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• **Supplementary File** •

Time-scheduled observer design for switched linear systems with unknown inputs

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Appendix A Notations

$\mathbb{R}^n$ represents the n-dimensional Euclidean space while $\mathbb{Z}_{\geqslant 0}$ stands for the set of nonnegative integers, and $\mathbb{Z}_{[k_1,k_2]}$ denotes the set $\{k \in \mathbb{Z}_{\geqslant 0} | k_1 \leqslant k \leqslant k_2\}$. $\lfloor q \rfloor$ denotes the nearest integer no more than q. The superscripts "-1", "$\top$" and "$\perp$" are used to denote matrix inverse, transposition and orthogonal complement, respectively. Meanwhile, $X^+ = (X^\top X)^{-1} X^\top$ stands for the left pseudoscience of X. $X \geqslant 0 (> 0)$ indicates that X is real symmetric and semi-positive definite (positive definite).

Appendix B The definition of AEDADT switching

Definition 1. [1, 2] For a switching signal σ and any $0 \leqslant t_0 \leqslant t$, let $N_{i,j}(t_0, t)$ denote the switching times that the system switches from the jth mode to the ith mode within the interval $[t_0, t)$, and $T_{i,j}(t_0, t)$ denote the total active time of the ith mode within $[t_0, t)$ whenever the jth mode switches to the ith mode, $(i, j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$. If there exist positive constants $N^0_{i,j}$ and $\tau^a_{i,j}$ such that

$$N_{i,j}(t_0, t) \leqslant N^0_{i,j} + \frac{T_{i,j}(t_0, t)}{\tau^a_{i,j}},$$

then $N^0_{i,j}$ and $\tau^a_{i,j}$ are called the admissible edge-dependent chatter bound and AEDADT, respectively.

Appendix C The system transformation

Consider the switched linear system with unknown inputs (1)-(2). As mentioned in [3], it is assumed that for $\forall i \in \mathcal{M}$, the invariant zeros of the matrix triplets (A_i, C_i, E_i) have negative real part, and $\mathrm{rank}(C_i E_i) = \mathrm{rank} E_i = n_w$. Introduce the following nonsingular matrices T_i and U_i, $i \in \mathcal{M}$,

$$T_i = \begin{bmatrix} E_i^\perp \\ (C_i E_i)^+ C_i \end{bmatrix}, U_i = \begin{bmatrix} (C_i E_i)^\perp \\ (C_i E_i)^+ \end{bmatrix}.$$

It can be obtained that

$$T_i^{-1} = \begin{bmatrix} (I - E_i (C_i E_i)^+ C_i)(E_i^\perp)^+ & E_i \end{bmatrix} \triangleq \begin{bmatrix} \bar{T}_i & E_i \end{bmatrix}.$$

Adopt the state and output transformation

$$\begin{aligned} \tilde{x}(t) =& T_{\sigma(t)} x(t) = \begin{bmatrix} E^\perp_{\sigma(t)} x(t) \\ (C_{\sigma(t)} E_{\sigma(t)})^+ C_{\sigma(t)} x(t) \end{bmatrix} \triangleq \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix}, \\ \tilde{y}(t) =& U_{\sigma(t)} y(t) = \begin{bmatrix} (C_{\sigma(t)} E_{\sigma(t)})^\perp y(t) \\ (C_{\sigma(t)} E_{\sigma(t)})^+ y(t) \end{bmatrix} \triangleq \begin{bmatrix} \tilde{y}_1(t) \\ \tilde{y}_2(t) \end{bmatrix}, \end{aligned}$$

where $\tilde{x}_1(t) \in \mathbb{R}^{n_x - n_w}$, $\tilde{x}_2(t) \in \mathbb{R}^{n_w}$, $\tilde{y}_1(t) \in \mathbb{R}^{n_y - n_w}$, and $\tilde{y}_2(t) \in \mathbb{R}^{n_w}$.

Then the switched system (1)-(2) transforms into the new coordinates, i.e., the system (3)-(6), where A_{1i}, A_{2i}, A_{3i}, A_{4i}, B_{1i}, B_{2i} and $\tilde{C}_i$, $i \in \mathcal{M}$ satisfy

$$\begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix} = T_i A_i T_i^{-1}, \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix} = T_i B_i, \tilde{C}_i = (C_i E_i)^\perp C_i (E_i^\perp)^+.$$

As indicated in [3], the strong detectability of the matrix triplets (A_i, C_i, E_i) is turned out to be equivalent to the simple detectability of the matrix pairs $(A_{1i}, \tilde{C}_i)$.

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Appendix D Stability analysis for the switched error system

In this subsection, we construct an improved MDLF for further proceedings.

For the switched error system (11)-(12), construct the following MDLF:

$$V_{\sigma(t)}(\tilde{e}_1(t),t)=V_{\sigma(t)}(\tilde{e}_1(t),q(t))=\tilde{e}_1^\top(t)P_{\sigma(t)}(q(t))\tilde{e}_1(t), \tag{D1}$$

where $q(t)$ is the time scheduler defined in (9), $P_i(\varphi_i)>0$ for $\varphi_i\in\mathbb{Z}_{[0,\Theta_i-1]}$, $\sigma(t)=i\in\mathcal{M}$.

It can be seen the MDLF (D1) exists some discontinuity points.

Firstly, the MDLF (D1) is discontinuous at the switching instant t_s. This relationship is described as follow:

$$V_{\sigma(t_s)}(\tilde{e}_1(t_s),t_s)\leqslant\mu_{\sigma(t_s),\sigma(t_s^-)}V_{\sigma(t_s^-)}(\tilde{e}_1(t_s^-),t_s^-), \tag{D2}$$

where $\mu_{i,j}>1$ for $(\sigma(t_s)=i,\sigma(t_s^-)=j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$.

In addition, it is noted that $q(t)$ updates at $\bar{t}_s^1$, $\ldots$, $\bar{t}_s^{\Theta_{\sigma(t_s)}-1}$. Accordingly, the Lyapunov matrix $P_{\hat{\sigma}(t)}(q(t))$ is changed at these instants. Consequently, the MDLF (D1) is also discontinuous at these instants:

$$V_{\sigma(\bar{t}_s^{l_{\sigma(t_s)}})}(\tilde{e}_1(\bar{t}_s^{l_{\sigma(t_s)}}),\bar{t}_s^{l_{\sigma(t_s)}})\leqslant\rho_{\sigma(t_s)}V_{\sigma((\bar{t}_s^{l_{\sigma(t_s)}})^-)}(\tilde{e}_1((\bar{t}_s^{l_{\sigma(t_s)}})^-),(\bar{t}_s^{l_{\sigma(t_s)}})^-), \tag{D3}$$

where $0<\rho_i<1$ and $l_i\in\mathbb{Z}_{[1,\Theta_i-1]}$ for $i\in\mathcal{M}$.

For $t\in[t_s,\bar{t}_s^1)\cup[\bar{t}_s^1,\bar{t}_s^2)\cup\ldots\cup[\bar{t}_s^{\Theta_{\sigma(t_s)}-1},t_{s+1})$, the MDLF (D1) is decreasing within these intervals:

$$\dot{V}_{\sigma(t)}(\tilde{e}_1(t),t)<-\lambda_{\sigma(t)}V_{\sigma(t)}(\tilde{e}_1(t),t) \tag{D4}$$

where $\lambda_i>0$ for $\sigma(t)=i\in\mathcal{M}$.

Lemma 1. Let $\lambda_i>0$, $0<\rho_i<1$, $\mu_{i,j}>1$ with $\mu_{i,j}\rho_i^{\Theta_i-1}>1$, $h>0$ be given constants, and Θ_i be a positive integer, $\forall(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$. The MDLF (D1) for the switched error system (11)-(12) converges to 0 as $t\to\infty$, if (D2)-(D4) hold, and the switching signal of the system (11)-(12) satisfies $\tau_i^d=\Theta_i h$, and

$$\tau_{i,j}^a>\tau_{i,j}^{a*}=\frac{\ln\mu_{i,j}\rho_i^{\Theta_i-1}}{\lambda_i}. \tag{D5}$$

Proof. According to (D2)-(D4), one can further deduce that

$$\begin{aligned}
V_{\sigma(t_s)}(\tilde{e}_1(t_s),t_s)\leqslant&\mu_{\sigma(t_s),\sigma(t_s^-)}\rho_{\sigma(t_{s-1})}^{\Theta_{\sigma(t_{s-1})}-1}\exp\{-\lambda_{\sigma(t_{s-1})}(t_s-t_{s-1})\}V_{\sigma(t_{s-1})}(\tilde{e}_1(t_{s-1}),t_{s-1})\\
\leqslant&\prod_{i\in\mathcal{M}}\prod_{\substack{j\in\mathcal{M}\\j\neq i}}\left(\mu_{i,j}\rho_i^{\Theta_i-1}\right)^{N_{i,j}(0,t_s)}\exp\Big\{\sum_{i\in\mathcal{M}}\sum_{\substack{j\in\mathcal{M}\\j\neq i}}(-\lambda_iT_{i,j}(0,t_s))\Big\}V_{\sigma(t_0)}(\tilde{e}_1(0),0)\\
=&\exp\Big\{\sum_{i\in\mathcal{M}}\sum_{\substack{j\in\mathcal{M}\\j\neq i}}\Big(N_{i,j}(0,t_s)\ln\left(\mu_{i,j}\rho_i^{\Theta_i-1}\right)-\lambda_iT_{i,j}(0,t_s)\Big)\Big\}V_{\sigma(t_0)}(\tilde{e}_1(0),0)\\
\leqslant&\exp\Big\{\sum_{i\in\mathcal{M}}\sum_{\substack{j\in\mathcal{M}\\j\neq i}}N_{i,j}^0\ln\left(\mu_{i,j}\rho_i^{\Theta_i-1}\right)\Big\}\\
&\times\exp\Big\{\sum_{i\in\mathcal{M}}\sum_{\substack{j\in\mathcal{M}\\j\neq i}}\Big(-\lambda_i^s+\frac{\ln\left(\mu_{i,j}\rho_i^{\Theta_i-1}\right)}{\tau_{i,j}^a}\Big)T_{i,j}(0,t_s)\Big\}V_{\hat{\sigma}(t_0)}(\tilde{e}_1(0),0).
\end{aligned}$$

When considering $t_s\to\infty$, $V_{\sigma(t_s)}(\tilde{e}_1(t_s),t_s)$ converges to 0 if the switching signal satisfies (D5). Consider $t\in[t_s,t_{s+1})$, it is noticed that $t\to\infty$. Combining with (D2)-(D4), one can further conclude that $V_{\sigma(t)}(\tilde{e}_1(t),t)$ converges to 0, which ends the proof.

In the following, we use this improved MDLF to analyze the stability of switched error system.

Lemma 2. Consider the switched error system (11)-(12). Let $\lambda_i>0$, $0<\rho_i<1$, $\mu_{i,j}>1$ with $\mu_{i,j}\rho_i^{\Theta_i-1}>1$, $h>0$ be given constants, and Θ_i be a positive integer, $\forall(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$. Suppose that there exist matrices $P_i(\varphi_i)>0$ for $\varphi_i\in\mathbb{Z}_{[0,\Theta_i-1]}$, $i\in\mathcal{M}$ such that for $\forall\varphi_i\in\mathbb{Z}_{[0,\Theta_i-1]}$, $l_i\in\mathbb{Z}_{[1,\Theta_i-1]}$, $(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$,

$$\mathcal{A}_i^\top P_i(\varphi_i)+P_i\mathcal{A}_i(\varphi_i)+\lambda_iP_i(\varphi_i)<0, \tag{D6}$$

$$(E_i^\perp\bar{T}_j)^\top P_i(0)E_i^\perp\bar{T}_j-\mu_{i,j}P_j(\Theta_j-1)\leqslant0, \tag{D7}$$

$$P_i(l_i)-\rho_iP_i(l_i-1)\leqslant0, \tag{D8}$$

then the system (11)-(12) is asymptotically stable for any switching signal with AEDADT satisfying (D5).

Proof. Select (D1) as the MDLF.

Assume that $\sigma(t_s^-)=j$, $\sigma(t_s)=i$, $(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$, i.e., at the switching instant t_s, the switched system switches from the jth mode to the ith mode. It is noticed that $t_{s+1}-t_s\geqslant\Theta_ih$ always holds. Therefore, $V_{\sigma(t_s)}(\tilde{e}_1(t_s),t_s)=\tilde{e}_1^\top(t_s)P_i(0)\tilde{e}_1(t_s)$ and $V_{\sigma(t_s^-)}(\tilde{e}_1(t_s^-),t_s^-)=\tilde{e}_1^\top(t_s^-)P_j(\Theta_j-1)\tilde{e}_1(t_s^-)$. Combining (12) with (D1), it can be obtained that (D7) can guarantee (D2). Meanwhile, (D3) can be ensured by (D8).

On the other hand, for $t\in[t_s,\bar{t}_s^1)$, combining (11) with (D1), it follows that

$$\dot{V}_i(\tilde{e}_1(t))+\lambda_iV_i(\tilde{e}_1(t))=\tilde{e}_1^\top(t)[\mathcal{A}_i^\top P_i(0)+P_i(0)\mathcal{A}_i+\lambda_iP_i(0)]\tilde{e}_1(t),$$

which indicates that for $t\in[t_s,\bar{t}_s^1)$, if (D6) holds, (D4) holds. By the similar manipulation, one can further deduce that (D4) holds during $[\bar{t}_s^1,\bar{t}_s^2)$, $\ldots$, $[\bar{t}_s^{\Theta_{\sigma(t_s)}-1},t_{s+1})$. Therefore, (D4) holds for $t\in[t_s,\bar{t}_s^1)\cup[\bar{t}_s^1,\bar{t}_s^2)\cup\ldots\cup[\bar{t}_s^{\Theta_{\sigma(t_s)}-1},t_{s+1})$.

According to Lemma 1, one can further deduce the asymptotic stability of the switched error system (11)-(12), which ends the proof.

Remark 1. The constraint conditions are with the form of linear matrix inequality (LMI). The computational complexity of LMI conditions is closely related to the number of decision variables and the size of LMIs. The number of decision variables and the size of LMIs of Lemma 2 are $\sum_{i=1}^{M}\Theta_i(n_x^2+n_w^2-2n_xn_w+n_x-n_w)/2$ and $M^2-M+\sum_{i=1}^{M}(3\Theta_i-1)(n_x-n_w)$, respectively.

Appendix E Discussions on Theorem 1

Proof. Consider $M_i(\varphi_i)=P_i(\varphi_i)L_i(\varphi_i)$ for $\varphi_i\in\mathbb{Z}_{[0,\Theta_i-1]}$, $i\in\mathcal{M}$. Then, using (11), one can obtain that (D6) can be ensured by (13). According to Lemma 2, one can conclude that the switched observer can asymptotically estimate the states of the system (3).

Remark 2. In Theorem 1, the AEDADT switching signal $\sigma(t)$ is adopted and the minimal AEDADT bound is given in (16), which is determined by some parameters λ_i, ρ_i and $\mu_{i,j}$, $(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$. Set $\mu_i=\max_{j\in\mathcal{M},j\neq i}\{\mu_{i,j}\}$. When replacing $\mu_{i,j}$ in constraint conditions of Theorem 1 with μ_i, respectively, Theorem 1 degrades into a observer design scheme with MDADT switching. Furthermore, set $\lambda=\min_{i\in\mathcal{M}}\{\lambda_i\}$, and $\mu=\max_{i\in\mathcal{M}}\{\mu_i\}$. By replacing λ_i and $\mu_{i,j}$ in constraint conditions of Theorem 1 with λ and μ_i, respectively, a observer design scheme is obtained with ADT switching. It is intuitive that the admissible minimal ADT bound under the ADT or MDADT scheme tends to be larger than the one obtained by Theorem 1. Therefore, compared with adopting ADT and MDADT switching, the developed state estimation method with AEDADT can further reduce conservatism.

When excluding the time scheduler $q(t)$, i.e., using $L_{\sigma(t)}$ to replace the observer gain $L_{\sigma(t)}(q(t))$ in (8), the QTD observer (8) is degrade into the time-independent one. The following corollary is presented.

Corollary 1. Let $\lambda_i>0$ and $\mu_{i,j}>1$ be given constants, $\forall(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$. Suppose that there exist matrices $P_i>0$ and M_i for $i\in\mathcal{M}$ such that for $\forall(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$,

$$A_{1i}^{\top}P_i+P_iA_{1i}-\tilde{C}_i^{\top}M_i^{\top}-M_i\tilde{C}_i+\lambda_i^sP_i<0, \tag{E1}$$
$$(E_i^{\perp}\bar{T}_j)^{\top}P_jE_i^{\perp}\bar{T}_j-\mu_{i,j}P_j\leqslant 0, \tag{E2}$$

then the observer (8) can asymptotically estimate the states of the system (3) for any switching signal with AEDADT satisfying

$$\tau_{i,j}^{a}>\tau_{i,j}^{a*}=\frac{\ln\mu_{i,j}}{\lambda_i}. \tag{E3}$$

Moreover, the observer gain is given by $L_i=P_i^{-1}M_i$ for $\forall i\in\mathcal{M}$.

Proof. Construct the following Lyapunov-like function:

$$V_{\sigma(t)}(\tilde{e}_1(t))=\tilde{e}_1^{\top}(t)P_{\sigma(t)}\tilde{e}_1(t), \tag{E4}$$

where $P_i>0$ for $\sigma(t)=i\in\mathcal{M}$.

We omit the subsequent proof since it follows a similar vein to the proof of Theorem 1.

Remark 3. When fixing the parameters λ_i and $\mu_{i,j}$ in Theorem 1 and Corollary 1, a smaller AEDADT bound $\tau_{i,j}^{a*}$ can be obtained by using Theorem 1, which can be obviously proved by comparing (D5) and (E3). This improvement is beneficial from the developed MDLF. Moreover, the increasing of Θ_i can further reduce the AEDADT bound.

Appendix F Discussions on Theorem 2

Proof. Combining (7) with (18) yields that

$$e(t)=T_{\sigma(t)}^{-1}\begin{bmatrix}\tilde{e}_1(t)\\0\end{bmatrix},$$

where $\tilde{e}_1(t)$ is the estimation error of $\tilde{x}_1(t)$. According to Theorem 1, $\tilde{e}_1(t)$ is asymptotically stable. Therefore, $e(t)$ is asymptotically stable, which ends the proof.

Remark 4. The number of decision variables and the size of LMIs of Theorem 2 are $\sum_{i=1}^{M}\Theta_i(n_x^2+2n_w^2-3n_xn_w+n_xn_y-n_yn_w+n_x-n_w)/2$ and $M^2-M+\sum_{i=1}^{M}(3\Theta_i-1)(n_x-n_w)$, respectively.

The designed QTD observer in Theorem 2 can degrade into the traditional time-independent one.

Corollary 2. Consider the unknown input switched system (1)-(2). Let $\lambda_i>0$ and $\mu_{i,j}>1$ be given constants, $\forall(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$. Suppose that there exist matrices $P_i>0$, M_i for $i\in\mathcal{M}$ such that for $(i,j)\in\mathcal{M}\times\mathcal{M}$, $i\neq j$, (E1) and (E2) hold, then for any switching signal with AEDADT properties satisfying (E3), the switched observer (18) can asymptotically estimate states of the system (1)-(2), where $\hat{\tilde{x}}_1(t)$ satisfies (8) and (10) with observer gains given by $L_i=P_i^{-1}M_i$ for $\forall i\in\mathcal{M}$.

Proof. Following the similar vein to the proof of Theorem 2, one can deduce it.

Remark 5. There exist some observer design results for continuous-time switched linear systems with unknown inputs [3–5]. The main developments of the proposed observer design scheme are twofold. Firstly, the AEDADT switching is adopted, which is more general than the DT or ADT switching using in aforementioned results. Secondly, the QTD observer is designed, which is helpful to achieve a lower AEDADT bound.

Appendix G An illustrative example

An example is provided to demonstrate the effectiveness of the developed state estimation scheme.

Consider the switched system (1)-(2) consisting of three subsystems with

$$A_1=\begin{bmatrix}-1&0&1&0&1\\0&-1&0&1&0\\1&-1&-2&-1&-1\\1&1&0&-1&0\\0&0&0&-1&-1\end{bmatrix},A_2=\begin{bmatrix}-2&0&0&1&1\\1&-2&0&1&1\\1&1&-2&1&1\\1&1&1&-2&1\\1&-1&0&-1&-1\end{bmatrix},A_3=\begin{bmatrix}-1&1&0&1&1\\0&-1&0&0&-1\\-1&-2&-1&0&-1\\0&-1&1&-1&1\\0&-1&0&-1&-1\end{bmatrix},$$

Table G1 The AEDADT bounds obtained by Corollary 2 and Theorem 2 with different $\Theta_i = \Theta$, $i \in \{1,2,3\}$

	Corollary 2	Theorem 2		
		$\Theta = 2$	$\Theta = 3$	$\Theta = 4$
$\tau_{1,2}^{a*}$	0.2915	0.2691	0.2466	0.2242
$\tau_{1,3}^{a*}$	0.2026	0.1801	0.1577	0.1352
$\tau_{2,1}^{a*}$	0.3739	0.3514	0.3290	0.3065
$\tau_{2,3}^{a*}$	0.2026	0.1801	0.1577	0.1352
$\tau_{3,1}^{a*}$	0.2915	0.2691	0.2466	0.2242
$\tau_{3,2}^{a*}$	0.4505	0.4281	0.4056	0.3832

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, B_1 = B_2 = B_3 = 0,$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}, C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}.$$

Assume that the unknown input satisfies

$$w(t) = \begin{bmatrix} 2\sin(0.75t) + 3 \\ -\sin(0.85t) - 5 \end{bmatrix},$$

Select $\alpha_i = 0.9$, $\rho_i = 0.98$, $i \in \{1,2,3\}$, $\mu_{1,2} = 1.3$, $\mu_{1,3} = 1.2$, $\mu_{2,1} = 1.4$, $\mu_{2,3} = 1.2$, $\mu_{3,1} = 1.3$, $\mu_{3,2} = 1.5$, and $h = 0.05$s.

Using (E3) in Corollary 2 and (16) in Theorem 2, the bounds of AEDADT are displayed in Table G1. It can be seen that the developed QTD technique in Theorem 2 is beneficial to achieve a lower bounds of AEDADT. As mentioned in Remark 2, when considering the MDADT scheme, accordingly, $\mu_1 = 1.3$, $\mu_2 = 1.4$ and $\mu_3 = 1.5$. Setting $\Theta_i = 2$ for $i \in \{1,2,3\}$, the corresponding bounds are $\tau_1^{a*} = 0.2691$, $\tau_2^{a*} = 0.3514$ and $\tau_3^{a*} = 0.4281$. The AEDADT scheme is more general since the switching direction is taken into consideration compared with MDADT.

Setting $\Theta_i = 2$ for $i \in \{1,2,3\}$, the observer gains can be deduced according to Theorem 1:

$$L_1(0) = \begin{bmatrix} 0.4899 \\ 0.3306 \\ 0.5811 \end{bmatrix}, L_1(1) = \begin{bmatrix} 0.2283 \\ 0.2403 \\ 0.4392 \end{bmatrix}, L_2(0) = \begin{bmatrix} 1.0677 \\ 1.3916 \\ 0.5319 \end{bmatrix}, L_2(1) = \begin{bmatrix} 0.9295 \\ 1.3978 \\ 0.5520 \end{bmatrix},$$

$$L_3(0) = \begin{bmatrix} -0.4346 \\ 1.1032 \\ 0.8690 \end{bmatrix}, L_3(1) = \begin{bmatrix} -0.5256 \\ 1.0843 \\ 0.8486 \end{bmatrix}.$$

A randomly generated switching signal is displayed in Figure G1, which satisfies the above constraints. Assume that the initial states of the given switched system are $\begin{bmatrix} 3.0150 & -5.0085 & -1.9935 & 0 & -5.0085 \end{bmatrix}^\top$, and the initial states of the designed observers are $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^\top$. With the obtained observers, the estimation errors $e(t)$ are plotted in Figure G2, which tend to converge to 0. The time evolution of the original states $x(t)$ and their estimations $\hat{x}(t)$ are plotted in Figures G3-G7. It can be intuitively viewed that the designed observer asymptotically estimates the state of the given switched system.

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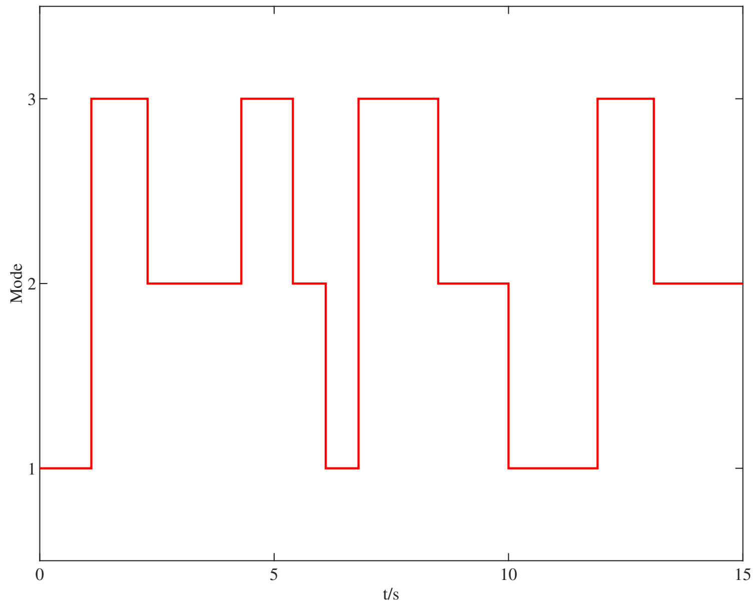

Figure G1 The switching signal.

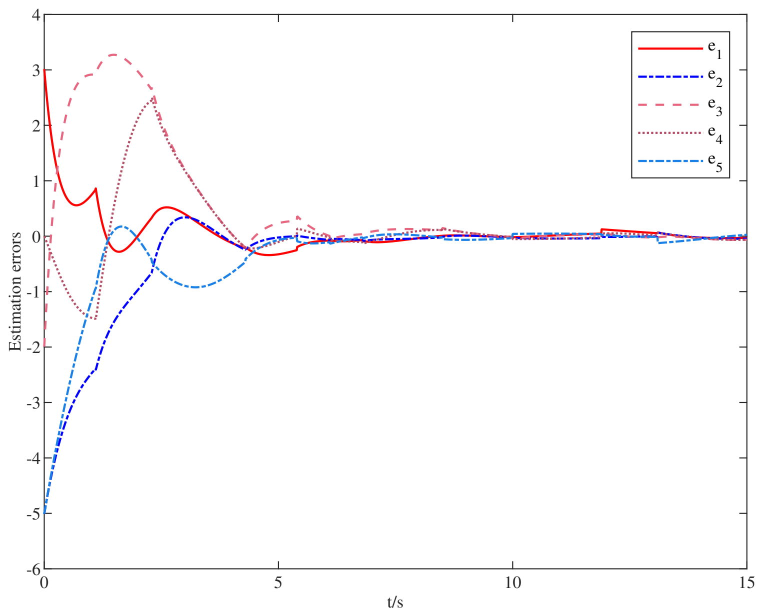

Figure G2 Estimation errors $e(t)$.

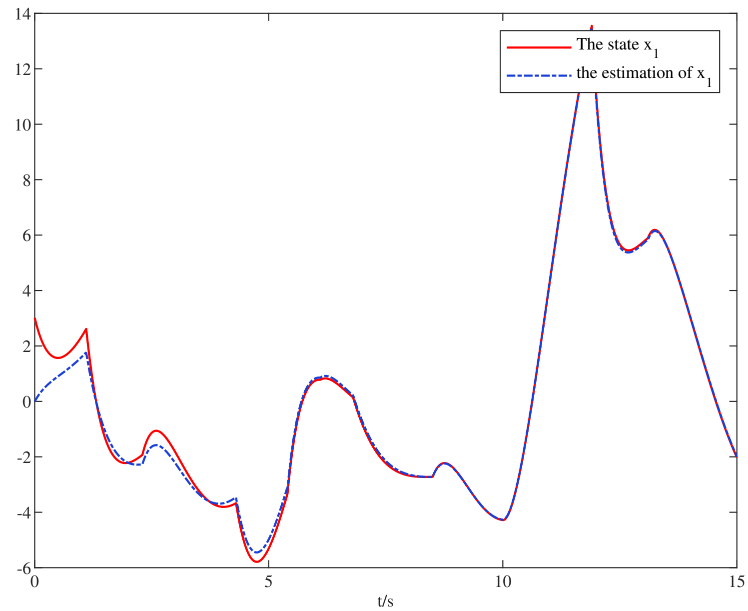

Figure G3 The original state $x_1(t)$ and its estimation $\hat{x}_1(t)$.

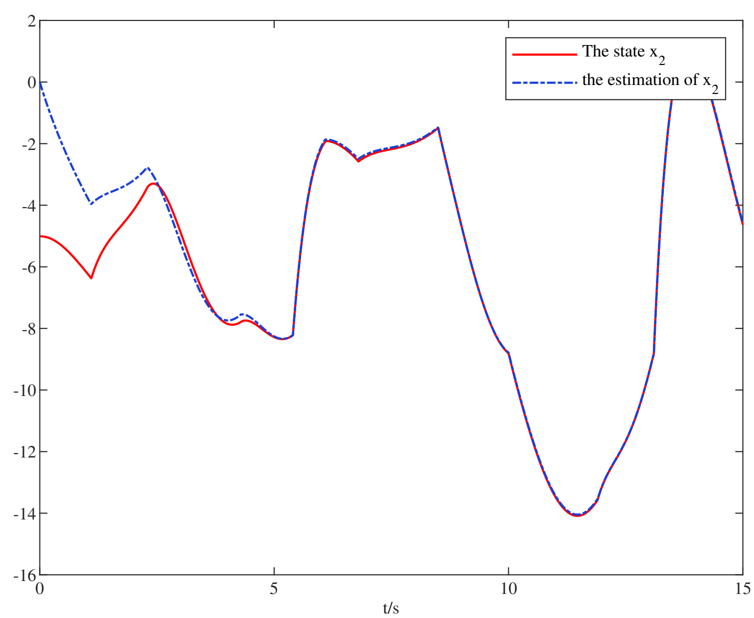

Figure G4 The original state $x_2(t)$ and its estimation $\hat{x}_2(t)$.

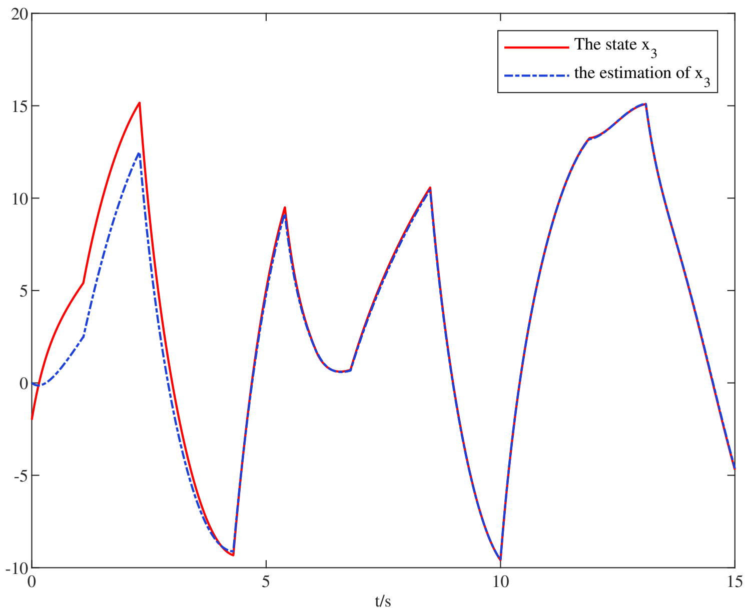

Figure G5 The original state $x_3(t)$ and its estimation $\hat{x}_3(t)$.

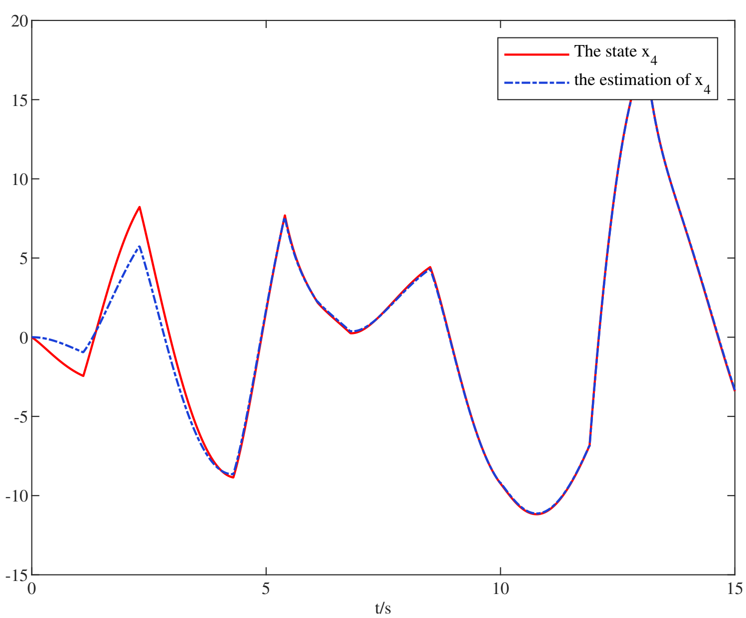

Figure G6 The original state $x_4(t)$ and its estimation $\hat{x}_4(t)$.

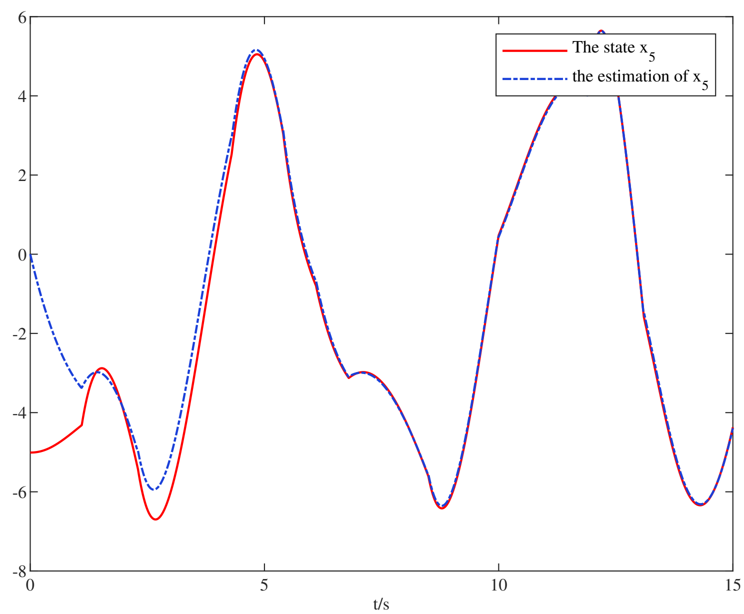

Figure G7 The original state $x_5(t)$ and its estimation $\hat{x}_5(t)$.