

# MPC-based strategy for longitudinal and lateral stabilization of a vehicle under extreme conditions

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Dear editor,

In recent decades, active safety-control systems have been developed to assist drivers. Wheel slip ratio, yaw rate, and side-slip angle are commonly considered as the control indices of longitudinal and lateral vehicle stabilities [1]. However, if systems that only contain vehicle dynamics in longitudinal or lateral directions work simultaneously, the system objectives may conflict. This conflict could cause degradation of the systems' overall performance, particularly under extreme driving conditions [2]. When this occurs, the vehicle and its tires work in a nonlinear region, and the tire forces tend to be saturated, suggesting that the highly coupled characteristics of dynamics cannot be ignored. Therefore, a certain safety system cannot effectively stabilize the vehicle, and control strategies are necessary to integrate multiple system objectives.

Model predictive control (MPC) is a suitable method to control longitudinal and lateral vehicle stabilities in a coordinated manner and has been widely utilized in this field [3]. Furthermore, to accurately express the nonlinearity of a tire, its forces are best described using the combined-slip tire model [4]. Four wheel independent motor drive (4WIMD) electric vehicles have presently become a key research topic with the advantages of adjusting vehicle motion without differential braking or intense steering interference from the driver [5], creating new possibilities for vehicle stability control under extreme conditions.

Herein, we present an MPC-based strategy for longitudinal and lateral vehicle stabilization under extreme driving conditions. A LuGre combined-slip tire model is developed to capture the variations in tire longitudinal and lateral dynamics motions. Then, the variation of longitudinal velocity is considered as a disturbance in the proposed strategy. In the corresponding multiple objective optimization problem, tracking the reference states and satisfying both safety and actuator constraints are accomplished for better handling performance and overall stability. The control scheme is illustrated in Figure 1.

*Related models.* The vehicle lateral and yaw motion dynamics are

$$m(\dot{V}_y + rV_x) = F_{yf} \cos \delta + F_{yr} + (T_{fl} + T_{fr}) \frac{\sin \delta}{R_e}, \quad (1)$$

$$I_z \dot{r} = F_{yf} L_f \cos \delta - F_{yr} L_r + (T_{fl} + T_{fr}) L_f \frac{\sin \delta}{R_e} - (T_{fl} - T_{fr}) \frac{L}{2} \frac{\cos \delta}{R_e} - (T_{rl} - T_{rr}) \frac{L}{2R_e}, \quad (2)$$

where  $V_x$ ,  $V_y$ ,  $r$  are the vehicle's longitudinal velocity, lateral velocity, and yaw rate, respectively;  $F_{yf}$ ,  $F_{yr}$  are the front and rear tire lateral resultant forces;  $\delta$  is the front wheel steering angle;  $\omega$  is the wheel rotational velocity;  $T_{ij}$  ( $i \in \{f, r\}$  represents the front or rear wheel,  $j \in \{l, r\}$  represents the left or right wheel) is the motor torque;  $L_f$ ,  $L_r$  and  $L$  are the distances from the front axle and rear axle to the vehicle center of gravity and wheel body distance, respectively;  $R_e$  is the effective tire radius;  $m$  is the vehicle mass;  $I_z$  is the vehicle yaw moment of inertia.

The longitudinal velocity of each wheel is assumed to be equal to  $V_x$ ; therefore, the relative longitudinal velocity of each wheel is defined as  $v_{rx} = R_e \omega - V_x$ , which reflects the wheel slip ratio. Therefore, according to the rotational dynamics  $I_w \dot{\omega} = T - F_x R_e$ , its variation is formulated as

$$\dot{v}_{rx} = \frac{R_e}{I_w} T - \left( \frac{m R_e^2}{4 I_w} + 1 \right) \dot{V}_x, \quad (3)$$

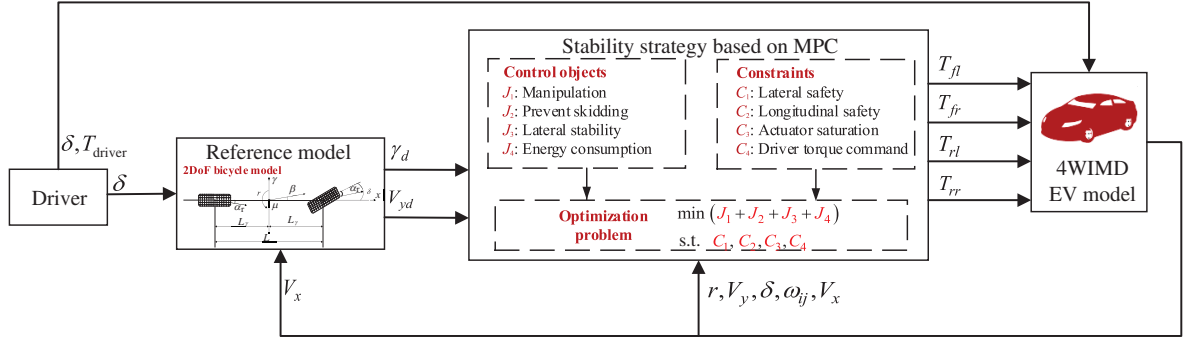
where  $I_w$  is the wheel moment of inertia.

Under extreme conditions, the effects of the slip ratio and slip angle should be considered simultaneously in the tire model for the coupled dynamics [6]. The combined-slip LuGre tire model is used to calculate  $F_y$  as follows:

$$F_y(v_{rx}, \alpha) = \left( \frac{\sigma_{0y} v_{ry}}{\frac{\sigma_{0y} |v_r|}{\mu \cdot g(v_r)} + \kappa_y R_e |\omega|} + \sigma_{2y} v_{ry} \right) F_z, \quad (4)$$

where  $F_z$  is the vertical load,  $\sigma_{0y}$  and  $\sigma_{2y}$  are the tire lateral rubber stiffness and relative viscous damping,  $\kappa_y$  is the

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**Figure 1** (Color online) Coordinated longitudinal and lateral vehicle stability control system.

load distribution factor, and  $|v_r| = \sqrt{v_{rx}^2 + V_x^2 \alpha^2}$ . The Stribeck function  $g(v_r)$  can also be approximated as the form of  $v_{rx}$  and  $\alpha$ , so the time derivative of  $F_y$  is given as  $\dot{F}_y(\dot{v}_{rx}, \dot{\alpha}) = \frac{\partial F_y}{\partial v_{rx}} \dot{v}_{rx} + \frac{\partial F_y}{\partial \alpha} \dot{\alpha}$ . Moreover,  $\alpha$  is commonly calculated with  $r$  and  $V_y$ , so the time derivative of  $\alpha$  can be calculated as  $\dot{\alpha}_{ij}(\dot{r}, \dot{V}_y) = \frac{\partial \alpha_{ij}}{\partial r} \dot{r} + \frac{\partial \alpha_{ij}}{\partial V_y} \dot{V}_y$ . Above all,  $\dot{F}_y$  can be described as the following form:

$$\begin{aligned} \dot{F}_{yf} &= f(\dot{v}_{rxfl}, \dot{v}_{rxfr}, \dot{r}, \dot{V}_y), \\ \dot{F}_{yr} &= f(\dot{v}_{rxrl}, \dot{v}_{rxrr}, \dot{r}, \dot{V}_y). \end{aligned} \quad (5)$$

By combining Eqs. (1)–(3) and (5), the prediction model can be given, in which the system states are chosen as  $x = [r \ V_y \ F_{yf} \ F_{yr} \ v_{rxfl} \ v_{rxfr} \ v_{rxrl} \ v_{rxrr}]^T$ . The control inputs are the torques  $T_{ij}$  on four independent motors, and  $\dot{V}_x$  is considered as disturbance.

*Control objectives.* To track the reference signals  $\gamma_d$  and  $V_{yd}$ , we define the following cost functions:

$$J_1 = \int_{t_k}^{t_k+t_p} \|r(t) - r_d(t)\|_Q^2 dt, \quad (6a)$$

$$J_2 = \int_{t_k}^{t_k+t_p} \|V_y(t) - V_{yd}(t)\|_R^2 dt. \quad (6b)$$

The cost function used for keeping the tires from skidding and locking during cornering is defined as follows:

$$J_3 = \int_{t_k}^{t_k+t_p} \|v_{rxij}(t)\|_W^2 dt. \quad (7)$$

In addition, the fourth term related to energy consumption is defined as

$$J_4 = \int_{t_k}^{t_k+t_p} \|T_{ij}(t)\|_P^2 dt. \quad (8)$$

In the above terms,  $t_k$  is the current time,  $t_p$  is the prediction horizon, and  $Q, R, W, P$  are the weighting factors.

*Constraints.* Moreover, vehicle safety constraints should be considered under extreme conditions. For lateral and longitudinal vehicle safety, the overlarge of the lateral velocity and wheel slip ratio will deteriorate the manipulation, so  $V_y$  and  $v_{rxij}$  are limited as follows:

$$V_y(t) \in [-V_{ylim} \ V_{ylim}], \quad (9a)$$

$$v_{rxij}(t) \in [-v_{rxlim} \ v_{rxlim}]. \quad (9b)$$

Two more constraints of the control inputs are from the actuator saturation and the driver's torque command, which are given as follows:

$$-T_{\max} \leq T_{ij}(t) \leq T_{\max}, \quad (10a)$$

$$T_{\text{driver}} = T_{fl}(t) + T_{fr}(t) + T_{rl}(t) + T_{rr}(t). \quad (10b)$$

According to the above objectives and constraints, the MPC strategy is described by the following optimization problem:

$$\begin{aligned} \min \quad & J_1 + J_2 + J_3 + J_4 \\ \text{s.t.} \quad & (9a), (9b), (10a), (10b). \end{aligned} \quad (11)$$

By solving Eq. (11), the optimal torques  $T_{ij}^*$  are obtained and are applied to the independent motors.

*Conclusion.* This study proposed a longitudinal and lateral dynamic control strategy for 4WIMD electric vehicles to enhance their handling and stability under extreme conditions. A developed LuGre combined-slip tire model was utilized to exactly represent the tire lateral force under extreme conditions. The control requirements include maintaining longitudinal and lateral vehicle stability and satisfying the driver's torque command. By solving the proposed optimization problem, the proper motor torques are obtained to achieve the vehicle's overall stability.

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