SCIENCE CHINA
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• **Supplementary File** •

Fault Estimation based on High Order Iterative Learning Scheme for Systems subject to Nonlinear Uncertainties

Feng Li[1], Xu Shuiqing[2*], Zhang Ke[3], Chai Yi[3*] & Huang Darong[1]

[1]*College of Traffic and Transportation, Chongqing Jiaotong University, Chongqing, 400074, China;*
[2]*College of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China;*
[3]*College of Automation, Chongqing University, Chongqing 400044, China*

Appendix A Proof of Theorem 1

Define that $\|\Delta \boldsymbol{x}_k\| = \boldsymbol{x}_d(t) - \boldsymbol{x}_k(t)$, $\boldsymbol{x}_d(0) = \boldsymbol{x}_k(0)$ and $u_d(t) = u_k(t)$, then substituting the reference trajectories to the tracking error, it can be deduced that

$$\begin{aligned}\|\Delta \boldsymbol{x}_k\| &= \left\| \int_0^t \left[g_d + \boldsymbol{B}_d u_d + \boldsymbol{E}_d f_d - g_k - \boldsymbol{B}_k u_k - \boldsymbol{E}_k f_k\right] d\tau \right\| \\ &\leqslant \int_0^t \|g_d - g_k\| + \|\boldsymbol{B}_d - \boldsymbol{B}_k\| \|u_d\| \\ &+ \|\boldsymbol{E}_d - \boldsymbol{E}_k\| \|f_d\| + \|\boldsymbol{E}_k\| \|\Delta f_k\| d\tau \\ &\leqslant \int_0^t (k_g + k_b b_{ud} + k_e b_{fd}) \|\Delta \boldsymbol{x}_k\| + b_e \|\Delta f_k\| d\tau\end{aligned} \tag{A1}$$

In which, k_g, k_b and k_e are the nonlinear parameters, b_{ud}, b_{fd} and b_e are the bounds of $u_d(t)$, $f_d(t)$ and $\boldsymbol{E}_k(\boldsymbol{x}_k(t), t)$, respectively.

Based on the Bellman-Gronwall theorem, it can be concluded that

$$\|\Delta \boldsymbol{x}_k\| \leqslant \int_0^t e^{\left(k_g + k_b b_{ud} + k_e b_{fd}\right)(t-\tau)} b_e \|\Delta f_k\| d\tau \tag{A2}$$

The output error can be written as

$$\begin{aligned}\Delta \boldsymbol{y}_k &= k_s \Delta \boldsymbol{x}_k + \boldsymbol{D}_k \Delta f_k + b_{fd} k_d \Delta \boldsymbol{x}_k \\ &= (k_s + b_{fd} k_d) \Delta \boldsymbol{x}_k + \boldsymbol{D}_k \Delta f_k\end{aligned} \tag{A3}$$

From the high order iterative learning law (5), one has

$$\begin{aligned}\Delta f_{k+1} &= f_d - f_{k+1} \\ &= f_d - \sum_{i=1}^{N} \boldsymbol{P}_i f_{k-i+1} - \sum_{i=1}^{N} \boldsymbol{Q}_i \Delta \boldsymbol{y}_{k-i+1}\end{aligned} \tag{A4}$$

Therefore, the equation (6) is hold and followed by that

$$\begin{aligned}\Delta f_{k+1} &= \sum_{i=1}^{N} \boldsymbol{P}_i \Delta f_{k-i+1} - \sum_{i=1}^{N} \boldsymbol{Q}_i \Delta \boldsymbol{y}_{k-i+1} \\ &= \sum_{i=1}^{N} \boldsymbol{P}_i \Delta f_{k-i+1} - (k_s + b_{fd} k_d) \\ &\times \sum_{i=1}^{N} \boldsymbol{Q}_i (\Delta \boldsymbol{x}_{k-i+1} + \boldsymbol{D}_{k-i+1} \Delta f_{k-i+1}) \\ &= \sum_{i=1}^{N} (\boldsymbol{P}_i - \boldsymbol{Q}_i \boldsymbol{D}_{k-i+1}) \Delta f_{k-i+1} \\ &- \sum_{i=1}^{N} (k_s + b_{fd} k_d) \Delta \boldsymbol{x}_{k-i+1}\end{aligned} \tag{A5}$$

Taking norm of the both sides, it can be derived that

$$\begin{aligned}\|\Delta f_{k+1}\| &\leqslant \sum_{i=1}^{N} \|\boldsymbol{P}_i - \boldsymbol{Q}_i \boldsymbol{D}_{k-i+1}\| \|\Delta f_{k-i+1}\| \\ &- \sum_{i=1}^{N} (k_s + b_{fd} k_d) \|\Delta \boldsymbol{x}_{k-i+1}\|\end{aligned} \tag{A6}$$

* Corresponding author (email: xsqanhui91@gmail.com, chaiyi@cqu.edu.cn)

Substituting (11) into (15), one can get that

$$\begin{aligned}\|\Delta f_{k+1}\| \leqslant & \sum_{i=1}^{N}\|\boldsymbol{P}_i-\boldsymbol{Q}_i\boldsymbol{D}_{k-i+1}\|\,\|\Delta f_{k-i+1}\| \\ & -\sum_{i=1}^{N}(k_s+b_{fd}k_d)\times \\ & \times\int_0^t e^{\left(k_g+k_b b_{ud}+k_e b_{fd}\right)(t-\tau)} b_e\,\|\Delta f_{k-i+1}\|\,d\tau \\ \leqslant & \sum_{i=1}^{N}\|\boldsymbol{P}_i-\boldsymbol{Q}_i\boldsymbol{D}_{k-i+1}\|\,\|\Delta f_{k-i+1}\| \\ & -\sum_{i=1}^{N}\alpha\int_0^t e^{\alpha(t-\tau)}\,\|\Delta f_{k-i+1}\|\,d\tau\end{aligned} \tag{A7}$$

where $\alpha=\max\{k_g+k_b b_{ud}+k_e b_{fd}, b_e(k_s+b_{fd}k_d)\}$, $b_e=\sup\limits_{t\in[0,T]}(\boldsymbol{E}_k)$. Multiplying both sides of (15) by $e^{-\lambda t}$ and defining that $\rho_i=\|\boldsymbol{P}_i-\boldsymbol{Q}_i\boldsymbol{D}_{k-i+1}\|$, it can be obtained that

$$\begin{aligned}e^{-\lambda t}\|\Delta f_{k+1}\| \leqslant e^{-\lambda t}&\sum_{i=1}^{N}\Big(\rho_i\|\Delta f_{k-i+1}\| \\ & -\alpha\int_0^t e^{\alpha(t-\tau)}\|\Delta f_{k-i+1}\|\,d\tau\Big)\end{aligned} \tag{A8}$$

Based on norm theorem, one can get that

$$\begin{aligned}\|\Delta f_{k+1}\|_\lambda \leqslant & \sum_{i=1}^{N}\left(\rho_i\|\Delta f_{k-i+1}\|_\lambda-\alpha\frac{1-e^{(\alpha-\lambda)T}}{\lambda-\alpha}\|\Delta f_{k-i+1}\|_\lambda\right) \\ \leqslant & \sum_{i=1}^{N}\left(\rho_i-\alpha\frac{1-e^{(\alpha-\lambda)T}}{\lambda-\alpha}\right)\|\Delta f_{k-i+1}\|_\lambda \\ \leqslant & \sum_{i=1}^{N}\left(\bar{\rho}_i\|\Delta f_{k-i+1}\|_\lambda\right)\end{aligned} \tag{A9}$$

In which $\bar{\rho}_i=\rho_i-\alpha\frac{1-e^{(\alpha-\lambda)T}}{\lambda-\alpha}$. Hence, the zero point of polynomials (18) is inside the unit circle and the fault estimating error satisfies that $\lim\limits_{k\to\infty}\|\Delta f_{k+1}\|_\lambda=0$.

$$\begin{aligned}R(z)=\;& z^n-\bar{\rho}_1 z^{n-1}-\bar{\rho}_2 z^{n-2}-\bar{\rho}_3 z^{n-3}-\cdots \\ & -\bar{\rho}_{n-1}z-\bar{\rho}_n\end{aligned} \tag{A10}$$

Namely, if the zero points of equation (8) are inside the unit circle and $\lambda\gg\alpha$ holds, the (18) could be represented by (8). From this base, one can further obtain that

$$\|\Delta \boldsymbol{x}_k\|_\lambda \leqslant b_e\frac{1-e^{(\alpha-\lambda)T}}{\lambda-\alpha}\|\Delta f_k\|_\lambda, \lambda>\alpha \tag{A11}$$

$$\|\Delta \boldsymbol{y}_k\|_\lambda \leqslant (k_s+b_{fd}k_d)\|\Delta \boldsymbol{x}_k\|_\lambda+b_d\|\Delta f_k\|_\lambda \tag{A12}$$

where $b_d=\sup\limits_{t\in[0,T]}(\boldsymbol{D}_k)$.

Appendix B Simulation Results

To demonstrate the effectiveness of the proposed method, this paper conducts three numerical simulations on a nonlinear system. Two numerical examples contain different kinds of fault signals, such as abrupt fault signal and sinusoidal fault signal. Simultaneously, one comparative example is expanded to verify the superiority of the proposed method. The nonlinear system is considered to have nonrepetitive uncertainties, that is, the uncertainties in kth trial is different from the uncertainties k+1th trial. In this paper, an high order iterative learning estimator is presented for fault estimation. This method can reasonably compensate for the effect of uncertainties on the tracking results without decreasing the estimating accuracy rates.

Case 1: Fault estimation based on high order scheme for nonlinear system with abrupt fault signal In this case, the experimental results are obtained by applying the iterative learning scheme to the nonlinear system. The corresponding system model is described in Section 2 and the iterative learning law is designed in Section 3, respectively. The reference trajectories used for the nonlinear system are chosen as abrupt fault signal that make the shift from the initial to the final desired values within 1 s.

Consider the system (1) with following matrices:

$$\begin{aligned}\dot{\boldsymbol{x}}(t)=&\begin{bmatrix}-3 & 2\sin(2\pi x_1(t)) \\ 3 & -5\end{bmatrix}\boldsymbol{x}(t) \\ &+\begin{bmatrix}\cos(2\pi x_1(t)) \\ 2\end{bmatrix}u(t)+\begin{bmatrix}1 \\ e^{-5x_1(t)}\end{bmatrix}f(t) \\ \boldsymbol{y}(t)=&\begin{bmatrix}1 & 2\sin(2\pi x_1(t))\end{bmatrix}\boldsymbol{x}(t)+f(t)\end{aligned} \tag{B1}$$

The desired fault signal is

$$f(t) = \begin{cases} 2 + 2e^{-4t}, t > 1 \\ 2t, t \leqslant 1 \end{cases} \tag{B2}$$

Additionally, the $N = 4$, $P = 0.25$, $Q = 0.125$ and fault updating law is designed as

$$f_{k+1}(t) = \frac{1}{4}\sum_{i=1}^{4} f_{k-i+1}(t) + \frac{1}{4}\sum_{i=1}^{4} 0.5\Delta y_{k-i+1}(t) \tag{B3}$$

Then set the initial value $\boldsymbol{x}_k(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $f_{k-1}(t) = 0, k < N$, $\Delta \boldsymbol{y}_{k-1}(t) = 0, k < N$. Hence, one can get the simulation results that is shown in figure B1-B2. One can find out that the proposed method can achieve a satisfactory performance after a few iterative iterations based on the designed fault estimation law. The maximum fault estimation errors are decreased with the iteration increasing.

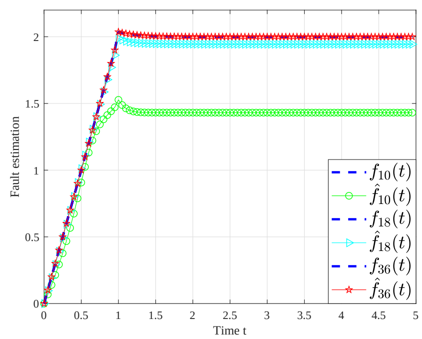

Figure B1 Fault tracking results in different iterations

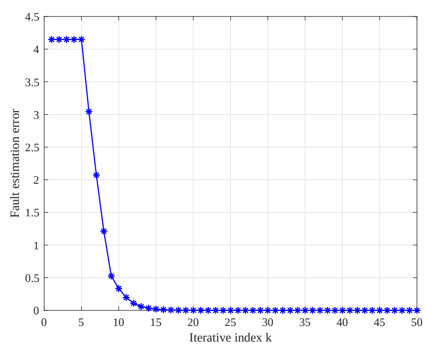

Figure B2 Maximum fault estimation error in different iterations

Case 2: Fault estimation based on high order scheme for nonlinear system with sinusoidal fault signal

This case exhibits the experimental results got by utilizing the high order iterative learning estimator designed in Section 3. Particularly, the system model is as same as the system described in Case 1. And the sinusoidal fault signal is employed to illustrate the soft fault occurring in the nonlinear system. To illustrate the validity of the high order iterative learning scheme, the desired fault signals are set as

$$f(t) = \begin{cases} 2 + 2\sin(2\pi t), t > 1 \\ 2t, t \leqslant 1 \end{cases} \tag{B4}$$

From the tracking results shown in figure B3 and figure B4, it can be concluded that the proposed method can successfully estimate the slowly varying fault signal.

It is clear the proposed method has such an expressiveness to the fault signal tracking results both in abrupt fault signal and in soft fault signal. The reason, as the simulating results displays, is not that the fault estimator is inadequately informed as often assumed, but because different fault signal reconstructions are shown in different perspectives and different contexts.

Case 3: Comparable results between fault estimation based on high order and P-type iterative learning schemes

To study the effect of nonrepetitive uncertainties, P-type iterative learning scheme and high order iterative learning fault estimator are applied to the same nonlinear system. One can find out the better fault signal tracking performance by comparing

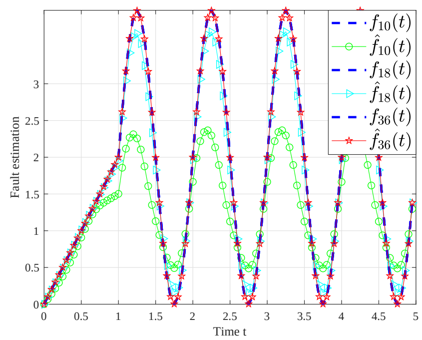

Figure B3 Fault tracking results in different iterations

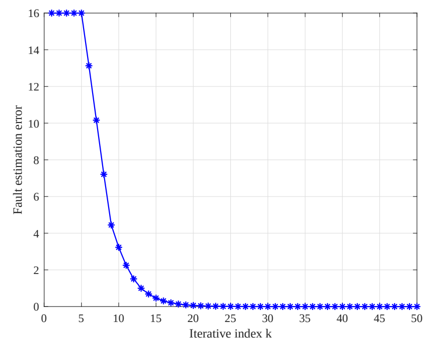

Figure B4 Maximum fault estimation error in different iterations

the simulating results. In order to demonstrate the superiority of proposed method in this paper, the P-type iterative learning scheme is given as follows:

$$f_{k+1}(t) = f_k(t) + 0.3\Delta y_k(t) \tag{B5}$$

and the high order iterative learning law is presented as

$$\begin{aligned} f_{k+1}(t) &= 0.6f_{k-1}(t) + 0.4f_k(t) \\ &+0.5\Delta y_{k-1}(t) + 0.5\Delta y_k(t) \end{aligned} \tag{B6}$$

The fault estimation errors are shown in the figure B5. One can find out that the fault estimation results based on P-type iterative learning scheme converge to zero after 10 iterations and the tracking trajectory using high order iterative learning scheme needs 10 iterations. However, one can find out that the convergence speed of high order method is faster than the P-type method. Namely, the high order approaches employ more prior information, which can improve the tracking speed.

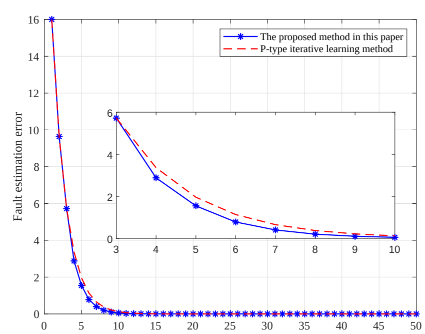

Figure B5 Comparable estimating errors between high order and P-type iterative learning methods

Figure B6 shows the fault signal tracking results of high order iterative learning scheme and the P-type method. It should be pointed out that $f(t)$ is the actual fault signal, $\hat{f}_4(t)$, $\hat{f}_8(t)$ and $\hat{f}_12(t)$ are the fault estimating results based on the proposed method in 4 iterative index, 8 iterative index and 12 iterative index, respectively. And the $\hat{F}_4(t)$, $\hat{F}_8(t)$ and $\hat{F}_12(t)$ represent the tracking results using P-type iterative learning scheme in 4 iterative index, 8 iterative index and 12 iterative index, respectively. One can conclude that the estimating results obtained by the proposed method are better than the results based on P-type iterative learning method.

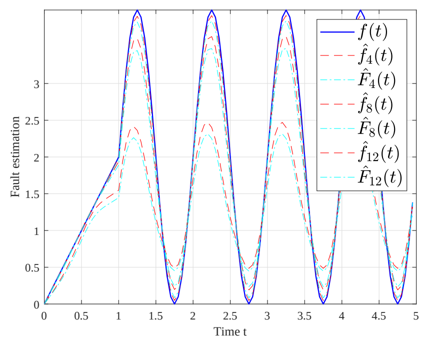

Figure B6 Comparable results between high order and P-type iterative learning methods in different iterations