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Time-scheduled observer design for switched linear systems with unknown inputs

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Dear editor,

Since uncertain disturbance, actuator faults and unmodeled dynamics can all be viewed as unknown inputs in various practical systems, the issue of unknown input observer design is of great significance. Meanwhile, switched systems have attracted extensive attention, which are prevailingly motivated by superior capabilities in modeling numerous practical systems possessing switching characteristics. Recently, the state observation for switched systems with unknown inputs has attracted considerable attention [1–5]. To mention a few, in [1], switched observers for continuous-time switched linear systems are developed based on both arbitrary and average dwell-time (ADT) switching. The common Lyapunov function is adopted to deduce the observer design scheme, which is fairly conservative. To overcome this restriction, a piecewise time-varying Lyapunov function approach is developed in [2] to analyze the switched error dynamics and design unknown input observers for switched linear systems. Furthermore, a dwell-time-based observer design scheme is proposed for switched systems with unknown inputs in [3], which no longer requires the stability of the estimation error dynamics of subsystems but asymptotically estimates the states by exploring the properties of switching signals. Here, we try to develop a more general observer design scheme for switched linear systems with unknown inputs. Firstly, the switching mechanism is considered to be with the admissible edge-dependent ADT (AEDADT) property, which is more flexible than ADT switching or mode-dependent ADT (MDADT) switching since it takes the switching relations among all subsystems into consideration [6]. Meanwhile, inspired by the quasi-time-dependent (QTD) technique for discrete-time switched systems in [7], we extend this method to construct an improved multiple discontinuous Lyapunov-like function (MDLF) for continuous-time switched systems. By adopting such an MDLF, the time-scheduled observer is designed, which is helpful to reduce conservatism compared with the traditional time-independent state observation scheme. See Appendix A for notations in this study.

Problem formulation. Consider the following switched linear system with unknown inputs:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}w(t), \tag{1}$$

$$y(t) = C_{\sigma(t)}x(t), \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, $y(t) \in \mathbb{R}^{n_y}$ is the system output, $u(t) \in \mathbb{R}^{n_u}$ and $w(t) \in \mathbb{R}^{n_w}$ are the known and unknown unknown inputs to the system, respectively. $\sigma(t)$ is the switching signal that takes its values on the discrete set $\mathcal{M} = \{1, 2, \dots, M\}$ with M denoting the total number of subsystems. The system matrix quadruples (A_i, B_i, C_i, E_i) are known for $i \in \mathcal{M}$.

The switching signal $\sigma(t)$ is considered to be with the AEDADT property (see Appendix B for the definition of AEDADT switching). Let $\{t_s, s \in \mathbb{Z}_{\geqslant 0}\}$ denote the switching sequence with t_s being the switching instant. When considering $\sigma(t_s) = i \in \mathcal{M}$, the switching signal $\sigma(t)$ is also required to satisfy $t_{s+1} - t_s \geqslant \tau_i^d$ with τ_i^d being a positive instant for $i \in \mathcal{M}$.

The objective is to deduce the asymptotic reconstruction of the state $x(t)$ of the system (1) and (2). Firstly, a system transformation is introduced by using transformation matrices T_i and U_i for $i \in \mathcal{M}$ (see Appendix C for details):

$$\dot{\tilde{x}}_1(t) = A_{1\sigma(t)}\tilde{x}_1(t) + A_{2\sigma(t)}\tilde{x}_2(t) + B_{1\sigma(t)}u(t), \tag{3}$$

$$\dot{\tilde{x}}_2(t) = A_{3\sigma(t)}\tilde{x}_1(t) + A_{4\sigma(t)}\tilde{x}_2(t) + B_{2\sigma(t)}u(t) + \omega(t), \tag{4}$$

$$\tilde{y}_1(t) = \tilde{C}_{\sigma(t)}\tilde{x}_1(t), \tag{5}$$

$$\tilde{y}_2(t) = \tilde{x}_2(t). \tag{6}$$

From (6), it can be seen that $\tilde{x}_2(t)$ is directly available by $\tilde{y}_2(t)$. Therefore, we only need to reconstruct $\tilde{x}_1(t)$. Once $\tilde{x}_1(t)$ is estimated, the original state $x(t)$ can be obtained through the inverse mapping:

$$x(t) = T_{\sigma(t)}^{-1}\begin{bmatrix}\tilde{x}_1(t)\\ \tilde{y}_2(t)\end{bmatrix} = T_{\sigma(t)}^{-1}\begin{bmatrix}\tilde{x}_1(t)\\ (C_{\sigma(t)}E_{\sigma(t)})^{+}y(t)\end{bmatrix}. \tag{7}$$

Next, we design the following Luenberger observer for $\tilde{x}_1(t)$ in (3):

$$\dot{\hat{\tilde{x}}}_1(t) = A_{1\sigma(t)}\hat{\tilde{x}}_1(t) + A_{2\sigma(t)}\tilde{y}_2(t) + B_{1\sigma(t)}u(t)$$

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$$+ L_{\sigma(t)}(q(t))(\tilde{C}_{\sigma(t)}\hat{\tilde{x}}_1(t) - \tilde{y}_1(t)), \quad (8)$$

where $L_i(q(t))$ for $i \in \mathcal{M}$ is the observer gain to be determined, and $q(t)$ is a time scheduler which can be calculated by

$$q(t) = \begin{cases} \left\lfloor \dfrac{t - t_s}{h} \right\rfloor, & t \in [t_s + \tau_{\max}, t_s + \tau^d_{\sigma(t_s)}), \\ \Theta_{\sigma(t_s)} - 1, & t \in [t_s + \tau^d_{\sigma(t_s)}, t_{s+1} + \tau_{s+1}). \end{cases} \quad (9)$$

Thereinto, $h > 0$ is a given constant, Θ_i is a given positive integer, and $\sigma(t_s) = i \in \mathcal{M}$. It indicates that the interval $[t_s, t_s + \tau^d_{\sigma(t_s)})$ is divided into $\Theta_{\sigma(t_s)}$ portions, and the length of each portion is h. Then, the constant h and positive integer Θ_i for the ith mode satisfy $\Theta_i h = \tau^d_i$ for $i \in \mathcal{M}$. The time scheduler $q(t)$ will reset at t_s. Let $\bar{t}^{l_i}_s = t_s + l_i h$, $l_i \in \mathbb{Z}_{[0,\Theta_i]}$, $i \in \mathcal{M}$. It can be seen $\bar{t}^{\Theta_i}_s = t_s + \tau^d_i$. $q(t)$ will update at the instants $\bar{t}^1_s, \ldots, \bar{t}^{\Theta_i - 1}_s$, and then remain unchanged until the next switching occurs. Consequently, for $t \in [t_s, t_{s+1})$, the value of $q(t)$ belongs to $\mathbb{Z}_{[0,\Theta_i - 1]}$.

The observer (8) is connected with the time scheduler $q(t)$. We call such a time-scheduled observer as a QTD one. The QTD observer (8) exists state jumps at the switching instant t_s:

$$\hat{\tilde{x}}_1(t_s) = E^{\perp}_{\sigma(t_s)}\hat{x}(t_s^-) = E^{\perp}_{\sigma(t_s)} T^{-1}_{\sigma(t_s^-)} \begin{bmatrix} \hat{\tilde{x}}_1(t_s^-) \\ \tilde{y}_2(t_s^-) \end{bmatrix}, \quad (10)$$

where $\hat{x}(t) \in \mathbb{R}^{n_x}$ is the estimation of $x(t)$.

Let $e(t) = x(t) - \hat{x}(t)$ and $\tilde{e}_1(t) = \tilde{x}_1(t) - \hat{\tilde{x}}_1(t)$ denote the estimation errors. According to (3), (8), and (10), the dynamics of the observation error can be obtained:

$$\dot{\tilde{e}}_1(t) = \mathcal{A}_{\sigma(t)}(q(t))\tilde{e}_1(t), \quad (11)$$

$$\tilde{e}_1(t_s) = E^{\perp}_{\sigma(t_s)} e(t_s) = E^{\perp}_{\sigma(t_s)} \bar{T}_{\sigma(t_s^-)} \tilde{e}_1(t_s^-), \quad (12)$$

where $\mathcal{A}_{\sigma(t)}(q(t)) = A_{1\sigma(t)} - L_{\hat{\sigma}(t)}(q(t))\tilde{C}_{\sigma(t)}$.

The state estimation issue of the system (1) is converted to find admissible observer gain $L_i(\varphi_i)$, $\varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $i \in \mathcal{M}$, such that the switched error system (11) and (12) is asymptotically stable.

Switched observer design. Using the improved multiple Lyapunov-like function, the stability analysis for the system (11) and (12) can be deduced (see Appendix D). On this basis, the switched observer (8) is designed for the system (3).

Theorem 1. Let $\lambda_i > 0$, $0 < \rho_i < 1$, $\mu_{i,j} > 1$ with $\mu_{i,j}\rho_i^{\Theta_i - 1} > 1$, $h > 0$ be given constants, and Θ_i be a positive integer, $\forall (i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$. If there exist matrices $P_i(\varphi_i) > 0$, $M_i(\varphi_i)$ for $\varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $i \in \mathcal{M}$ such that $\forall \varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $l_i \in \mathbb{Z}_{[1,\Theta_i - 1]}$, $(i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$,

$$\Xi_i(\varphi_i) < 0, \quad (13)$$

$$(E^{\perp}_i \bar{T}_j)^{\top} P_i(0) E^{\perp}_i \bar{T}_j - \mu_{i,j} P_j(\Theta_j - 1) \leqslant 0, \quad (14)$$

$$P_i(l_i) - \rho_i P_i(l_i - 1) \leqslant 0, \quad (15)$$

where $\Xi_i(\iota) = A^{\top}_{1i} P_i(\iota) + P_i(\iota) A_{1i} - \tilde{C}^{\top}_i M^{\top}_i(\iota) - M_i(\iota)\tilde{C}_i + \lambda_i P_i(\iota)$, then, for any switching signal $\sigma(t)$ satisfying $\tau^d_i = \Theta_i h$, and

$$\tau^a_{i,j} > \tau^{a*}_{i,j} = \frac{\ln \mu_{i,j}\rho_i^{\Theta_i - 1}}{\lambda_i}, \quad (16)$$

the observer (8) can asymptotically estimate the states of the system (3). Moreover, for $\varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $\forall i \in \mathcal{M}$, the observer gain is given by

$$L_i(\varphi_i) = P_i(\varphi_i)^{-1} M_i(\varphi_i). \quad (17)$$

Proof. See Appendix E.

In Theorem 1, the time-scheduled state estimation for the transformed system (3) is studied. Next, the observer for all states of the switched system with unknown inputs (1) and (2) is presented.

Theorem 2. Consider the unknown input switched system (1) and (2). Let $\lambda_i > 0$, $0 < \rho_i < 1$, $\mu_{i,j} > 1$ with $\mu_{i,j}\rho_i^{\Theta_i - 1} > 1$, $h > 0$ be given constants, and Θ_i be a positive integer, $\forall (i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$. If there exist matrices $P_i(\varphi_i) > 0$, $M_i(\varphi_i)$ for $\varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $i \in \mathcal{M}$ such that $\forall \varphi_i \in \mathbb{Z}_{[0,\Theta_i - 1]}$, $l_i \in \mathbb{Z}_{[1,\Theta_i - 1]}$, $(i,j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$, (13)–(15) hold, then for any switching signal satisfying $\tau^d_i = \Theta_i h$ and (16), the switched observer

$$\hat{x}(t) = T^{-1}_{\sigma(t)} \begin{bmatrix} \hat{\tilde{x}}_1(t) \\ \tilde{y}_2(t) \end{bmatrix} \quad (18)$$

can asymptotically estimate states of the system (1) and (2), where $\hat{\tilde{x}}_1(t)$ satisfies (8) and (10) with the observer gain given in (17).

Proof. See Appendix F.

Simulation results are provided in Appendix G.

Conclusion. This study focus on the state estimation for switched linear systems with unknown inputs. An improved MDLF is constructed to analyze the continuous-time switched systems, which is helpful to further reduce conservatism compared with the traditional multiple Lyapunov-like function. The stability criterion is constructed for the switched error system with AEDADT switching. Based on the coordinate transformation technique, observers for unknown input switched systems are designed to guarantee the asymptotic state reconstruction.

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Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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