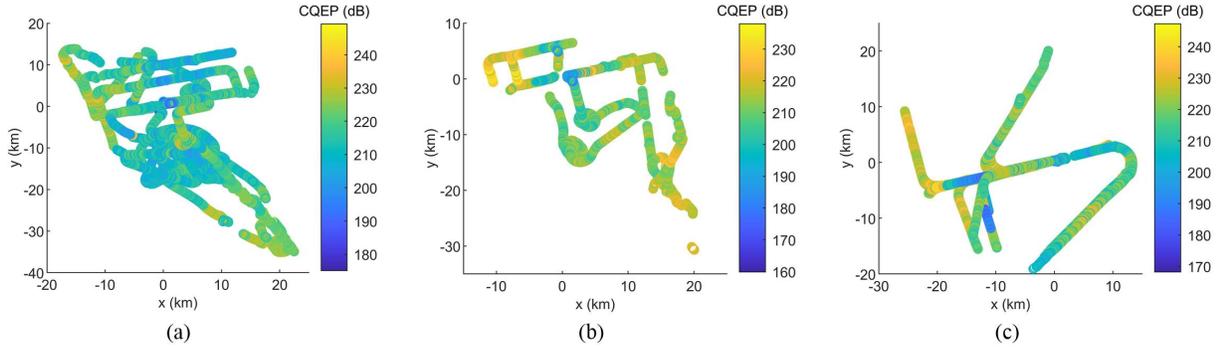
## Appendix D The experimental scene configuration and measured raw data

We conducted a field experiment in Luoyang Beijiao Airport in Henan Province of China with the multistatic passive radar developed in Wuhan University. Figure D1 (a) shows the sketch map of the station layout of the multistatic passive radar. It consists of one transmitting station and three receiving stations. The transmitting station is located at the Luoyang City, which is a television station and transmits the digital television terrestrial multimedia broadcast (DTMB) signals. The three receiving stations are located nearby the Luoyang Beijiao Airport. The array antennas of the three receiving stations point to different directions to observe target from different angles simultaneously. Thus, more comprehensive target data could be obtained. In the experiment, we collect data of three cooperative targets, i.e., Cirrus SR-20, Piper PA44, and Airbus A320, to evaluate performance of the proposed ATR method. These cooperative targets are all confirmed by the Automatic Dependent Surveillance-Broadcast (ADS-B). Figure D1 (b) – Figure D1 (d) show their appearance. It should be noted that the feature size of A320 is the largest, which is much larger than the other two targets. The SR20 and PA44 have the similar feature size, but their detailed structure is quite different. As shown in Figure D1 (b) and Figure D1 (c), the SR20 is a single engine propeller aircraft with a traditional tail, while PA44 is a twin engine propeller aircraft with a T-tail.



**Figure D1** Sketch map of the multistatic passive radar and three cooperative targets. (a) Configuration of the multistatic passive radar. (b) General aviation plane Cirrus SR20. (c) General aviation plane Piper PA44. (d) Civil aviation plane Airbus A320. The array antennas of the three receiving stations point to different directions to obtain target information from different angles simultaneously. Three cooperative targets are confirmed by the Automatic Dependent Surveillance-Broadcast (ADS-B). Among three target classes, feature size of A320 is the largest, which is much larger than those of the other two targets. The SR20 and PA44 have the similar feature size, but their detailed structure is quite different.

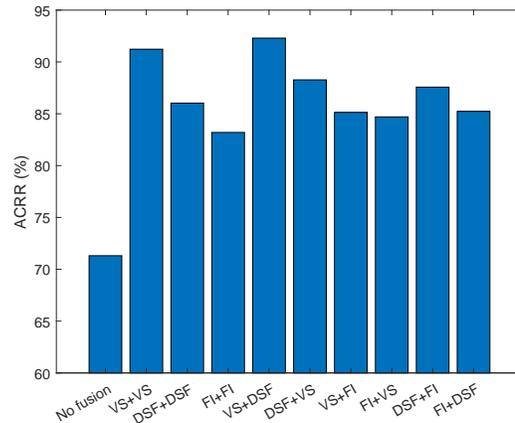
For each target class, we collect a large amount of data and pick 30000 sets of data for target recognition experiment. Each set of data contains the coordinates of target in the radar coordinate system, target velocity, target acceleration, and the compensated quasi-echo-power (CQEP) (i.e., a kind of quasi-RCS). Figure D2 shows the measured target trajectories of SR20, PA44, and A320. The CQEP corresponding to each point in the trajectory is indicated by color. The data of A320 is mainly collected during the take-off and landing stages, whereas the data of the SR20 and PA44 are also collected during the cruise stage in addition to the take-off and landing stages. We then divide these data into two sets: 70% of them as the training data set and the rest 30% as the test data set. The recognition results shown in the Letter are all obtained based on the data of the test set.



**Figure D2** Target trajectories detected by the multistatic passive radar. (a) SR20, (b) PA44, (c) A320. The color of each point in the trajectory indicates the amplitude of CQEP.

## Appendix E Comparison with other methods of fusion recognition

A comparison of the ACRRs among recognizers using the voting strategy (VS) and other fusion methods for station fusion and time fusion is made in this Appendix. The other fusion methods used for comparison are the Dempster-Shafer fusion (DSF) and fuzzy integral (FI). Besides the VS method, these two fusion methods are also typically used when the recognizer output is the fuzzy membership grade. Figure E1 shows the comparison results. The x-tick label denotes the combination of fusion methods used in the recognizer. Notice that the order of fusion methods in each combination indicates the order in which it is used in recognition. That is, the former one is used in the station fusion, and the latter one is used in the time fusion. These recognition results are all obtained by fusing three receiving stations and three time points.



**Figure E1** A comparison of the ACRRs among recognizers using three fusion methods. VS denotes the fusion method of voting strategy, DSF denotes Dempster-Shafer fusion, and FI denotes fuzzy integral. These three fusion methods are typically used when the recognizer output is fuzzy membership grade.

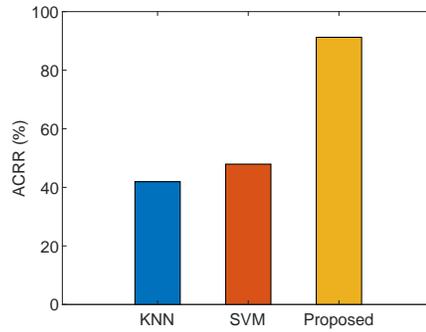
It can be seen from the figure that when the fusion methods are used in the recognition, the ACRR is significantly improved, which validates the benefit of the fusion method to improve the recognizers performance. However, the benefits of these fusion methods are different. As can be seen from the figure, when we use the same fusion methods in both the station fusion and the time fusion, the ACRR of the VS+VS combination is the highest whereas that of the FI+FI combination is the lowest. This result indicates that in order to design a good recognizer, it is worth evaluating the performance of different fusion methods before choosing one for recognition. Moreover, we also investigate the performance of the recognizer using different fusion methods in the station fusion and time fusion. As can be seen from the figure, the combination of the VS+DSF outperforms other combinations, whereas the FI+VS combination performs worst among these combinations. Moreover, the ACRR obtained with the VS+DSF combination is also larger than the one obtained with the VS+VS combination. These results indicate that the VS+DSF combination is the best one to be used in the recognizer. In addition, we can also find from the figure that the order of combining different fusion methods

in the station fusion and time fusion matters. As can be seen from the figure that the ACRRs corresponding to the VS+DSF and the DSF+VS combinations are quite different.

Although the VS+DSF combination outperforms other combinations, we do not follow this combination to use the fusion methods in our recognizer. Instead, we adopt the VS+VS combination. The main reason lies in two aspects. For one thing, the ACRR of the VS+VS combination is only slightly lower than that of the VS+DSF combination. Thus, we can obtain similar recognition performance using the VS+VS combination as using the VS+DSF combination. For another, the application scene of the VS+DSF combination is limited due to the limitation of DSF method. On the contrary, the VS+VS combination is not only simple to implement, but also applicable to a wider range of scenarios.

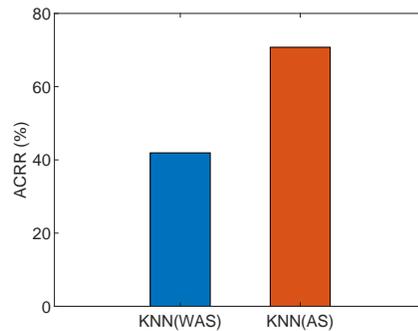
## Appendix F Comparison with other ATR methods

A comparison of the proposed method with other commonly used ATR methods is made in this Appendix. The methods used to make comparison are k-nearest neighbor (KNN) and support vector machine (SVM). These two methods are widely studied and used in the field of target recognition. Figure F1 shows the results of ACRRs obtained with these ATR methods. Note that the training data and test data used for the three methods are the same. What is different is that the data for the KNN and SVM methods are those in the whole angular space (WAS), whereas those for the proposed method are in the angular subspaces (ASs). The reason for this difference is that dividing the angular space of the target coordinate system into multiple subspaces is part of the design of the proposed method. Moreover, the ACRR of the proposed method is obtained by fusing three receiving stations and three time points.



**Figure F1** A comparison of the proposed method with the KNN and SVM methods. The ACRRs of the KNN and SVM methods are obtained with the data in the WAS. The ACRR of the proposed method is obtained with the data in the ASs and by fusing three receiving stations and three time points.

It can be seen from the figure that the proposed method possesses the highest ACRR among three methods, whereas the KNN method possesses the lowest ACRR, which indicates that the proposed method outperforms the other two methods. The main reason for the superiority of the proposed method to the other two methods lies in two aspects. On the one hand, the proposed method adopts the angular diversity and time diversity in the recognition, which could provide more information for the recognition and is beneficial to the improvement of correct recognition rate. On the other hand, building and training sub-recognizers in the ASs can keep the local characteristics of the target from being averaged. Compared to the averaged target characteristics in the WAS, the local characteristics possess more obvious differences among targets, which is good to the target recognition. To verify this point, another comparison of recognizers using data in the WAS and in the AS is made. Figure F2 shows the results for comparison. The x-tick labels indicate the method and data type for obtaining the ACRRs. The KNN(WAS) means that the KNN method uses the data in the whole angular space to recognize targets, whereas the KNN(AS) means that the KNN method uses the data in each angular subspace separately to recognize targets.



**Figure F2** A comparison of recognizers using data in the whole angular space and in the angular subspaces. The KNN(WAS) means that the KNN method uses the data in the whole angular space to recognize targets, whereas the KNN(AS) means that the KNN method uses the data in each angular subspace separately to recognize targets.

As can be seen from Figure F2, the ACRR of the KNN recognizer using the data in the ASs is much larger than that of the

recognizer using the data in the WAS. This comparison indicates that there is a significant impact of the division of angular space upon the ACRR, which also verifies the analysis made above.