

Fault estimation based on high order iterative learning scheme for systems subject to nonlinear uncertainties

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Dear editor,

Due to the high load and continuous work, some equipment such as sensors and actuators in the system will inevitably degrade as the aging time increases. Also, they may evolve into faults causing the decline of system control performance. However, the stability of the system cannot be guaranteed due to the large loss of property and casualties caused by faults. Therefore, fault diagnosis and fault-tolerant control [1, 2] are widely applied in the industrial system to solve these problems.

Fault estimation [3] is the foundation work of fault-tolerant control, which can reconstruct the detailed information such as magnitude and period. Recently, fault estimation based on iterative learning schemes [4, 5] has attracted much attention since the previous information in repetitive systems can enhance the tracking precision. Several studies [6] showed that high order iterative learning schemes are more suitable for the nonlinear systems subject to the uncertainties varying both in the time direction and iterative direction.

This study investigates the fault estimation problem for a class of nonlinear systems subject to nonlinear uncertainties. In this study, a high order iterative learning scheme is used to design the fault estimation law by introducing the mean operator. In particular, the norm theory is employed to ensure sufficient convergence conditions and obtain the gain matrices' solutions for leveraging updating model continuity.

Model and methodology. Consider the continuous-time system with nonlinear uncertainties in the following form:

$$\begin{aligned} \dot{\mathbf{x}}_k(t) &= g(\mathbf{x}_k(t), t) + \mathbf{B}(\mathbf{x}_k(t), t)u_k(t) + \mathbf{E}(\mathbf{x}_k(t), t)f_k(t), \\ \mathbf{y}_k(t) &= s(\mathbf{x}_k(t), t) + \mathbf{D}(\mathbf{x}_k(t), t)f_k(t), \end{aligned} \quad (1)$$

where $t \in [0, T]$ is the time index with period T , $u_k(t) \in \mathbb{R}^r$ is the input, $f_k(t) \in \mathbb{R}^r$ represents the fault signal, $\mathbf{x}_k(t) \in \mathbb{R}^n$ denotes the state vector, and $\mathbf{y}_k(t) \in \mathbb{R}^m$ represents

the output vector. The nonlinear parameter uncertainties $\mathbf{B}_k \in \mathbb{R}^{n \times r}$, $\mathbf{D}_k \in \mathbb{R}^{m \times r}$, $\mathbf{E}_k \in \mathbb{R}^{n \times r}$ are with appropriate dimensions.

Remark 1. For consistent description, the independent variables for objective function are omitted, such as $\mathbf{x}_d(t) \rightarrow \mathbf{x}_d$, $\mathbf{x}_k(t) \rightarrow \mathbf{x}_k$, $g(\mathbf{x}_k(t), t) \rightarrow g_k$, $\mathbf{B}(\mathbf{x}_k(t), t) \rightarrow \mathbf{B}_k$, $\mathbf{E}(\mathbf{x}_k(t), t) \rightarrow \mathbf{E}_k$, $s(\mathbf{x}_k(t), t) \rightarrow s_k$, $\mathbf{D}(\mathbf{x}_k(t), t) \rightarrow \mathbf{D}_k$.

Assumption 1. Assume that the function \mathbf{B}_K , \mathbf{D}_K , \mathbf{E}_K , g_K and s_K are global uniform Lipschitz functions subject to $\mathbf{x}_k(t)$, such that there exists a function $\alpha \in \{B, D, E, s, g\}$ that satisfies

$$\|\alpha(x_1, t) - \alpha(x_2, t)\| \leq k_\alpha \|x_1 - x_2\|, \quad (2)$$

where k_α is the nonlinear parameter.

It should be known that the desired output $\mathbf{y}_d(t)$ and the initial state value $\mathbf{x}_d(0)$ are obtainable if and only if there exists a desired fault signal $f_d(t)$ such that

$$\begin{aligned} \dot{\mathbf{x}}_d(t) &= g(\mathbf{x}_d(t), t) + \mathbf{B}(\mathbf{x}_d(t), t)u_d(t) + \mathbf{E}(\mathbf{x}_d(t), t)f_d(t), \\ \mathbf{y}_d(t) &= s(\mathbf{x}_d(t), t) + \mathbf{D}(\mathbf{x}_d(t), t)f_d(t), \end{aligned} \quad (3)$$

where the desired input is $u_d(t) = u_k(t)$ and the desired state is $\mathbf{x}_d(t)$.

The satisfactory tracking performance of this paper is

$$\lim_{k \rightarrow \infty} f_{k+1}(t) = f_d(t), \quad (4)$$

where $f_{k+1}(t)$ is the estimating fault signal.

To solve the problem of nonlinear uncertainties, a high order learning scheme is proposed to reconstruct the fault signal in this study.

$$f_{k+1}(t) = \sum_{i=1}^N \mathbf{P}_i f_{k-i+1}(t) + \sum_{i=1}^N \mathbf{Q}_i \Delta y_{k-i+1}(t), \quad (5)$$

where N represents the iterative learning order, \mathbf{P}_i and \mathbf{Q}_i are the gain matrices of the i -th iteration, respectively.

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Remark 2. Different from the conventional iterative learning scheme-based fault estimating method, this study introduces the mean operator and presents a high order learning scheme. It should be pointed out that the high order learning law employs more potential information in previous trials, thereby improving the tracking speed.

The following are our main results.

Theorem 1. Consider the system described by (1) and assume that the high order iterative learning scheme (5) is applied. Then, to achieve the tracking objective, the leaning gain matrices should satisfy the following conditions:

$$\sum_{i=1}^N \mathbf{P}_i = I, \tag{6}$$

and there exists parameter ρ_i ($i = 1, 2, 3, \dots, N$) that satisfies

$$\|\mathbf{P}_i - \mathbf{Q}_i \mathbf{D}_{k-i+1}\|_\lambda \leq \rho_i, \tag{7}$$

where \mathbf{D}_{k-i+1} is the abbreviation of $\mathbf{D}(x_{k-1+i}(t), t)$, and λ is set by the experts' experience. Simultaneously, the zero point of polynomial (8) is inside the unit circle.

$$R(z) = z^n + \rho_1 z^{n-1} + \rho_2 z^{n-2} + \rho_3 z^{n-3} + \dots + \rho_{n-1} z + \rho_n. \tag{8}$$

Then, the fault estimating errors $\lim_{k \rightarrow \infty} \|\Delta f_{k+1}(t)\|_\lambda = 0$ and $\lim_{k \rightarrow \infty} \|\Delta \mathbf{y}_{k+1}(t)\|_\lambda = 0$ hold.

Based on the difference inequality, it can be further obtained that the inequality

$$\sum_{i=1}^N \rho_i = \rho_1 + \rho_2 + \rho_3 + \dots + \rho_n < 1 \tag{9}$$

holds if $\rho_i \geq 0, i = 1, 2, \dots, N$.

Remark 3. Although the designed high order learning scheme can improve the fault estimating accuracy and tracking speed, only the spans of gain parameter matrices can be obtained. In future work, we will propose computational methods for obtaining the precision values of the gain parameter matrices.

Proof. The proof is shown in Appendix A.

Simulation results. To demonstrate the effectiveness of the proposed method, three numerical simulations are conducted on a nonlinear system. Two numerical examples contain different kinds of fault signals, such as abrupt fault signal and sinusoidal fault signal. Simultaneously, one comparative example is expanded to verify the superiority of the proposed method. The nonlinear system is considered to have nonrepetitive uncertainties, that is, the uncertainties in the k -th trial are different from the uncertainties in the

$(k + 1)$ -th trial. The high order iterative learning estimator can reasonably compensate for the effect of uncertainties on the tracking results without decreasing the estimating accuracy rate.

More details of simulation results can be found in Appendix B.

Conclusion. We proposed a high order iterative learning scheme based on the fault estimation method for nonlinear systems subject to varying parameter uncertainties. We discussed both the fault estimation problem and the uncertain varying parameters. Also, we illustrated the effectiveness of the proposed method by theoretical analysis and simulation results simultaneously. With further investigation, the high order iterative learning scheme is inherited by the tracking error in the previous iterations. Hence, the tracking performs better as the number of iterations increases.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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