SCIENCE CHINA
Information Sciences

• LETTER •

July 2022, Vol. 65 179201:1–179201:2
https://doi.org/10.1007/s11432-020-3086-5

Robust fixed-time output-feedback control for linear systems without chattering: an exact uncertainty compensation method

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Received 3 June 2020/Revised 5 August 2020/Accepted 1 October 2020/Published online 17 March 2021

Citation Shi S, Sun Y H, Hu Y L, et al. Robust fixed-time output-feedback control for linear systems without chattering: an exact uncertainty compensation method. Sci China Inf Sci, 2022, 65(7): 179201, https://doi.org/10.1007/s11432-020-3086-5

Dear editor,

The development of robust controllers for systems subject to uncertainties is always one of the hottest issues in modern control field [1, 2]. When only the output signals are measurable, the design of robust output-feedback controllers is required, which is more difficult [3]. Recently, the development of finite-time exact observer has provided us an alternative way to solve this problem [4, 5]. The interest is to use the finite-time exact observer to estimate the system states and uncertainties, simultaneously. Then, by replacing the unmeasurable states with the estimated states as well as an exact uncertainty compensation, robust output feedback controllers can be finally implemented.

Note that most of the existing controllers can only achieve asymptotic or finite-time control whose convergence time depends on the system initial values and will grow as the initial values grow. To overcome this drawback, the notation of fixed-time stability is developed [6]. As an extension of finite-time stability, the settling time of fixed-time stability is uniformly bounded by a constant independent of the system initial conditions. For this remarkable property, many results about fixed-time control have been developed.

Inspired by the aforementioned studies, we designed a robust fixed-time output-feedback controller for uncertain linear systems. Firstly, a composite observer is proposed to identify the system states and uncertainties, simultaneously. Then, based on the proposed composite observer, a robust fixed-time output-feedback controller with exact uncertainty compensation is proposed. Compared with the existing robust controllers, the proposed controller is completely continuous and thus is free of chattering, and can guarantee the fixed-time convergence of system states to zero independent of the system initial values.

Problem statement and preliminaries. In this study, we consider the following linear system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + \psi(t, x), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), \dots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}$ are the system state vector and input, respectively; $y(t) \in \mathbb{R}$ is the output; the function $\psi(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is the system uncertainty; $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times 1}$ and $C \in \mathbb{R}^{1\times n}$ are known matrices.

Assumption 1. The pair (A, B) is controllable and the function $\psi(t, x)$ fulfills the matched condition with $\psi(t, x) = B\varphi(t, x), |\dot{\varphi}(t, x)| \leqslant \varphi_{\max}$, where $\varphi_{\max}$ is a known constant.

Assumption 2. System (1) is strongly observable.

Remark 1. Assumption 1 is a very classical assumption for the control of linear systems. Assumption 2 is equivalent to the statement that the pair (A, C, B) has no invariant zeros or the relative degree of the system output y with respect to the matched uncertainty $\varphi(t, x)$ is n.

The goal of the study is to develop an observer-based output-feedback controller for system (1) under Assumptions 1 and 2 with exact uncertainty compensation such that the origin of system (1) is globally fixed-time stable.

Design of composite observer. A composite observer composed of a Luenberger observer (LO) and a high-order sliding mode observer (HOSMO) will be constructed to identify the system states and uncertainties in a fixed time.

(1) Design of LO. The following LO is designed to approximately estimate the system states:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + L(y(t) - C\tilde{x}(t)), \tag{2}$$

where $\tilde{x}(t) = [\tilde{x}_1(t), \dots, \tilde{x}_n(t)]^{\mathrm{T}}$ is the state vector of the LO. Define the observation error $e(t) = x(t) - \tilde{x}(t)$. Then, the error system can be obtained from (1) and (2) as

$$\dot{e}(t) = \tilde{A}e(t) + B\varphi(t, x),\ e_y(t) = Ce(t), \tag{3}$$

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where $\tilde{A} = A - LC$ is Hurwitz and $e_y(t) = y(t) - C\hat{x}(t) = Ce(t)$. Note that Assumption 2 implies that we can always find some matrix L such that $\tilde{A}$ is Hurwitz.

(2) Design of HOSMO. For error system (3), there exists a coordinate transformation $\bar{e}(t) = [\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n]^{\mathrm{T}} = \Phi e(t)$ with Φ defined as

$$\Phi = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & \alpha_1 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & \alpha_1 & \cdots & \alpha_{n-2} & \alpha_{n-1} \end{bmatrix} \begin{bmatrix} C\tilde{A}^{n-1} \\ C\tilde{A}^{n-2} \\ \vdots \\ C\tilde{A}^{0} \end{bmatrix}, \quad (4)$$

such that the following canonical form can be obtained:

$$\dot{\bar{e}}(t) = \bar{A}\bar{e}(t) + \bar{B}\varphi(t,x), \ \bar{e}_y(t) = \bar{C}\bar{e}(t), \quad (5)$$

where $\bar{A} = \Phi\tilde{A}\Phi^{-1} = [\begin{smallmatrix} -[\alpha_1,\dots,\alpha_{n-1}]^{\mathrm{T}} & \mathbf{I}_{n-1} \\ -\alpha_n & \mathbf{0}_{1\times(n-1)} \end{smallmatrix}]$, $\bar{B} = \Phi B = [0,\dots,0,\bar{C}\tilde{A}^{n-1}B]^{\mathrm{T}}$, $\bar{C} = C\Phi^{-1} = [1,0,\dots,0]$, and $\alpha_1, \alpha_2, \dots, \alpha_n$ are coefficients of the characteristic polynomial $\det(sI - \tilde{A}) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n$. For system (5), we can construct the following HOSMO:

$$\begin{aligned} \dot{z}_1 &= v_0 - k_1(1-\theta(t))\lfloor \tilde{z}_1 \rceil^{\frac{n+1+\alpha}{n+1}} - \alpha_1 \bar{e}_y(t), \quad (6) \\ \dot{z}_i &= v_{i-1} - k_i(1-\theta(t))\lfloor \tilde{z}_1 \rceil^{\frac{n+1+\alpha i}{n+1}} - \alpha_i \bar{e}_y(t), i = 2,\dots,n, \\ \dot{z}_{n+1} &= -\kappa_{n+1}\Gamma\theta(t)\mathrm{sign}(z_{n+1} - v_{n-1}) \\ &\quad - k_{n+1}(1-\theta(t))\lfloor \tilde{z}_1 \rceil^{1+\alpha}, \end{aligned}$$

where $v_0 = -\kappa_1 \Gamma^{\frac{1}{n+1}}\theta(t)\lfloor \tilde{z}_1 \rceil^{\frac{n}{n+1}} + z_2$, $v_{i-1} = -\kappa_i \Gamma^{\frac{1}{n+2-i}}\theta(t)\lfloor z_i - v_{i-2} \rceil^{\frac{n-i+1}{n-i+2}} + z_{i+1}$, $i = 2,\dots,n$, $z(t) = [z_1, z_2, \dots, z_{n+1}]^{\mathrm{T}}$ is the state vector of system (6), $\tilde{z}_1$ is defined as $\tilde{z}_1 = z_1 - \bar{e}_1$, $\theta(t)$ is a switching function defined as $\theta(t) = \frac{\mathrm{sign}(t-T_u)+1}{2}$ with $T_u > 0$ being an arbitrarily chosen constant, and the parameters $\Gamma, \alpha > 0$ and $\{\kappa_i, k_i\}_{i=1}^{n+1}$ are design constants to be determined later.

(3) State and uncertainty identification. The system state and uncertainty can be identified as

$$\begin{aligned} \hat{x} &= \tilde{x}(t) + \Phi^{-1}[z_1, z_2, \dots, z_n]^{\mathrm{T}}, \\ \hat{\varphi} &= \left((\bar{C}\tilde{A}^{n-1}B)^{\mathrm{T}}(\bar{C}\tilde{A}^{n-1}B)\right)^{-1}(\bar{C}\tilde{A}^{n-1}B)^{\mathrm{T}} z_{n+1}. \quad (7) \end{aligned}$$

Design of output-feedback controller. Because (A,B) is controllable and the uncertainty $\psi(t,x)$ satisfies the matched condition, by the coordinate transformation

$$S = \Psi x = [s_1, \dots, s_n]^{\mathrm{T}}, \quad \Psi = [A^{n-1}B, \dots, AB, B]^{-1},$$

system (1) can be transformed as a Brunovsky form:

$$\dot{s}_1 = s_2, \dots, \dot{s}_{n-1} = s_n, \quad \dot{s}_n = u + \varphi(t,x) + \sum_{i=1}^{n} a_i s_i. \quad (8)$$

Note that the vector $S = \Psi x$ is unmeasurable. We define the estimation of S as

$$\hat{S} = \Psi\hat{x} = [\hat{s}_1, \dots, \hat{s}_n]^{\mathrm{T}}, \quad \Psi = [A^{n-1}B, \dots, AB, B]^{-1}. \quad (9)$$

Then, with the definition of (9), the output-feedback controller can be finally designed for system (8) as

$$u = -\sum_{i=1}^{n}\varrho_i|\hat{s}_i|^{\rho_i}\mathrm{sign}(\hat{s}_i) - \sum_{i=1}^{n}\bar{\varrho}_i|\hat{s}_i|^{\bar{\rho}_i}\mathrm{sign}(\hat{s}_i) - \hat{\varphi} - \sum_{i=1}^{n} a_i\hat{s}_i, \quad (10)$$

where $\{\varrho_i, \bar{\varrho}_i, \rho_i, \bar{\rho}_i\}_{i=1}^{n}$ are design constants.

Theorem 1. Suppose that Assumptions 1 and 2 hold. The origin of the system (1) under the LO (2), HOSMO (6) and the controller (9) is globally fixed-time stable when their parameters are selected as follows: (i) For the LO (2), the matrix L is selected such that $A - LC$ is Hurwitz; (ii) For the HOSMO (6), the parameter Γ is selected to satisfy $\Gamma \geqslant \|\bar{C}\tilde{A}^{n-1}B\|\varphi_{\max}$, the parameters $\{\kappa_i\}_{i=1}^{n+1}$ are selected according to the original HOSMO [7], and the parameters $\{k_i\}_{i=1}^{n+1}$ are selected such that the polynomial $s^{n+1} + k_1 s^n + \cdots + k_n s + k_{n+1}$ is Hurwitz and $\alpha > 0$ is chosen small enough; (iii) For the controller (9), the parameters $\{\varrho_i, \bar{\varrho}_i\}_{i=1}^{n}$ are selected such that the polynomials $s^n + \varrho_n s^{n-1} + \cdots + \varrho_2 s + \varrho_1$ and $s^n + \bar{\varrho}_n s^{n-1} + \cdots + \bar{\varrho}_2 s + \bar{\varrho}_1$ are Hurwitz, and the parameters $\{\rho_i, \bar{\rho}_i\}_{i=1}^{n}$ are selected to satisfy $\rho_{i-1} = \frac{\rho_i\rho_{i+1}}{2\rho_{i+1}-\rho_i}, \bar{\rho}_{i-1} = \frac{\bar{\rho}_i\bar{\rho}_{i+1}}{2\bar{\rho}_{i+1}-\bar{\rho}_i}$, for $i = 2,\dots,n$, where $\rho_{n+1} = \bar{\rho}_{n+1} = 1, \rho_n = \rho_0 \in (1-\varepsilon, 1)$ and $\bar{\rho}_n = \bar{\rho}_0 \in (1, 1+\varepsilon)$ with $\varepsilon > 0$ being sufficiently small.

Proof. According to [8], the system state $x(t)$ and uncertainty $\varphi(t,x)$ can be identified by $\hat{x}, \hat{\varphi}$ defined in (7) in a fixed-time $T_{\max}^1$ independent of the system initial values, which implies that $\hat{s}_i = s_i$, $i = 1,\dots,n$ holds for $\forall t \geqslant T_{\max}^1$. Therefore, when $t \geqslant T_{\max}^1$, system (7) under the controller (9) is globally fixed-time stable [9]. Note that $s_i = 0, i = 1,\dots,n$ implies $x(t) \equiv 0$, which completes the proof.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 62003131, 62073121) and Natural Science Foundation of Jiangsu Province (Grant No. BK20200520).

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