SCIENCE CHINA
Information Sciences

• **Supplementary File** •

Locating multiple roots of nonlinear equation systems via Multi-strategy Optimization Algorithm with Sequence Quadratic Program

Jing Liang[1], Boyang Qu[1,2], Baolei Li[1,2,4*], Kunjie Yu[1] & Caitong Yue[1]

[1]*School of Electrical Engineering , Zhengzhou University, Zhengzhou* 450001*, China;*
[2]*School of Physics and Electronic Engineering, Nanyang Normal University, Nanyang* 473061*, China;*
[3]*Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, Kunming* 650011*, China;*
[4] *School of Electronic and Information Engineering, Zhongyuan University of Technology, Zhengzhou* 450007*, China*

Appendix A Experiment and Results

Appendix A.1 Testing problems

Firstly, experiments on frequently-used NES (F01-F42) benchmark problems [1] are carried out. The local traps of these frequently-used NES problems are too rare to challenge the proposed methods. To test the ability of the proposed methods in complex problems, ten complex NES benchmark problems (F43-F52), having many local traps, are proposed. At last, four real-world engineering problems are tested to show the strength of SQP-MOA in finding the distribution regularity of optima. Formulations of F01-F42 and four real-world engineering problems can be found in [1]. Details of F43-F52 can be found in the Appendix B.

Appendix A.2 Performance Metrics for NES problems

Peak ratio (PR) and success rate (SR) are used to assess the effectiveness of algorithms.

Peak ratio is the ratio of located roots number to total roots number. A root is said to be found if the Euclidean distance between it and one of the stagnant and present global searchers is smaller than a threshold value ε. On F43-F52, $\varepsilon = 0.01$, $maxFES$=200000. On, other problems, ε and $maxFES$ are set to be the same as that in [1]. If $d \leqslant 5$, $\varepsilon = 0.01$, otherwise, $\varepsilon = 0.1$. Details on $maxFES$ can be found in [1].

Success rate is the ratio of successful runs to total runs. A run is said to be successful if the peak ratio equals to 100%.

The success performance (SP) is used to evaluate the efficiency of algorithms. The SP denotes the mean number of FEs in successful runs divided by SR [2].

Appendix A.3 Methods in Comparison and Experimental Setup

For SQP-MOA, $N_{\mathrm{g}} = 3, N_{\mathrm{l}} = 2d, \mu_1 = 0.1, \alpha = 5,\ \beta = d.$ The options for $fmincon$ are set as follows: $Algorithm$='sqp', $maxN$=5000, $OptTol$=0.0000001, $StepTol$=0.00001.

The results of the six compared methods (i.e.,DR-JADE, A-WeB, NCDE, NSDE, MONES, GA-SQP) on F01-F42 are directly taken from [1]. The parameters of DR-JADE, NCDE and NSDE are set to be the same as used in their original literature when solving F43-F52.

Appendix A.4 Results and Discussions

The statistical test results are obtained by the Wilcoxon signed-rank test [3]. Generally, the best results are highlighted in boldface in all tables, R^+ means the sum of ranks that a method performs better than its competitor, and R^- is the sum of ranks for the opposite in tables. The sum of ranks is calculated by using the Wilcoxon test methods [3]. p-value generally is probability in the tails of the distribution under the null hypothesis. Thus, a small p-value means that the methods in comparison have an obvious difference.

Table A1 Results of SR and PR obtained by the wilcoxon test for the six compared methods on F01-F42

SQP-MOA VS	SR			PR		
	R^+	R^-	p-value	R^+	R^-	p-value
DR-JADE	670.5	232.5	< 0.05	670.5	232.5	< 0.05
A-WeB	817.5	85.5	< 0.05	817.5	85.5	< 0.05
NCDE	822.0	39.0	< 0.05	822.0	39.0	< 0.05
NSDE	822.0	39.0	< 0.05	822.0	39.0	< 0.05
MONES	775.5	85.5	< 0.05	775.5	85.5	< 0.05
GA-SQP	861.0	0.0	< 0.05	861.0	0.0	< 0.05

Appendix A.4.1 *Comparison on frequently-used NES problems*

The statistical results obtained by the Wilcoxon test are reported in Table A1. SQP-MOA provided higher R^+ values than R^- values in all the cases for both the SR and PR metrics. SQP-MOA significantly outperforms the other six methods, since all p values are less than 0.05. Therefore, we can conclude that the proposed method can be an effective alternative to simultaneously locate multiple roots of NES problems.

Details of SR and PR values obtained by using SQP-MOA and other six compared methods are summarized in Table A2. SQP-MOA obtains the highest average SR value (100%) and the highest average PR value (100%), i.e., it succeeds in locating all the roots. It is worth noting that none of the compared methods obtain 100% SR on F11, F14, F22, F24, and F40, but SQP-MOA obtains 100% SR. Therefore, we can conclude that SQP-MOA is an effective alternative to locate multiple roots of NES problems simultaneously. The 100% success rates on all problems prove that SQP-MOA overcomes the difficulties which limit the performance of these compared algorithms, details are analyzed as follows.

Results on F09 and F24, which have 20 variables, show that DR-JADE and A-web obtain rather low SR and PR on F09, and all algorithms except SQP-MOA obtain rather low SR and PR on F24. Obviously, the compared algorithms are weak in refining the solutions in high dimensional search space. Thus, they obtain low SR and PR due to the low quality of the obtained solutions. Evidently, SQP-MOA is more powerful in solving high dimensional NES problems, SQP in MOA enable SQP-MOA to effectively refine the solutions in high dimensional search space.

Results on F11, F14, F19, F28, F30 and F34, where close roots exist, show that DR-JADE and A-web obtain rather low SR but the PR is higher than 50%, NCDE and NSDE perform better, MONES attain 100% SR only on F14, F30, and F34. The reason is that the repulsion technique makes DR-JADE face the risk of omitting close roots. The non one-to-one mapping between the roots and the location function in A-web results in the risk of losing some roots. The niching methods in NCDE and NSDE also have the possibility of losing some close roots. MONES can not work when the first variable of roots is the same, which is the case of F19 and F28. We can conclude that SQP-MOA has the ability to effectively locate all roots when close roots exist.

Results on F22, where 8 variables and 16 roots exist, show that only SQP-MOA achieve 100% SR. The reason is that the increment in numbers of variables and roots make it difficult for the compared algorithms to find all the roots by using 100,000 FEs. We can conclude that SQP-MOA can locate all roots efficiently when the numbers of variables and roots are both high.

A comparison between SQP-MOA and GA-SQP shows that GA-SQP performs worse than SQP-MOA, and obtains lower SR and PR on most problems. The reason is that GA not only can not simultaneously locate multiple approximate roots but also frequently switches between different basins of roots. Thus, we need better EAs which can fast locate multiple promising sub-areas and escape from local traps. In this work, we use MOA for fast finding multiple approximate solutions of NES due to the characters of MOA.

Appendix A.4.2 *Comparison on Complex NES problems*

To study the performance of SQP-MOA on Complex NES problems, the results of Wilcoxon test on SR and PR are summarized in Table A3. It can be seen that SQP-MOA provided higher R^+ values than R^- values for both the SR and PR metrics. SQP-MOA significantly outperforms DR-JADE, NCDE and NSDE, all p values are less than 0.05. We can conclude that SQP-MOA is an effective alternative for complex NES problems.

The detailed results of SR and PR values are listed in Table A4. SQP-MOA obtained much higher mean values of SR and PR compared with DR-JADE, NCDE and NSDE. Many crowded local traps bring new challenges to these peer algorithms in these complex testing problems. DR-JADE obtain low peak ratio and 0% success rate. The reason is that the distances among roots and local traps are close, which makes it difficult for the dynamic repulsion technique in DR-JADE to adjusting the repulsion parameter. NSDE and NCDE perform better than DR-JADE because the crowding and speciation scheme is not sensitive to that the distances among roots and local traps are close or far. But they are still much less efficient than SQP-MOA in solving problems having many local traps. It can be seen that SQP-MOA has a stronger ability to escape from local traps and converging to the roots of NES. Moreover, these new testing problems are complex enough to challenge new methods.

* Corresponding author (email: bl_li@qq.com)

Table A2 Comparison of different methods on test cases F01-F42 with respect to the success rate and peak ratio

Prob.	Success Rate							Peak Ratio						
	SQP -MOA	DR -JADE	A -WeB	NCDE	NSDE	MONES	GA -SQP	SQP -MOA	DR -JADE	A -WeB	NCDE	NSDE	MONES	GA -SQP
F01	**1.000**	**1.000**	0.530	**1.000**	**1.000**	**1.000**	0.533	**1.000**	**1.000**	0.725	**1.000**	**1.000**	**1.000**	0.767
F02	**1.000**	**1.000**	0.680	**1.000**	0.933	0.933	0.367	**1.000**	**1.000**	0.880	**1.000**	0.978	0.978	0.700
F03	**1.000**	0.667	0.200	0.867	0.367	**1.000**	0.000	**1.000**	0.817	0.545	0.990	0.926	**1.000**	0.200
F04	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.333	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.667
F05	**1.000**	**1.000**	0.970	0.267	0.800	0.433	0.000	**1.000**	**1.000**	0.990	0.656	0.922	0.811	0.456
F06	**1.000**	**1.000**	0.650	**1.000**	**1.000**	**1.000**	0.800	**1.000**	**1.000**	0.830	**1.000**	**1.000**	**1.000**	0.900
F07	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.400	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.767
F08	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.600	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.800
F09	**1.000**	0.000	0.360	0.967	**1.000**	0.967	0.000	**1.000**	0.000	0.620	0.983	**1.000**	0.983	0.000
F10	**1.000**	0.967	**1.000**	0.833	0.567	0.767	0.000	**1.000**	0.997	**1.000**	0.985	0.949	0.976	0.242
F11	**1.000**	0.467	0.580	0.800	0.633	0.433	0.000	**1.000**	0.958	0.957	0.978	0.964	0.942	0.200
F12	**1.000**	**1.000**	**1.000**	0.000	0.000	0.000	0.000	**1.000**	**1.000**	**1.000**	0.464	0.544	0.592	0.151
F13	**1.000**	**1.000**	**1.000**	**1.000**	0.933	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.933	**1.000**	**1.000**
F14	**1.000**	0.667	0.600	0.300	0.267	**1.000**	0.000	**1.000**	0.958	0.940	0.871	0.883	**1.000**	0.188
F15	**1.000**	**1.000**	0.420	0.067	0.133	0.033	0.600	**1.000**	**1.000**	0.420	0.067	0.133	0.033	0.600
F16	**1.000**	**1.000**	0.120	0.433	0.500	0.000	0.000	**1.000**	**1.000**	0.837	0.919	0.924	0.776	0.438
F17	**1.000**	**1.000**	0.680	0.000	0.000	0.000	0.067	**1.000**	**1.000**	0.893	0.000	0.011	0.000	0.633
F18	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.933	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.933
F19	**1.000**	0.033	0.280	0.933	0.800	0.233	0.000	**1.000**	0.877	0.888	0.993	0.953	0.833	0.113
F20	**1.000**	**1.000**	0.760	0.400	0.467	0.000	0.000	**1.000**	**1.000**	0.973	0.919	0.933	0.196	0.226
F21	**1.000**	**1.000**	**1.000**	0.767	0.433	**1.000**	0.000	**1.000**	**1.000**	**1.000**	0.982	0.936	**1.000**	0.115
F22	**1.000**	0.033	0.000	0.867	0.900	0.000	0.000	**1.000**	0.840	0.669	0.992	0.990	0.083	0.138
F23	**1.000**	**1.000**	0.660	0.967	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.943	0.994	**1.000**	**1.000**	0.489
F24	**1.000**	0.000	0.240	0.000	0.000	0.000	0.000	**1.000**	0.000	0.620	0.000	0.000	0.000	0.000
F25	**1.000**	**1.000**	0.700	0.833	0.967	0.033	0.000	**1.000**	**1.000**	0.951	0.976	0.995	0.719	0.529
F26	**1.000**	**1.000**	0.980	**1.000**	**1.000**	**1.000**	0.033	**1.000**	**1.000**	0.995	**1.000**	**1.000**	**1.000**	0.500
F27	**1.000**	**1.000**	**1.000**	**1.000**	0.967	0.967	0.000	**1.000**	**1.000**	**1.000**	**1.000**	0.994	0.989	0.439
F28	**1.000**	0.433	0.140	**1.000**	**1.000**	0.033	0.000	**1.000**	0.913	0.855	**1.000**	**1.000**	0.733	0.400
F29	**1.000**	**1.000**	**1.000**	**1.000**	0.967	0.033	0.233	**1.000**	**1.000**	**1.000**	**1.000**	0.983	0.500	0.617
F30	**1.000**	0.267	0.000	0.767	0.467	**1.000**	0.000	**1.000**	0.917	0.093	0.975	0.939	**1.000**	0.283
F31	**1.000**	**1.000**	**1.000**	0.567	0.833	0.667	0.367	**1.000**	**1.000**	**1.000**	0.783	0.917	0.833	0.633
F32	**1.000**	**1.000**	**1.000**	0.667	0.967	0.467	0.000	**1.000**	**1.000**	**1.000**	0.917	0.992	0.858	0.467
F33	**1.000**	**1.000**	**1.000**	0.033	0.767	**1.000**	0.000	**1.000**	**1.000**	**1.000**	0.450	0.925	**1.000**	0.458
F34	**1.000**	0.000	**1.000**	0.000	0.000	**1.000**	0.367	**1.000**	0.500	**1.000**	0.000	0.000	**1.000**	0.550
F35	**1.000**	**1.000**	**1.000**	0.000	0.000	0.000	0.133	**1.000**	**1.000**	**1.000**	0.000	0.000	0.000	0.133
F36	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.433	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.650
F37	**1.000**	**1.000**	0.880	0.367	0.867	0.167	0.167	**1.000**	**1.000**	0.880	0.367	0.867	0.167	0.167
F38	**1.000**	**1.000**	**1.000**	0.433	0.967	**1.000**	0.233	**1.000**	**1.000**	**1.000**	0.789	0.989	**1.000**	0.722
F39	**1.000**	**1.000**	0.880	0.000	0.000	0.000	0.000	**1.000**	**1.000**	0.940	0.300	0.500	0.000	0.000
F40	**1.000**	0.767	0.660	0.333	0.500	0.000	0.000	**1.000**	0.953	0.932	0.807	0.860	0.140	0.220
F41	**1.000**	**1.000**	**1.000**	0.967	**1.000**	**1.000**	0.033	**1.000**	**1.000**	**1.000**	0.992	**1.000**	**1.000**	0.525
F42	**1.000**	**1.000**	0.990	0.967	**1.000**	**1.000**	0.400	**1.000**	**1.000**	0.990	0.983	**1.000**	**1.000**	0.567
Mean	**1.000**	0.817	0.737	0.652	0.690	0.599	0.191	**1.000**	0.922	0.890	0.789	0.832	0.742	0.442

Table A3 Results of SR and PR obtained by the Wilcoxon test for SQP-MOA vuse DR-JADE, NSDE and NCDE on F43-F52

SQP-MOA VS	SR			PR		
	R^+	R^-	p-value	R^+	R^-	p-value
DR-JADE	45.0	0.0	< 0.05	55.0	0.0	< 0.05
NSDE	53.5	1.5	< 0.05	45.0	0.0	< 0.05
NCDE	45.0	0.0	< 0.05	55.0	0.0	< 0.05

Table A4 Comparison of different methods on test cases F43-F52 with respect to the success rate and peak ratio

Prob.	Success Rate				Peak Ratio			
	SQP-MOA	DR-JADE	NSDE	NCDE	SQP-MOA	DR-JADE	NSDE	NCDE
F43	**0.980**	0.000	0.200	0.000	**0.999**	0.240	0.931	0.415
F44	**0.960**	0.000	0.000	0.000	**0.999**	0.374	0.848	0.265
F45	0.000	0.000	0.000	0.000	**0.910**	0.167	0.800	0.196
F46	**1.000**	0.000	0.000	0.000	**1.000**	0.536	0.622	0.140
F47	**0.840**	0.000	0.000	0.000	**0.988**	0.366	0.778	0.096
F48	**0.960**	0.000	0.000	0.000	**0.999**	0.374	0.406	0.113
F49	**1.000**	0.040	0.020	0.000	**1.000**	0.703	0.668	0.090
F50	**1.000**	0.120	0.980	0.280	**1.000**	0.605	0.995	0.745
F51	**1.000**	0.000	**1.000**	0.000	**1.000**	0.604	**1.000**	0.423
F52	**1.000**	0.000	0.100	0.000	**1.000**	0.430	0.938	0.271
Mean	**0.874**	0.016	0.230	0.028	**0.989**	0.440	0.799	0.276

The SR value for F45 is 0% for all methods, but the PR obtained by SQP-MOA is considerable high (91%). The reason for low SR is that 200,000 FEs is too little to obtain 83 roots. Assuming that the probability that a global searcher searches for any root is p, the probability that a new global searcher searches for one of N_f located root is $N_f \times p$. Obviously, the risk of searching in these exploited areas became larger when the number of located roots (N_f) increase. As a result, SQP-MOA did not find all the roots when 200,000 FEs are used up.

Although, SQP-MOA has the ability to locate all roots, including close roots, on most problems. It has a risk that multiple global searchers search near the same or located roots. Indeed, it is difficult to distinguish close roots and the same root. This motivates us to develop more efficient global searcher selection methods in future work.

Appendix A.4.3 *Application of SQP-MOA in real-world engineering design problems*

To study the performance of SQP-MOA in real-world engineering design problems, experiments are carried out on four well-studied problems, i.e., chemical equilibrium application model, neurophysiology application model, combustion theory application model and economic modeling system.The results obtained by SQP-MOA have been compared with those reported in [1].

Because the roots of these four models are unknown, the SR and PR criteria cannot be used. To show the regularity of distribution of roots, some dimensions of obtained solutions are shown in Fig.A1. The fitness values of these roots are smaller than 0.01. Note that, since SQP-MOA is able to obtain multiple roots in each run, we only report the result in a random run.

The results in Fig.A1 indicate that SQP-MOA is able to obtain much more roots compared with DR-JADE. It is worth noticing that the roots obtained by SQL-MOA are able to show the regularity of distribution of roots clearly on chemical equilibrium application, neurophysiology application and combustion theory application problems. The distribution of roots is crucial for users to analyze the problem and to make a decision.

Appendix A.5 Further Analysis

We have shown so far that in most cases, SQP-MOA achieves much better performance than the compared methods in terms of SR and PR. In this section, we will further investigate their performance to reveal the contributions of different components to improving SQP-MOA search capability and the influence of different parameter settings on the performance of SQP-MOA.

Appendix A.5.1 *Effectiveness of Each Component*

SQP started to run from a random initial guess, and it is restarted from another random initial guess when it stopped. The stop criteria are the same as that in SQP-MOA. The solutions obtained at each stop are saved for evaluating its performance.

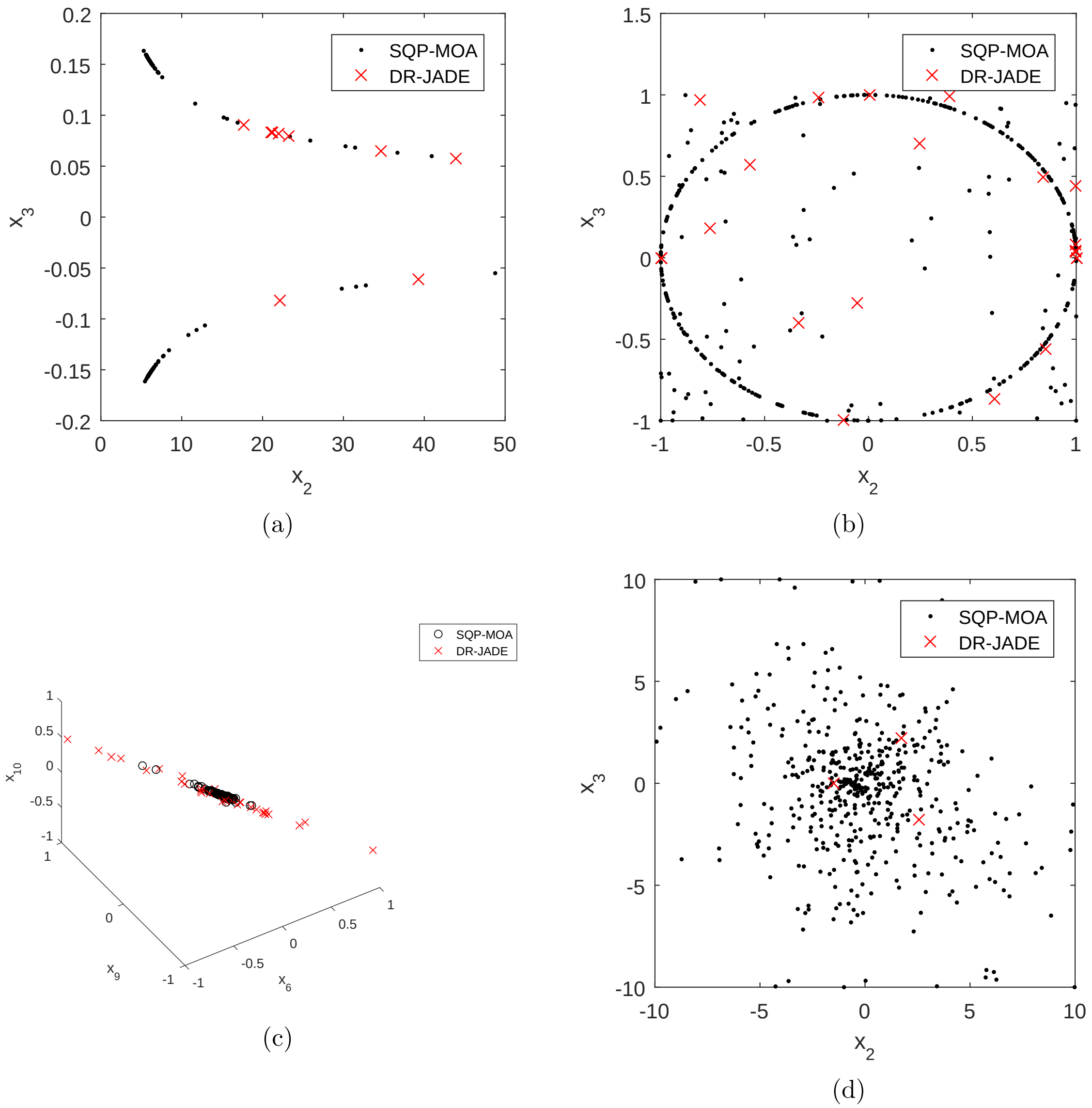


Figure A1 The second dimension and third dimension of roots for four real engineering design problems obtained by SQP-MOA and DR-JADE. (a) chemical equilibrium application model; (b) neurophysiology application model; (c) combustion theory application model;(d) economic modeling system.

The statistical results obtained by the Wilcoxon test are shown in Table A5. SQP-MOA provide larger R^+ than R^- on SR and PR. SQP-MOA significantly outperforms SQP and MOA, since all p values are less than 0.05. We can conclude that combining MOA and SQP can not only bring their advantages into full play but also offset their disadvantages.

The details on SR and PR are shown in Table A6. To investigate the effectiveness of MOA, we compared the SR and PR obtained by SQP-MOA and SQP. The results showed that both of them can achieve good SR and PR on F01-F42. But, SQP obtained nearly 0% SR and rather lower PR on F43-F52, on the opposite, SQP-MOA obtained considerable high SR and PR. It is clear that MOA contributed to improving SQP-MOA search capability in complex problems.

To investigate the effectiveness of SQP, we compare the SR and PR obtained by using SQP-MOA and MOA. The results show that MOA obtains rather lower SR and PR on F01-F52. Extra experiments indicate that MOA can achieve comparative result compared with SQP-MOA, when the accuracy requirement is relaxed, i. e., ε is set at 10 times of the original ε. It is obvious that SQP contributed to improving the quality of solutions in an effective way.

To sum up, MOA and SQP strategies are all effective in the proposed methods.

Appendix A.5.2 *Efficiency of Each Component*

To investigate the efficiency of each component, the statistical results on SP obtained by the Wilcoxon test are shown in Table A7. SQP-MOA provides higher R^+ values than R^- values. This indicates that SQP-MOA are more efficient than

Table A5 Results of SR and PR obtained by the wilcoxon test for SQP-MOA versus SQP and MOA

SQP-MOA VS	SR			PR		
	R^+	R^-	p-value	R^+	R^-	p-value
SQP	968.0	410.0	< 0.05	968.0	410.0	< 0.05
MOA	1360.0	18.0	< 0.05	1315.5	10.5	< 0.05

MOA and SQP. Because the p-value on SQP is larger than 0.05, we can conclude that SQP-MOA is not obviously more efficient than SQP. It is reasonable because SQP is efficient on problems with rare local traps and most of these 52 testing problems have rare local traps.

The details on SP values are shown in Table A8. To investigate the efficiency of MOA, we compare the SP obtained by using SQP-MOA and SQP. The results show that SQP obtains smaller SP on F01,F03, F09, F10, F13, F14, F16, F18, F20, F21, F23, F25, F26, F28, F32-F36, F38, F39 and F41. We find that there exist rare local traps in these functions by drawing the landscapes of 2-d functions. This indicates that SQP is more efficiency than MOA if it is not trapped in local traps. On the opposites, SQP-MOA achieved better SP on the other test problems, where SQP is easily trapped. It can be concluded that MOA contributes to improving the effectiveness of SQP-MOA by helping SQP-MOA jump out of local traps and focus its search on a more fruitful area in complex problems.

To investigate the effectiveness of SQP, we compare the SP obtained by using SQP-MOA and MOA. The results show that MOA achieves worse SP on almost all problems. It is because that MOA costs redundant fitness evaluations in exploitation and SQP contributes to improving the effectiveness of SQP-MOA.

Appendix A.5.3 *Influence of Different Radius*

The radius (μ_1) is one of the parameters in SQP-MOA. In the previous experiments, the default setting $\mu_1 = 0.1$. In this section, its influence on the performance of SQP-MOA is investigated by experiments. In these experiments, $N_{\mathrm{g}} = 20$, $N_{\mathrm{l}} = 2d$, $\alpha = 5$, $\beta = d$. The statistical results obtained by using the Wilcoxon test are reported in Table A9.

On SR and PR in Table A9, the p-values are larger than 0.05, so there is no significant difference between the results obtained by $\mu_1 = 0.1$ and those obtained by other μ_1 values. The effectiveness of SQP-MOA is insensitive to μ_1.

On SP, $\mu_1 = 0.1$ only outperforms $\mu_1 = 0.01$ and $\mu_1 = 0.001$ with rather low p-values. This indicates that too small μ_1 go against the efficiency of SQP-MOA. SQP-MOA with μ_1=0.001 has low exploration ability. So, the results also prove the efficiency of MOA in locating approximate solutions near roots fast by exploration, which is one of our motivation.

Given effectiveness and efficiency, the value of μ_1 can be chosen from a broad range, for example, from 0.1 to 0.25.

The detailed experimental results on SR, PR and SP are, respectively, given in Tables A10 and A11.

From Table A10, it can be seen that the SR and PR are high for different values of μ_1 on almost all problems. The settings of μ_1 value have little influence on the effectiveness of SQP-MOA. On F47 and F48, the SR and PR increase when μ_1 raise. On F16, the SR and PR decline when μ_1 raise. This means that the optimal setting of μ_1 value is related to the problem. This motivates us to develop adaptive parameter methods in future work.

From Table A11, it can be seen that there are small differences between SP obtained by using different μ_1. The settings of μ_1 value have little influence on the efficiency of SQP-MOA.

In summary, SQP-MOA is insensitive to μ_1, and the value of μ_1 can be chosen from a vast range, for example, from 0.1 to 0.25.

Appendix A.5.4 *Influence of Different Number of Global Searchers*

The number Of global searchers N_{g} is one of the parameters in SQP-MOA. In the previous experiments, the default setting $N_{\mathrm{g}} = 3$. In this section, its influence on the performance of SQP-MOA is investigated by experiments. In these experiments, $\mu_1 = 0.1$, $N_{\mathrm{l}} = 2d$, $\alpha = 5$, $\beta = d$. The statistical test results obtained by using the Wilcoxon test are reported in Table A12.

The p-values on SR and PR in Table A12 are all smaller than 0.05. So there are no significant difference between results obtained when N_{g} equals 3 and that obtained when N_{g} equals other values. Thus, the effectiveness of SQP-MOA is insensitive to N_{g}

However, on SP , the p-values are smaller than 0.05 when $N_{\mathrm{g}} = 7, 10, 20, 30, 40, 50$.Obviously, N_{g} has an influence on the efficiency of SQP-MOA. Moreover, N_{g} equals 3 obtained higher R^- values than R^+ values when compared N_{g} equals 1. The reasons would be that the number of roots (NOR) of these NES problems are small. The relationship between the optimal N_{g} and NOR would be discussed by analyzing the details of results, which are listed in Tables A13 and A14 on SR, PR, and SP.

From Table A13, it can be seen that the SR and PR are all high for different values of N_{g} on almost all problems. The settings of N_{g} have little influence on the effectiveness of SQP-MOA.

The results in Table A14 show a trend that the optimal N_{g} increase as the NOR raise. On problems with rare roots, small N_{g} gets smaller SP compared with large N_{g} . When the roots are rare, the large N_{g} may make many sub-groups of local searchers exploit the same area. On problems with many local traps and roots, large N_{g} gets better SP than small N_{g}. The reason is that multiple global searchers are generated at each iteration to located multiple promising areas fast, and hence, improved the performance. This indicates that the optimal setting of N_{g} value is related to the NOR.

Table A6 Comparison of different components of SQP-MOA with respect to the success rate and peak ratio

Prob.	Success Rate			Peak Ratio		
	SQP-MOA	SQP	MOA	SQP-MOA	SQP	MOA
F01	**1.000**	**1.000**	0.220	**1.000**	**1.000**	0.420
F02	**1.000**	**1.000**	0.020	**1.000**	**1.000**	0.287
F03	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.000
F04	**1.000**	**1.000**	0.620	**1.000**	**1.000**	0.800
F05	**1.000**	**1.000**	0.920	**1.000**	**1.000**	0.973
F06	**1.000**	**1.000**	0.420	**1.000**	**1.000**	0.670
F07	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F08	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F09	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.000
F10	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F11	**1.000**	**1.000**	0.460	**1.000**	**1.000**	0.948
F12	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.146
F13	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.000
F14	**1.000**	**1.000**	0.600	**1.000**	**1.000**	0.940
F15	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.000
F16	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.731
F17	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.000
F18	**1.000**	**1.000**	0.460	**1.000**	**1.000**	0.460
F19	**1.000**	**1.000**	0.500	**1.000**	**1.000**	0.938
F20	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.382
F21	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.603
F22	**1.000**	0.996	0.000	**1.000**	**1.000**	0.063
F23	**1.000**	**1.000**	0.160	**1.000**	**1.000**	0.787
F24	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.500
F25	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.443
F26	**1.000**	**1.000**	0.940	**1.000**	**1.000**	0.985
F27	**1.000**	**1.000**	0.840	**1.000**	**1.000**	0.973
F28	**1.000**	**1.000**	0.100	**1.000**	**1.000**	0.863
F29	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.010
F30	**1.000**	0.860	0.000	**1.000**	0.988	0.055
F31	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F32	**1.000**	**1.000**	0.780	**1.000**	**1.000**	0.935
F33	**1.000**	**1.000**	0.600	**1.000**	**1.000**	0.890
F34	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F35	**1.000**	**1.000**	0.760	**1.000**	**1.000**	0.760
F36	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F37	**1.000**	**1.000**	0.500	**1.000**	**1.000**	0.500
F38	**1.000**	**1.000**	0.780	**1.000**	**1.000**	0.920
F39	**1.000**	**1.000**	0.580	**1.000**	**1.000**	0.790
F40	**1.000**	**1.000**	0.000	**1.000**	**1.000**	0.060
F41	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F42	**1.000**	0.800	0.000	**1.000**	0.800	0.000
F43	**0.980**	0.000	0.000	**0.999**	0.129	0.226
F44	**0.960**	0.000	0.000	**0.999**	0.126	0.278
F45	0.000	0.000	0.000	**0.910**	0.040	0.162
F46	**1.000**	0.000	0.000	**1.000**	0.054	0.474
F47	**0.840**	0.000	0.000	**0.988**	0.028	0.213
F48	**0.960**	0.000	0.000	**0.999**	0.021	0.275
F49	**1.000**	0.000	0.000	**1.000**	0.130	0.518
F50	**1.000**	0.000	0.100	**1.000**	0.180	0.540
F51	**1.000**	0.000	0.000	**1.000**	0.126	0.408
F52	**1.000**	0.000	0.000	**1.000**	0.065	0.348

Table A7 Results of SP obtained by the wilcoxon test for SQP-MOA versus SQP and MOA

SQP-MOA VS	R^+	R^-	p-value
SQP	866.0	460.0	0.056
MOA	1326.0	0.0	< 0.05

Appendix A.5.5 *Influence of Different Number of Local Searchers*

The number Of local searchers N_l is one of the parameters in SQP-MOA. In the previous experiments, the default setting $N_\mathrm{l} = 2d$. In this section, its influence on the performance of SQP-MOA is investigated by experiments. In these experiments, $\mu_1 = 0.1$, $N_\mathrm{g} = 20$, $\alpha = 5$, $\beta = d$. The statistical test results obtained by using the Wilcoxon test, are reported in Table A15.

The results indicate that there are no significant differences among the values of SR and PR when the values of N_l change from d to $3d$. There are no significant differences among the values of SP when the values of N_l change from d to $2d$.

The detailed results on SR, PR, and SP are given in Tables A16 and A17.

From Table A16, it can be seen that the SR and PR are all high for different values of N_l on almost all problems. The settings of N_l have little influence on the effectiveness of SQP-MOA. The SR and PR on F09, F16 and F22 decline when N_l raise. The reason would be that the large N_l may decrease the probability of keeping unchanged in d continuous iterations for a global searcher. As a result, SQP-MOA distributed many FEs to MOA for exploiting. However, SQP is more efficient in exploiting, which has been proved in the section on the efficiency of each component. It is clear that large N_l make SQP have no chance to come into play. Thus, the performance decrease.

From Table A17, it can be seen that small N_l obtains better PF on F01-F42, and medium N_l obtains better SP on F43-F52. The reason is that large N_l makes SQP-MOA distribute too many FEs to MOA for exploiting on F01-F42, SQP has no chance to come into play. On F43-52, medium N_l gets better results than small N_l and large N_l. It is reasonable, Too small N_l makes SQP-MOA can not converge toward the optimum fast. Meanwhile, too large N_l makes SQP-MOA cost redundant fitness evaluations on local search. This indicates that the optimal setting of N_l value is related to problems.

As mentioned all, the effectiveness of SQP-MOA is insensitive to N_l, and the value of N_l can be chosen from a large range, for example, from d to $2d$. Given the efficiency, the optimal N_g depends on the problem. This motivates us to develop adaptive parameter methods in future work.

Appendix A.5.6 *Influence of Different β values*

The value Of β determines when to switch MOA to SQP. In the previous experiments, the default setting $\beta = 1$. In this section, its influence on the performance of SQP-MOA is investigated by experiments. In these experiments, $\mu_1 = 0.1$, $N_\mathrm{g} = 3$, $N_\mathrm{l} = 2d$, $\alpha = 5$. The statistical test results obtained by using the Wilcoxon test, are reported in Table A18.

The results indicate that there is no significant difference among the values of SR and PR when the values of β change from d to $3d$. There is no significant difference among the values of SP when the values of β change from d to $5d$. $\beta = d$ is better than other values, especially $5d$.

The detailed results on SR, PR, and SP are given in Tables A19 and A20.

From Table A19, it can be seen that the SR and PR are all high for different values of β on most problems. The settings of β have little influence on the effectiveness of SQP-MOA. The SR and PR on F16, F22, F42, and F44 decline when β raise. The reason would be that the large β makes SQP-MOA distribute many FEs to MOA for exploiting. But SQP is more efficient than MOA in exploiting. Thus, the performance decreases.

From Table A20, it can be seen that $\beta = d$ and $\beta = 2d$ obtain lower PF value. The reason is that large β makes SQP-MOA wast too many FEs in refining the solutions by using MOA, even the solution has standstill for many generations. It is reasonable, small β makes SQP-MOA switch to SQP as soon as MOA has difficulty in refining the solution.

As mentioned all, the effectiveness of SQP-MOA is insensitive to β, and the value of β can be chosen from a large range, for example, from d to $3d$. Given the efficiency, β is suggested to set at d or $2d$.

Table A8 Comparison of different components of SQP-MOA with respect to the success performance

Prob.	SQP-MOA	SQP	MOA
F01	164	**156**	22200
F02	**822**	2997	120000
F03	10161	**2160**	NaN
F04	**300**	6647	16089
F05	**3552**	5088	38194
F06	**734**	960	39512
F07	**469**	2027	6722
F08	**362**	731	1830
F09	8365	**2902**	NaN
F10	4090	**2167**	11926
F11	**5124**	6105	43276
F12	**3998**	5142	NaN
F13	1860	**958**	NaN
F14	3913	**2016**	31236
F15	**562**	872	NaN
F16	13201	**2434**	620000
F17	**11101**	8297	NaN
F18	1002	**549**	38177
F19	**7641**	11026	56060
F20	5174	**1810**	NaN
F21	3412	**2513**	NaN
F22	**53445**	42170	NaN
F23	1971	**899**	128021
F24	**25130**	32080	NaN
F25	4017	**3239**	NaN
F26	724	**533**	21938
F27	**1398**	3466	26950
F28	15894	**3214**	452143
F29	**1478**	2680	NaN
F30	**10728**	34348	NaN
F31	**617**	763	12284
F32	765	**569**	32391
F33	667	**503**	45298
F34	323	**198**	3360
F35	257	**122**	19134
F36	264	**218**	2508
F37	**1223**	1756	31336
F38	572	**528**	26888
F39	400	**216**	34749
F40	**5347**	6130	NaN
F41	707	**459**	8598
F42	**3632**	35518	626250
F43	**67688**	NaN	NaN
F44	**126980**	NaN	NaN
F45	NaN	NaN	NaN
F46	**50236**	NaN	NaN
F47	**149910**	NaN	NaN
F48	**116910**	NaN	NaN
F49	**57325**	NaN	NaN
F50	**11067**	NaN	1586200
F51	**46338**	NaN	NaN
F52	**33292**	NaN	NaN

Table A9 Results of SR, PR and SP obtained by the Wilcoxon test for SQP-MOA with $\mu_1 = 0.1$ versus other values

$\mu_1 = 0.1$ VS	SR			PR			SP		
	R^+	R^-	p-value	R^+	R^-	p-value	R^+	R^-	p-value
μ_1 =0.001	789.5	588.5	0.35	781.5	544.5	0.26	793.0	482.0	0.13
μ_1 =0.01	740.5	637.5	0.64	735.0	591.0	0.50	981.5	344.5	< 0.05
μ_1 =0.05	687.0	639.0	0.82	687.0	691.0	1.00	796.0	530.0	0.21
μ_1 =0.15	789.5	588.5	0.35	784.0	542.0	0.25	748.0	578.0	0.42
μ_1 =0.2	741.0	637.0	0.63	789.0	589.0	0.35	626.0	649.0	1.00
μ_1 =0.25	689.5	636.5	0.80	740.5	637.5	0.64	721.0	605.0	0.58

Table A10 Influence of the parameter of radius (μ_1) on the performance of SQP-MOA with respect to the success rate and the peak ratio

μ_1 / Prob.	Success Rate							Peak Ratio						
	0.001	0.01	0.05	0.1	0.15	0.2	0.25	0.001	0.01	0.05	0.1	0.15	0.2	0.25
F01	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F02	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F03	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F04	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F05	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F06	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F07	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F08	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F09	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F10	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F11	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F12	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F13	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F14	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F15	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F16	**1.000**	0.920	0.920	0.800	0.560	0.640	0.580	**1.000**	0.989	0.989	0.971	0.929	0.949	0.937
F17	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F18	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F19	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F20	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F21	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F22	0.980	0.920	0.960	0.960	0.960	**1.000**	**1.000**	0.999	0.995	0.998	0.998	0.998	**1.000**	**1.000**
F23	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F24	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F25	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F26	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F27	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F28	**1.000**	**1.000**	**1.000**	0.980	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.998	**1.000**	**1.000**	**1.000**
F29	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F30	0.960	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.997	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F31	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F32	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F33	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F34	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F35	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F36	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F37	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F38	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F39	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F40	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F41	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F42	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.980	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.990	**1.000**
F43	0.960	**1.000**	**1.000**	**1.000**	0.980	**1.000**	**1.000**	0.998	**1.000**	**1.000**	**1.000**	0.999	**1.000**	**1.000**
F44	0.880	0.920	0.880	**0.960**	**0.960**	0.900	**0.960**	0.997	0.998	0.997	**0.999**	**0.999**	0.997	**0.999**
F45	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.906	0.906	**0.923**	0.911	0.907	0.907	0.898
F46	0.960	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.996	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F47	0.420	0.520	0.840	**0.860**	0.800	**0.860**	0.780	0.946	0.960	0.988	**0.991**	0.984	0.989	0.984
F48	0.560	0.660	0.860	**0.960**	0.900	0.880	0.920	0.985	0.990	0.996	**0.999**	0.997	0.997	0.998
F49	**1.000**	**1.000**	**1.000**	**1.000**	0.980	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.998	**1.000**	**1.000**
F50	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F51	0.960	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.998	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F52	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

Table A11 Influence of the parameter of radius (μ_1) on the performance of SQP-MOA with respect to the success performance

Prob.	μ_1=0.001	μ_1=0.01	μ_1=0.05	μ_1=0.1	μ_1=0.15	μ_1=0.2	μ_1=0.25
F01	**378**	415	411	414	413	424	415
F02	1045	1667	1243	1143	966	1018	**906**
F03	**7488**	10235	10641	7941	10724	9709	9422
F04	655	663	656	630	628	**600**	659
F05	4234	3968	3933	3549	3849	**2942**	3555
F06	1436	1418	1360	**1349**	1470	1573	1432
F07	767	854	858	871	892	782	**759**
F08	736	628	**618**	660	631	660	776
F09	**21696**	30163	26163	25130	24650	24267	23156
F10	3947	4675	4766	4271	3652	3698	**3183**
F11	6104	5299	5093	5314	5150	5282	**5088**
F12	4384	4676	4702	4374	4352	**4229**	4252
F13	7050	10338	7204	**6735**	6760	7420	8593
F14	2930	**2913**	3732	3825	4252	3445	4128
F15	1742	**1470**	1527	1553	1646	1686	1514
F16	**7172**	9308	15686	20417	22378	27495	28146
F17	14725	13438	14177	11768	16683	**11615**	12454
F18	3196	3702	3005	2814	3070	2934	**2786**
F19	8448	6842	8137	**6284**	6976	7094	6612
F20	4038	**3517**	4937	6156	4875	5489	5807
F21	**3144**	4207	3358	3524	3352	3284	3557
F22	57224	61445	54675	56496	54417	**52919**	55306
F23	1767	**1610**	1929	2053	1842	1845	2364
F24	47846	56732	48443	43105	39719	40847	**38211**
F25	4448	4550	4750	4584	**4141**	4969	4576
F26	**950**	1104	1133	1090	1064	987	989
F27	2840	2706	1887	**1778**	1810	1869	1858
F28	10656	18266	23144	19335	16872	9422	**6765**
F29	1969	2077	2537	1843	**1582**	1584	1692
F30	17941	13702	9404	11044	8884	10760	**8426**
F31	962	983	1157	920	**917**	990	924
F32	**1001**	1240	1124	1122	1135	1120	1148
F33	**993**	1159	1184	1099	1043	962	1146
F34	612	722	661	612	650	**611**	638
F35	1036	888	865	**854**	893	978	958
F36	606	**568**	597	656	589	601	611
F37	3864	2863	2751	**2632**	3014	2775	3317
F38	867	**875**	960	898	912	1030	920
F39	699	**670**	691	723	671	776	747
F40	5218	**4130**	5736	5795	7264	7124	6635
F41	894	1106	1014	962	986	1019	**946**
F42	**1070**	1865	2641	3090	4441	3112	2884
F43	76350	67635	61846	65180	68630	**63065**	63374
F44	139640	141660	149090	**114750**	116920	129763	123480
F45	NaN	NaN	NaN	NaN	NaN	NaN	NaN
F46	93073	75100	**49770**	52329	52557	52650	49817
F47	360500	269380	153790	141000	142550	**137716**	164060
F48	265720	207780	139640	**112640**	134110	138512	131210
F49	**61284**	71878	67175	62477	64171	68000	64211
F50	15666	13617	10132	12151	10004	**9064**	10373
F51	93605	46538	**39820**	59340	58838	61185	61664
F52	36646	35101	**31382**	31888	33748	33468	34199

Table A12 Results of SR, PR and SP obtained by the Wilcoxon test for SQP-MOA with $N_g = 3$ versus other values

$N_g = 3$	SR			PR			SP		
VS	R^+	R^-	p-value	R^+	R^-	p-value	R^+	R^-	p-value
N_g =1	687.5	638.5	0.81	684.0	642.0	0.84	390.0	936.0	1.00
N_g =5	691.5	634.5	0.78	697.0	681.0	0.94	782.5	595.5	0.39
N_g =7	784.5	541.5	0.25	785.0	541.0	0.25	921.0	405.0	< 0.05
N_g =10	784.5	541.5	0.25	783.5	542.5	0.26	999.0	327.0	< 0.05
N_g =20	689.5	636.5	0.80	692.0	686.0	0.97	1010.0	316.0	< 0.05
N_g =30	784.0	542.0	0.24	790.0	588.0	0.35	1098.0	228.0	< 0.05
N_g =40	691.0	635.0	0.79	743.0	635.0	0.62	1193.0	185.0	< 0.05
N_g =50	785.0	541.0	0.25	745.0	633.0	0.61	1284.0	42.0	< 0.05

Table A13 Influence of the number of global atoms (n_{g}) on the performance of SQP-MOA with respect to the success rate and peak ration

Prob. \ N_{g} / NOR		Success Rate									Peak Ratio								
		1	3	5	7	10	20	30	40	50	1	3	5	7	10	20	30	40	50
F13	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F15	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F18	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F35	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F37	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F01	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F03	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F04	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F06	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F08	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F09	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.400	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.680
F24	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F29	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F31	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F34	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F36	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F39	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F42	2	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.990	1.00	1.00	1.00	1.00
F02	3	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00
F05	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F07	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F17	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F38	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F26	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F32	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F33	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F41	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F50	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F40	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F23	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F27	6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F16	7	1.00	1.00	0.80	0.86	0.82	0.80	0.80	0.64	0.72	1.00	1.00	0.97	0.98	0.97	0.97	0.97	0.95	0.96
F25	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F14	8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F28	8	1.00	1.00	1.00	0.98	1.00	0.98	0.98	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F49	8	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F20	9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F19	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F46	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F10	11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F30	12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F12	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F21	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F11	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F22	16	0.98	1.00	0.94	0.96	0.98	0.96	0.96	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F47	16	0.90	0.84	0.82	0.82	0.84	0.86	0.82	0.78	0.80	0.99	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99
F51	18	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F43	24	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F48	37	0.88	0.96	0.98	0.94	0.90	0.96	0.94	0.96	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F52	44	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F44	46	0.96	0.96	0.98	0.84	0.92	0.96	0.94	1.00	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
F45	83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.92	0.91	0.90	0.91	0.91	0.91	0.92	0.91	0.91

Table A14 Influence of the number of global atoms (n_g) on the performance of SQP-MOA with respect to the success performance

Prob.	NOR	N_g=1	N_g=3	N_g=5	N_g=7	N_g=10	N_g=20	N_g=30	N_g=40	N_g=50
F13	1	**1260**	1860	2459	2880	3786	6735	9742	12717	16486
F15	1	**525**	562	718	843	1030	1553	2060	2735	3162
F18	1	**755**	1002	1398	1515	1844	2814	4094	4932	6186
F35	1	**166**	257	334	406	512	854	1259	1650	2047
F37	1	**1061**	1223	1595	1569	2036	2632	3478	3834	4702
F01	2	**151**	164	193	214	254	414	565	703	865
F03	2	10895	**10161**	8943	9599	11889	7941	8118	10603	10692
F04	2	**280**	300	342	425	463	630	835	987	1238
F06	2	**724**	734	895	924	1141	1349	1904	2424	2492
F08	2	355	362	**337**	399	442	660	872	951	1089
F09	2	**7544**	8365	9106	11394	14153	25130	35519	45978	132330
F24	2	**23233**	25130	24522	28282	26620	43105	52456	60783	75274
F29	2	**1067**	1478	1369	1201	1435	1843	2479	2291	2896
F31	2	**576**	617	665	752	695	920	1436	1465	1523
F34	2	**302**	323	343	379	450	612	764	979	1136
F36	2	**243**	264	310	323	438	656	846	980	1241
F39	2	**382**	400	457	459	493	723	835	951	1181
F42	2	**2053**	3632	4389	3919	3037	3090	2465	3390	3653
F02	3	**745**	822	869	791	880	1143	1410	1655	1711
F05	3	3467	3552	**2434**	4066	3063	3549	3120	3867	3618
F07	3	**455**	469	589	575	624	871	1065	1094	1384
F17	3	11651	**11101**	13298	11251	13371	11768	14195	16004	15356
F38	3	**549**	572	576	593	853	898	1027	1292	1446
F26	4	746	**724**	761	761	885	1090	1314	1472	1499
F32	4	803	**765**	888	883	929	1122	1291	1536	1545
F33	4	678	**667**	748	765	818	1099	1304	1302	1685
F41	4	**599**	707	744	687	798	962	1088	1206	1408
F50	4	**8044**	11067	10708	10053	9668	12151	9983	11370	11167
F40	5	5710	5347	**5045**	5746	5919	5795	6076	7311	7072
F23	6	**1477**	1971	2059	2186	1849	2053	2216	2392	2165
F27	6	1774	1398	**1360**	1733	1540	1778	2132	2355	2621
F16	7	**10389**	13201	18713	21933	17027	20417	23613	26607	23866
F25	7	**3659**	4017	4077	4001	4422	4584	4741	5501	5805
F14	8	3845	3913	3676	**3407**	4145	3825	3846	3726	4318
F28	8	**10984**	15894	17500	18891	16252	19335	17588	22800	19428
F49	8	59644	57325	**53659**	65819	68605	62477	64265	71991	67183
F20	9	5042	5174	4729	5658	5726	6156	**4527**	5299	5898
F19	10	7565	7641	7333	7314	8067	**6284**	7224	7319	9244
F46	10	56157	**50236**	53141	54183	47568	52329	53888	43638	50682
F10	11	4010	4090	**3705**	4043	4587	4271	4344	4403	4826
F30	12	**10056**	10728	10935	10259	11191	11044	10741	11288	10497
F12	13	4887	**3998**	4230	4165	4156	4374	4392	5009	5398
F21	13	3757	3412	**3200**	3393	3460	3524	3478	3807	3954
F11	15	5048	5124	5133	**4953**	5269	5314	5543	5639	5264
F22	16	54059	**53445**	55700	55434	56379	56496	54367	58940	60632
F47	16	**134730**	149910	159200	142778	143970	141000	156880	154940	158130
F51	18	**42727**	46338	45040	60602	43858	59340	53983	52068	57168
F43	24	**59520**	67688	64229	61603	65002	65180	67287	66610	72524
F48	37	134570	116910	116580	115919	120420	**112640**	127170	120410	114580
F52	44	32055	33292	31732	**31326**	35645	31888	32792	36076	33716
F44	46	123650	126980	125550	127672	126030	114750	115680	114310	142360
F45	83	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**6844200**	NaN

Table A15 Results of SR, PR and SP obtained by the Wilcoxon test for SQP-MOA with $N_1 = 2d$ versus other values

$N_1 = 2d$	SR			PR			SP		
VS	R^+	R^-	p-value	R^+	R^-	p-value	R^+	R^-	p-value
$N_1 = d$	686.0	640.0	0.82	685.0	641.0	0.83	340.0	986.0	1.00
$N_1 = 3d$	687.5	638.5	0.81	689.5	636.5	0.80	1102.0	224.0	< 0.05
$N_1 = 5d$	874.5	451.5	< 0.05	926.5	451.5	< 0.05	1178.0	148.0	< 0.05
$N_1 = 10d$	916.0	410.0	< 0.05	968.0	410.0	< 0.05	1251.0	75.0	< 0.05
$N_1 = 20d$	1044.5	333.5	< 0.05	1028.0	298.0	< 0.05	1321.0	5.0	< 0.05

Table A16 Influence of the number of local atoms (n_l) on the performance of SQP-MOA with respect to the success rate and peak ratio

Prob.	Success Rate						Peak Ratio					
	$N_l = d$	$N_l = 2d$	$N_l = 3d$	$N_l = 5d$	$N_l = 10d$	$N_l = 20d$	$N_l = d$	$N_l = 2d$	$N_l = 3d$	$N_l = 5d$	$N_l = 10d$	$N_l = 20d$
F01	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F02	**1.000**	**1.000**	**1.000**	0.980	**1.000**	0.960	**1.000**	**1.000**	**1.000**	0.993	**1.000**	0.987
F03	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F04	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F05	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F06	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F07	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F08	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F09	**1.000**	**1.000**	**1.000**	0.500	0.000	0.000	**1.000**	**1.000**	**1.000**	0.730	0.000	0.000
F10	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F11	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F12	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F13	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.180	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.180
F14	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F15	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F16	**0.900**	0.800	0.500	0.480	0.180	0.120	**0.986**	0.971	0.923	0.923	0.851	0.826
F17	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F18	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F19	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.980	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.998
F20	**1.000**	**1.000**	**1.000**	**1.000**	0.980	0.940	**1.000**	**1.000**	**1.000**	**1.000**	0.998	0.993
F21	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F22	**1.000**	0.960	0.960	0.860	0.240	0.020	**1.000**	0.998	0.998	0.991	0.940	0.783
F23	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F24	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.740	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.870**
F25	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F26	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F27	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F28	**1.000**	0.980	**1.000**	0.980	0.980	**1.000**	**1.000**	0.998	**1.000**	0.998	0.998	**1.000**
F29	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F30	**1.000**	**1.000**	**1.000**	**1.000**	0.980	0.920	**1.000**	**1.000**	**1.000**	**1.000**	0.998	0.993
F31	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F32	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F33	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F34	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F35	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F36	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F37	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F38	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F39	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F40	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.940	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.988
F41	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F42	**1.000**	**1.000**	0.980	0.420	0.000	0.000	**1.000**	**1.000**	0.990	0.640	0.000	0.000
F43	**1.000**	**1.000**	**1.000**	0.980	0.960	0.740	**1.000**	**1.000**	**1.000**	0.999	0.997	0.988
F44	0.940	**0.960**	0.940	0.840	0.580	0.340	**0.999**	**0.999**	**0.999**	0.996	0.988	0.977
F45	0.000	0.000	0.000	0.000	0.000	0.000	0.906	**0.911**	0.908	0.895	0.871	0.820
F46	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F47	0.740	0.860	**0.960**	0.800	0.820	0.420	0.984	0.991	**0.998**	0.986	0.989	0.958
F48	0.920	**0.960**	**0.960**	0.840	0.860	0.520	0.998	**0.999**	**0.999**	0.996	0.996	0.981
F49	**1.000**	**1.000**	**1.000**	**1.000**	0.980	0.980	**1.000**	**1.000**	**1.000**	**1.000**	0.998	0.998
F50	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F51	0.980	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.999	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F52	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.880	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.997

Table A17 Influence of the number of local atoms (n_l) on the performance of SQP-MOA with respect to the success performance

Prob.	$N_l = d$	$N_l = 2d$	$N_l = 3d$	$N_l = 5d$	$N_l = 10d$	$N_l = 20d$
F01	**251**	414	589	783	1338	2603
F02	**797**	1143	1392	1776	2826	4874
F03	9243	**7941**	9172	12946	14205	24649
F04	**464**	630	795	1186	2124	4086
F05	3840	3549	**3494**	3694	4275	6286
F06	**1117**	1349	1842	2426	4440	8422
F07	**691**	871	1011	1541	2508	3744
F08	**523**	660	779	1087	1612	2591
F09	**15332**	25130	35635	105783	NaN	NaN
F10	**3498**	4271	4369	4866	6950	8801
F11	**5163**	5314	5541	6456	8731	15133
F12	**3852**	4374	4911	6110	10169	17816
F13	**5864**	6735	8769	14342	29789	295700
F14	**2625**	3825	5019	5843	8448	14633
F15	**1196**	1553	1943	2923	4937	10056
F16	**19971**	20417	29122	30090	53933	88193
F17	12932	11768	15170	19003	20609	37703
F18	**2038**	2814	4096	6059	12279	23734
F19	**5866**	6284	8809	9035	12039	16974
F20	**4118**	6156	6654	8452	15560	24792
F21	**3340**	3524	3948	5386	8168	14576
F22	**52767**	56496	62052	83970	357587	4749100
F23	**1493**	2053	2356	3755	5651	11906
F24	**31291**	43105	49463	69195	118338	284830
F25	**3649**	4584	6202	7152	11762	23034
F26	**961**	1090	1269	1646	2691	5054
F27	**1652**	1778	1890	2511	4124	6512
F28	19478	**19335**	19403	23530	27300	32596
F29	**1382**	1843	2636	3437	5416	9978
F30	**9091**	11044	12338	13699	19797	33600
F31	**771**	920	1283	1585	2696	4599
F32	**895**	1122	1505	1781	3401	5977
F33	**883**	1099	1286	1644	2480	4833
F34	**463**	612	837	1103	1707	3249
F35	**611**	854	1143	1653	2951	6200
F36	**453**	656	750	991	1701	2639
F37	2747	2632	**2581**	3408	4860	7164
F38	**737**	898	1156	1639	2791	4714
F39	**484**	723	744	1155	2070	3662
F40	**5499**	5795	6372	9977	12518	20328
F41	**727**	962	1171	1572	2577	4333
F42	**1405**	3090	4523	13666	133749	NaN
F43	**63853**	65180	72262	75009	106346	172047
F44	124515	**114750**	133310	144857	260725	499117
F45	NaN	NaN	NaN	NaN	NaN	NaN
F46	50915	52329	47883	**40258**	53658	68807
F47	182195	141000	**123853**	148082	164998	347707
F48	116777	**112640**	119630	134571	150450	291269
F49	69948	62477	54318	60204	**51768**	59838
F50	10526	12151	**9162**	9605	12753	21573
F51	70442	59340	52522	**42575**	50929	67770
F52	**31873**	31888	36490	47539	72877	146080

Table A18 Results of SR, PR and SP obtained by the Wilcoxon test for SQP-MOA with $\beta = d$ versus other values

$\beta = d$	SR			PR			SP		
VS	R^+	R^-	p-value	R^+	R^-	p-value	R^+	R^-	p-value
$\beta = 2d$	741.0	637.0	0.63	789.0	589.0	0.36	698.0	628.0	0.74
$\beta = 3d$	782.0	544.0	0.26	785.0	541.0	0.25	737.0	589.0	0.48
$\beta = 4d$	692.0	634.0	0.78	738.5	587.5	0.48	813.0	513.0	0.16
$\beta = 5d$	836.5	541.5	0.17	882.5	495.5	0.08	896.5	429.5	< 0.05

Table A19 Influence of β on the performance of sqp-moa with respect to the success rate and peak ratio

Prob.	Success Rate					Peak Ratio				
	$N_1 = d$	$N_1 = 2d$	$N_1 = 3d$	$N_1 = 4d$	$N_1 = 5d$	$N_1 = d$	$N_1 = 2d$	$N_1 = 3d$	$N_1 = 4d$	$N_1 = 5d$
F1	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F2	**1.00**	0.98	**1.00**	**1.00**	**1.00**	**1.000**	0.993	**1.000**	**1.000**	**1.000**
F3	**1.00**	**1.00**	**1.00**	**1.00**	0.98	**1.000**	**1.000**	**1.000**	**1.000**	**0.990**
F4	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F5	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F6	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F7	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F8	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F9	**1.00**	**1.00**	0.98	**1.00**	**1.00**	**1.000**	**1.000**	**0.990**	**1.000**	**1.000**
F10	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F11	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F12	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F13	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F14	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F15	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F16	**0.80**	0.64	0.34	0.30	0.18	**0.971**	0.949	0.897	0.880	0.869
F17	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F18	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F19	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F20	**1.00**	**1.00**	**1.00**	**1.00**	0.96	**1.000**	**1.000**	**1.000**	**1.000**	**0.996**
F21	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F22	0.96	**1.00**	0.90	0.66	0.56	0.998	**1.000**	0.993	0.974	0.971
F23	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F24	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F25	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F26	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F27	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F28	0.98	**1.00**	0.98	**1.00**	0.94	0.998	**1.000**	0.998	**1.000**	0.993
F29	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F30	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F31	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F32	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F33	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F34	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F35	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F36	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F37	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F38	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F39	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F40	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F41	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F42	**1.00**	0.84	0.38	0.02	0.00	**1.000**	**0.920**	**0.630**	**0.190**	**0.040**
F43	**1.00**	**1.00**	**1.00**	**1.00**	0.96	**1.000**	**1.000**	**1.000**	**1.000**	**0.998**
F44	**0.96**	0.90	0.94	0.88	0.82	**0.999**	0.997	**0.999**	0.997	0.994
F45	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.911**	0.560	0.499	0.446	0.405
F46	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F47	0.86	0.86	0.94	**0.98**	0.90	0.991	0.991	0.995	**0.999**	0.994
F48	0.96	**0.98**	0.92	**0.98**	**0.98**	**0.999**	**0.999**	0.998	**0.999**	**0.999**
F49	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F50	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
F51	**1.00**	0.98	**1.00**	**1.00**	**1.00**	**1.000**	0.999	**1.000**	**1.000**	**1.000**
F52	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

Table A20 Influence of β on the performance of sqp-moa with respect to the success performance

Prob.	$\beta = d$	$\beta = 2d$	$\beta = 3d$	$\beta = 4d$	$\beta = 5d$
F1	414	**239**	302	356	402
F2	1143	924	**899**	994	1266
F3	**7941**	9008	12043	10554	16765
F4	630	**368**	451	511	618
F5	3549	3957	**2339**	2750	3150
F6	1349	**974**	1137	1266	1342
F7	871	**623**	718	898	942
F8	660	**383**	444	585	584
F9	25130	**14685**	16669	21485	24493
F10	4271	**4239**	4484	4294	5076
F11	5314	**5075**	6424	6817	7446
F12	**4374**	4885	5967	6932	7474
F13	6735	**2652**	3304	4115	4669
F14	**3825**	4870	5427	6600	6729
F15	1553	**789**	864	1064	1089
F16	**20417**	22627	37076	38265	34680
F17	**11768**	14144	16551	15998	19734
F18	2814	**1374**	1562	1923	2253
F19	**6284**	7175	7984	8547	8669
F20	**6156**	7061	9046	11723	11781
F21	**3524**	4012	4572	5579	6227
F22	**56496**	59500	79454	105699	134605
F23	**2053**	2452	3044	5445	5364
F24	43105	**24632**	26994	32952	42438
F25	**4584**	4940	6598	7051	7375
F26	1090	**883**	1085	1309	1321
F27	1778	**1466**	1648	2032	2159
F28	**19335**	24216	29672	26891	33151
F29	1843	**1530**	1966	1890	1851
F30	**11044**	11695	12755	13944	14548
F31	920	866	**769**	963	990
F32	1122	**1053**	1204	1511	1779
F33	1099	**809**	1015	1142	1355
F34	612	400	**387**	523	609
F35	854	**339**	430	516	621
F36	656	**348**	421	562	494
F37	2632	**975**	1128	1198	1163
F38	898	**686**	824	1049	1250
F39	723	**554**	752	1045	1092
F40	**5795**	7126	7041	9369	10560
F41	962	**789**	948	1195	1333
F42	**3090**	6177	12294	25582	38472
F43	**65180**	73781	76799	82709	96685
F44	**114750**	133276	125720	150477	162332
F45	NaN	NaN	NaN	NaN	NaN
F46	52329	42377	39687	**38860**	39329
F47	141000	133144	120712	**112604**	112707
F48	112640	115494	117005	**107086**	107364
F49	62477	64031	55523	57360	**54511**
F50	12151	**10593**	10956	11129	10770
F51	59340	**42318**	44614	46435	47465
F52	**31888**	38903	42521	56475	57247

Appendix B New Testing Functions

$f(x_1, x_2) = |f_1(x_1, x_2)| + |f_2(x_1, x_2)|$

F43:

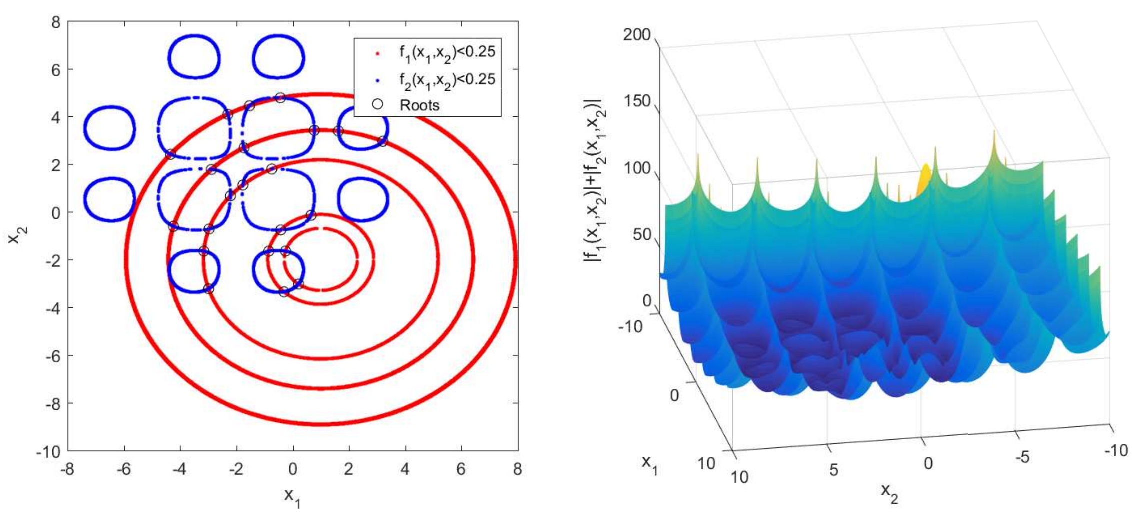
$$\begin{cases} f_1(x_1,x_2) = 100 \times (0.05 - \frac{1+\cos\left(2\sqrt{(x_1-1)^2+(x_2+2)^2}\right)}{0.5\left((x_1-1)^2+(x_2+2)^2\right)+2}) = 0 \\ f_2(x_1,x_2) = 100 \times (-0.0001(\left|(\sin(x_1+2)\sin(x_2-2)\exp(|100 - \frac{\sqrt{(x_1+2)^2+(x_2-2)^2}}{\pi}|)\right| + 1)^{0.1} + 1.8) = 0 \end{cases} \tag{B1}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 24 roots as shown in Table B1. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B1. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B1.

Table B1 The roots of F43

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
1.02E-06	-4.336363124	2.402609879	4.42E-07	-1.767968137	1.11761607	8.88E-06	-0.253675835	-1.653095838
7.31E-07	-4.233371046	-0.614169683	4.85E-07	-1.728925712	2.675645579	4.62E-06	0.207429212	-3.031444645
4.16E-06	-3.154860357	-1.656031027	1.25E-06	-1.529653954	4.438990552	1.97E-06	0.651971698	-0.149487294
2.80E-08	-2.983214909	-3.230925006	2.02E-06	-0.851556123	-1.65756627	1.10E-06	0.771146672	3.408911392
1.66E-05	-2.969376772	-0.725159458	7.15E-09	-0.739316553	1.788925349	1.76E-06	1.625369961	3.377509456
1.80E-06	-2.886777615	1.768508372	1.28E-07	-0.435397463	4.767523957	1.19E-07	3.20301237	2.945243573
9.08E-06	-2.301405038	4.079513612	2.83E-06	-0.422960237	-0.766831216			
9.61E-07	-2.211739418	0.658177379	1.10E-06	-0.306533973	-3.355909229			

Figure B1 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F43



F44:

$$\begin{cases} f_1(x_1,x_2) = 2(0.1((x_1+3)^2 + (x_2+6)^2) - 10\cos(\pi(x_1+3)) - 10\cos(\pi(x_2+6)) - 5) = 0 \\ f_2(x_1,x_2) = 100 \times (-0.0001(\left|(\sin(x_1+2)\sin(x_2-2)\exp(|100 - \frac{\sqrt{(x_1+2)^2+(x_2-2)^2}}{\pi}|)\right| + 1)^{0.1} + 1.8) = 0 \end{cases} \tag{B2}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 46 roots as shown in Table B2. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B2. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B2.

F45:

$$\begin{cases} f_1(x_1,x_2) = 2(0.1((x_1+3)^2 + (x_2+6)^2) - 10\cos(\pi(x_1+3)) - 10\cos(\pi(x_2+6)) - 5) = 0 \\ f_2(x_1,x_2) = 100 \times (0.05 - \frac{1+\cos\left(2\sqrt{(x_1-1)^2+(x_2+2)^2}\right)}{0.5\left((x_1-1)^2+(x_2+2)^2\right)+2}) = 0 \end{cases} \tag{B3}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 83 roots as shown in Table B3. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B3. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B3.

F46:

$$\begin{cases} f_1(x_1,x_2) = 0.5(\left(\sum_{i=1}^{5} i\cos((i+1)x_1+i)\right)\left(\sum_{i=1}^{5} i\cos((i+1)x_2+i)\right) - 70) = 0 \\ f_2(x_1,x_2) = 100 \times (-0.0001(\left|(\sin(x_1+2)\sin(x_2-2)\exp(|100 - \frac{\sqrt{(x_1+2)^2+(x_2-2)^2}}{\pi}|)\right| + 1)^{0.1} + 1.8) = 0 \end{cases} \tag{B4}$$

Table B2 The roots of F44

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
2.90E-06	-7.374723303	0.583296186	0.000206	-3.690246394	2.232356632	6.60E-06	-0.307388102	1.767396329
1.56E-06	-7.373605265	3.57078715	3.93E-08	-3.51612125	-3.376649148	1.50E-06	-0.254051455	-1.653016807
4.00E-06	-7.048869521	1.201820628	2.00E-05	-3.346494513	5.627306668	4.87E-06	0.057472983	-1.772808436
4.64E-06	-6.71728438	1.350591658	6.81E-06	-3.324772572	-0.769196557	8.51E-06	0.385090433	-2.470556518
1.16E-05	-6.603313812	4.373499912	1.77E-05	-2.618399065	6.276772714	3.66E-07	0.443404588	-0.467634046
7.79E-07	-6.35762299	-0.382495471	4.26E-05	-2.447164379	2.379221077	4.89E-06	0.55857225	4.329412825
2.08E-06	-5.872183868	-0.181684683	3.66E-06	-2.425036076	1.601843929	6.08E-06	0.766706661	0.724858987
3.52E-06	-5.633397792	0.375002487	1.23E-06	-2.235633126	0.279570309	5.62E-06	0.771202581	3.428959568
1.87E-05	-4.770949873	3.389905904	4.42E-06	-1.763980413	0.276168551	8.48E-06	1.641435439	0.316794182
6.89E-06	-4.763635392	0.768735252	1.06E-05	-1.571923602	1.604600527	4.44E-06	1.821003599	-0.125266687
3.18E-05	-4.544726383	4.349652445	6.05E-05	-1.549592123	2.376651325	3.65E-07	2.421839384	-0.385403394
1.02E-06	-4.42672263	-0.483156493	1.62E-05	-1.380640811	6.268386707	4.37E-06	2.659201843	4.36384984
1.13E-06	-4.383846285	-2.500484021	6.47E-06	-0.694429944	-0.768354171	7.13E-06	3.365581571	3.639575533
1.69E-06	-4.005218992	-1.742916682	3.56E-06	-0.671455987	5.628744436	4.29E-07	3.3770838	0.514509392
6.43E-06	-3.794826205	-1.664339098	9.58E-07	-0.50253982	-3.377022422			
1.21E-06	-3.717344215	1.76471435	6.65E-06	-0.33276137	2.230038045			

Figure B2 The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$, and the landscape of $f(x_1,x_2)$ for F44

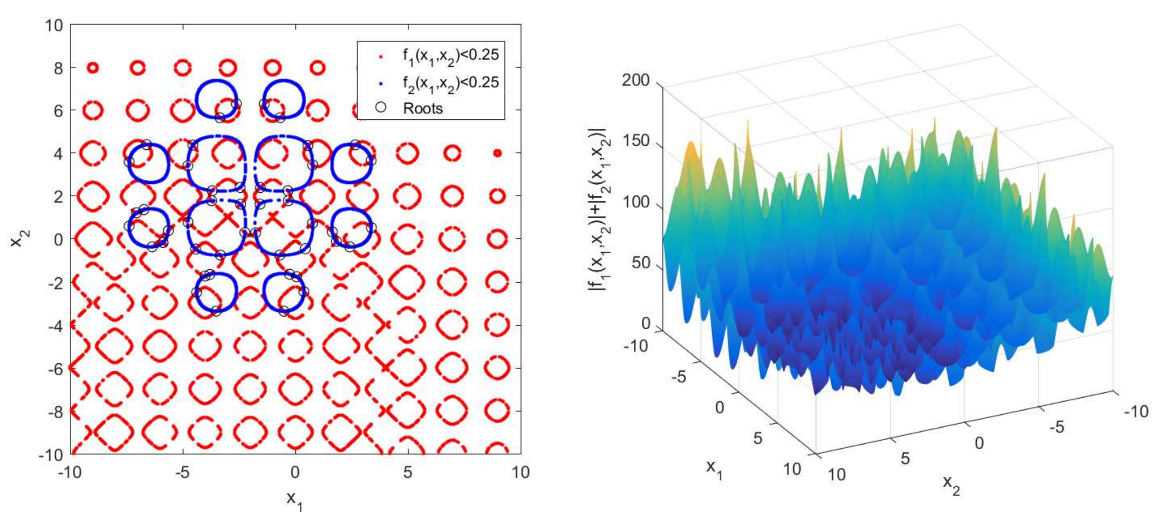


Table B3 The roots of F45

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
2.46E-06	-5.913899628	-1.759711624	4.29E-05	-0.703480131	-1.197705692	2.95E-05	2.78934569	-7.109495026
1.82E-05	-5.913242165	-2.258513713	3.56E-05	-0.681610901	-2.847175007	6.82E-06	3.15580896	-6.966002933
4.12E-06	-5.73145167	-3.596027877	8.13E-06	-0.648584173	-7.156633362	4.81E-06	3.312883132	1.468681162
8.05E-06	-5.712893932	-0.327635916	1.04E-05	-0.596027677	-8.731451717	0.0002192	3.421586967	-5.393684419
1.52E-06	-5.416555187	0.586032888	0.0003228	-0.253727626	-1.653282172	1.43E-05	3.586032554	-8.416555323
4.19E-06	-5.142466907	1.182742036	0.0002963	-0.249094822	-2.363054161	6.64E-05	4.182743579	-8.142466108
4.00E-05	-4.399576619	2.324849005	1.82E-05	-0.21341665	1.988583383	0.0005228	4.302969966	-6.289415686
1.06E-05	-4.392144123	-1.51680319	5.65E-06	0.152825743	-3.68161128	0.0002335	4.41571636	-4.390409824
8.54E-08	-4.390019388	-2.506348096	2.51E-05	0.245584067	-0.27478195	2.31E-05	4.454953712	2.167972199
0.0001303	-4.290119147	-6.458069765	1.34E-05	0.259584023	2.102799234	4.23E-05	4.468681493	0.312882633
4.06E-05	-4.156633144	-3.648584855	6.05E-05	0.493652806	-7.390019473	2.66E-06	4.521253363	3.954873745
3.86E-05	-4.109497635	-0.21065608	0.0003259	0.636946534	-3.249086883	7.59E-05	4.959835081	-0.695820754
1.08E-05	-3.966002669	0.155809568	0.0007662	0.699228168	-0.734445038	7.42E-05	4.988584079	-3.213414365
3.27E-05	-3.878723272	-4.346647959	1.82E-05	0.741486287	-8.913242165	2.00E-05	5.102799263	-2.740415814
1.95E-05	-3.458068322	-7.290120359	7.95E-06	0.79983929	3.410049269	3.45E-05	5.111634745	-1.310335516
2.19E-05	-3.383570761	-5.176948975	2.42E-05	1.183874154	3.410627292	9.15E-05	5.130360594	3.549762872
1.39E-05	-3.289410902	1.30297616	5.77E-06	1.2402883	-8.913899631	8.69E-05	5.167973233	1.454952465
8.92E-06	-3.289354618	3.4278161	0.0004004	1.285252648	-0.73087545	0.0002405	5.263076638	-5.33689601
1.96E-05	-2.393685763	0.421585083	1.67E-05	1.483196722	-7.392144131	0.0003479	5.324852724	-7.39957364
1.60E-07	-2.344567156	4.055874595	5.29E-06	1.689665283	2.111634611	0.0001212	6.410049157	-2.200163727
0.0001888	-2.339045411	-4.496388014	1.28E-05	1.718409862	-0.259480541	2.07E-05	6.410626577	-1.816127288
7.10E-05	-2.336892689	2.263079237	2.11E-05	1.802294655	-3.703479968	0.0001089	6.427815051	-6.28935547
7.58E-05	-2.176947691	-6.383571691	0.000207	2.265540822	-2.300753432	2.63E-05	6.549764097	2.130358949
0.0001111	-1.496387264	-5.339045972	0.0002723	2.269127034	-1.714758402	3.16E-06	6.954873685	1.521253465
2.47E-05	-1.390407183	1.415718141	3.76E-05	2.304176863	1.959835868	3.42E-05	7.055874909	-5.344566626
2.67E-07	-1.346647203	-6.878723635	2.88E-05	2.672364659	-8.712893789	9.27E-07	7.572414553	-4.159424348
5.40E-07	-1.159424399	4.572414536	2.33E-05	2.72521771	-2.754416708	5.07E-06	7.603126204	-4.06360629
2.14E-07	-1.06360687	4.603126022	0.0001292	2.740520315	-1.281592211			

Figure B3 The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$, and the landscape of $f(x_1,x_2)$ for F45

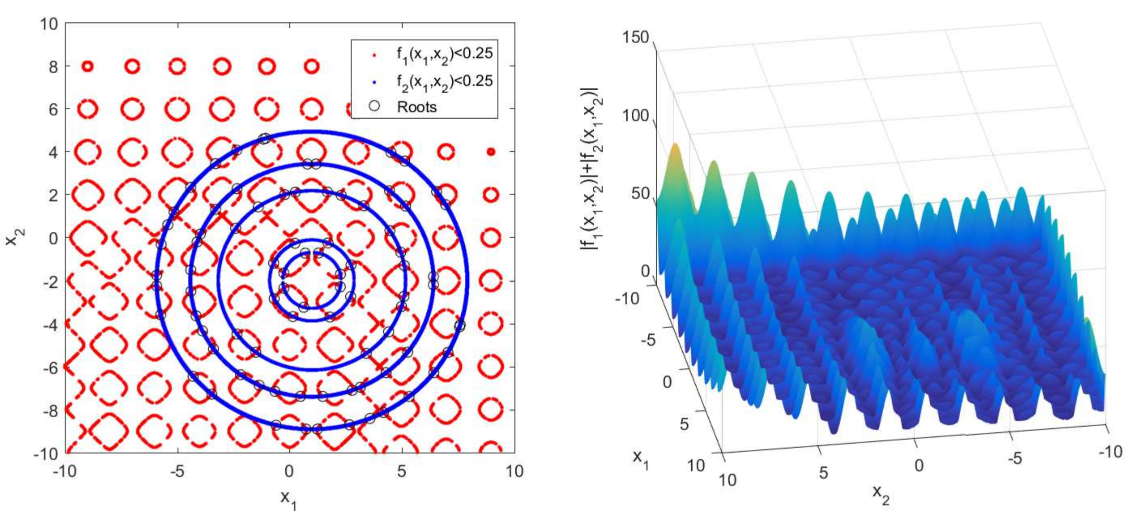
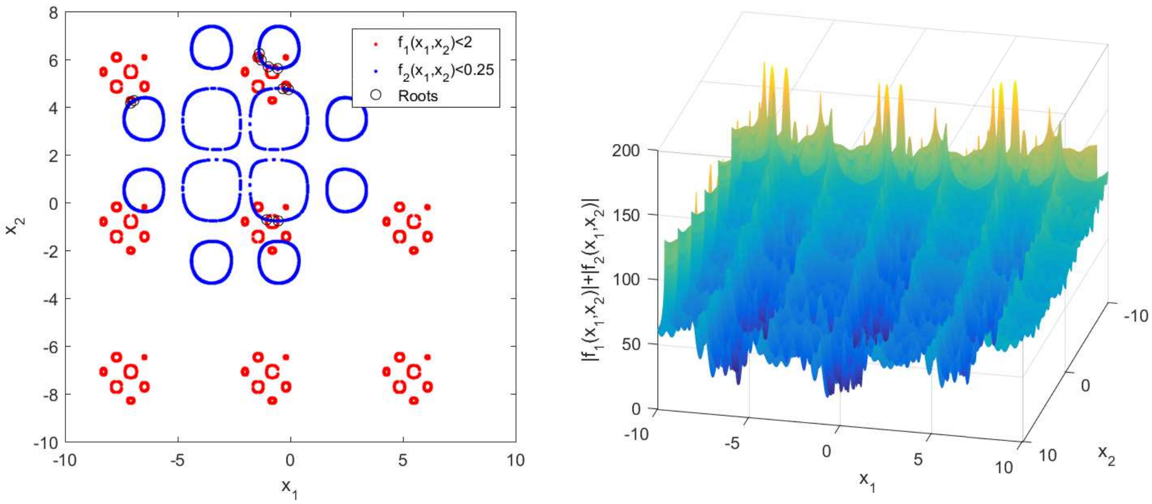


Figure B4 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F46



where $x_i \in [-10, 10], i = 1, 2$. It has 10 roots as shown in Table B4. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B4. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B4.

Table B4 The roots of F46

$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2
1.80E-06	-7.089828011	4.161361768	2.97E-05	-1.046360472	-0.721506419	9.11E-07	-0.328606118	4.759566677
1.13E-06	-6.944071566	4.262279529	6.87E-05	-0.980085671	5.698630697	1.53E-05	-0.079413799	4.721148389
6.63E-05	-1.378233591	6.249487291	7.39E-06	-0.560117197	5.623500564			
3.67E-05	-1.295417236	5.97732307	2.34E-05	-0.538130037	-0.770958863			

F47:

$$\begin{cases} f_1(x_1, x_2) = 0.5(\left(\sum_{i=1}^{5} i\cos\left((i+1)x_1 + i\right)\right)\left(\sum_{i=1}^{5} i\cos\left((i+1)x_2 + i\right)\right) - 70) = 0 \\ f_2(x_1, x_2) = 100 \times (0.05 - \frac{1+\cos\left(2\sqrt{(x_1-1)^2+(x_2+2)^2}\right)}{0.5\left((x_1-1)^2+(x_2+2)^2\right)+2}) = 0 \end{cases} \tag{B5}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 16 roots as shown in Table B5. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B5. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B5.

Table B5 The roots of F47

$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2
2.74E-05	-1.047360422	-7.011687611	8.23E-05	-0.087849258	-1.286816861	4.01E-05	5.409960776	-7.330290098
6.13E-05	-0.881143418	-1.917415773	6.50E-05	-0.029508992	4.841042158	1.31E-05	5.743658989	-7.035617734
2.97E-05	-0.880153986	-2.102672797	5.45E-05	4.663102633	-7.868681766	8.61E-06	6.086660641	-0.14673776
9.63E-06	-0.549021016	-7.187410847	2.78E-05	4.708098104	-0.094432939	3.28E-05	6.117343184	-0.233224365
9.45E-06	-0.338394513	4.787374065	0.000149	4.826834887	-0.345759353			
1.48E-05	-0.238382614	-1.601936223	4.23E-05	5.063072964	-7.599212861			

F48:

$$\begin{cases} f_1(x_1, x_2) = 0.5(\left(\sum_{i=1}^{5} i\cos\left((i+1)x_1 + i\right)\right)\left(\sum_{i=1}^{5} i\cos\left((i+1)x_2 + i\right)\right) - 70) = 0 \\ f_2(x_1, x_2) = 2(0.1((x_1+3)^2 + (x_2+6)^2) - 10\cos(\pi(x_1+3)) - 10\cos(\pi(x_2+6)) - 5) = 0 \end{cases} \tag{B6}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 37 roots as shown in Table B6. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B6. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B6.

F49:

$$\begin{cases} f_1(x_1, x_2) = 10 \times (-20\exp\left(-0.2\sqrt{\frac{1}{2}\sum_{i=1}^{2}(x_i+3)^2}\right) - \exp(\frac{1}{2}\sum_{i=1}^{2}\cos(\pi \times (x_i+3))) + 10 + \exp(1)) = 0 \\ f_2(x_1, x_2) = 0.5(\left(\sum_{i=1}^{5} i\cos\left((i+1)x_1 + i\right)\right)\left(\sum_{i=1}^{5} i\cos\left((i+1)x_2 + i\right)\right) - 70) = 0 \end{cases} \tag{B7}$$

where $x_i \in [-10, 10], i = 1, 2$. It has 8 roots as shown in Table B7. The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are illustrated in the left part of Fig.B7. The landscape of $f(x_1, x_2)$ is illustrated in the right part of Fig.B7.

Figure B5 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F47

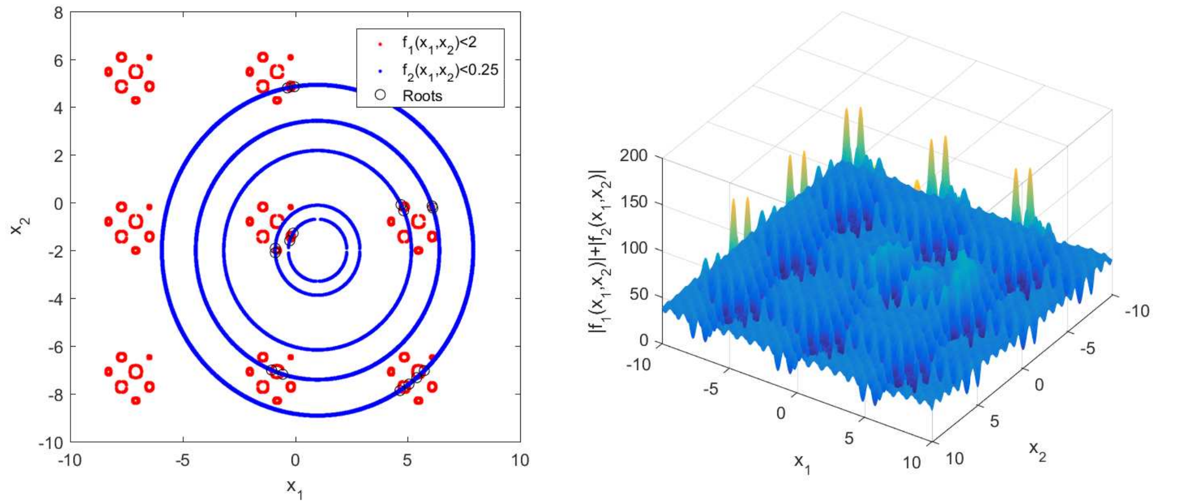


Table B6 The roots of F48

$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2
8.17E-06	-7.941734863	-7.774303415	1.93E-06	-6.851333787	-0.955086091	6.02E-06	-0.57623592	-0.627466791
7.91E-07	-7.845065769	-0.079257184	0.000218	-6.823759529	5.540691815	4.57E-06	-0.292039713	-7.565941728
1.91E-05	-7.788874424	-6.326534467	8.49E-06	-6.613428599	-1.333153454	1.04E-06	-0.197820626	-0.242761518
2.20E-07	-7.698912466	-1.664389125	1.42E-06	-6.368610169	-1.567697938	8.41E-07	-0.146769615	-0.197465646
1.95E-05	-7.646612669	-0.340380336	1.85E-06	-1.641879185	-1.547228924	3.60E-06	-0.02891363	-7.710198096
1.08E-04	-7.592024735	-7.505933326	1.46E-05	-1.511937404	5.949895641	8.20E-06	4.621663463	-1.472247858
2.59E-05	-7.548322888	-6.54436818	3.47E-06	-1.39464234	6.252299606	1.87E-05	4.683727077	6.036398163
4.69E-05	-7.488707701	-1.459209838	5.46E-06	-1.321838816	-1.218192062	3.10E-06	4.8741065	6.25373788
8.86E-06	-7.323675609	-7.1912259	8.84E-06	-1.051371945	5.456401087	1.46E-06	5.050984037	-1.288340082
7.52E-06	-7.323136423	-0.688614561	1.64E-06	-0.785138326	-1.051781658	2.76E-06	5.50524938	-8.403792006
9.73E-06	-7.301792495	5.651978577	4.54E-05	-0.648776089	-6.847966566	6.91E-06	5.620781622	-8.268413498
4.27E-06	-7.28334308	-6.887281591	1.32E-05	-0.612561154	5.696469146			
1.27E-05	-7.140138287	-1.049110774	2.74E-06	-0.582681455	-7.260889386			

Figure B6 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F48

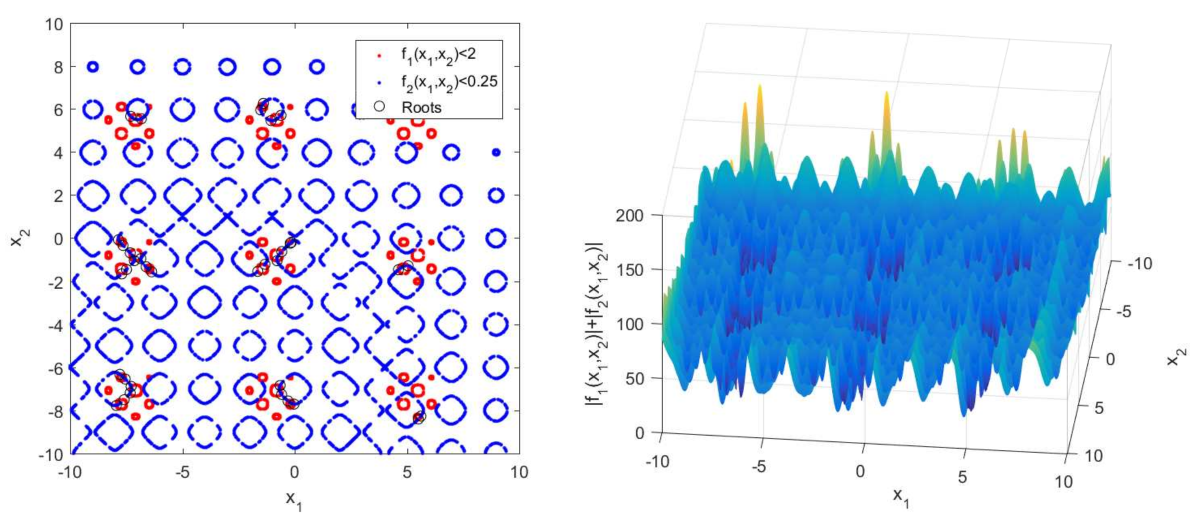
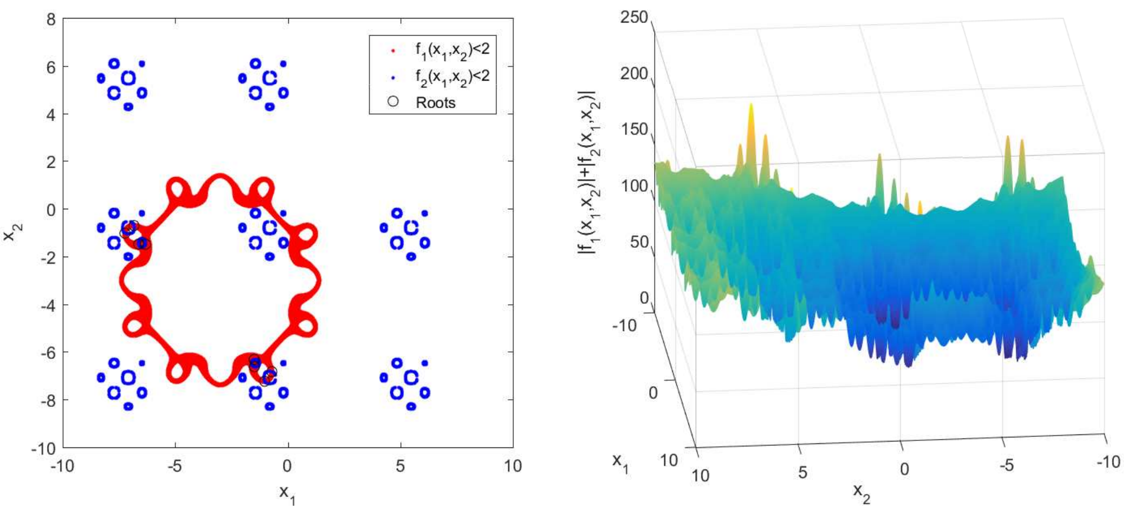


Table B7 The roots of F49

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
1.80E-05	-7.233232878	-1.026323681	2.40E-07	-6.320614194	-1.48797987	2.57E-06	-1.026323559	-7.233233108
3.85E-06	-6.829503323	-0.706683976	1.67E-06	-1.487979612	-6.320614121	3.21E-06	-0.706683937	-6.829503336
5.17E-08	-6.624952122	-1.481772356	2.23E-07	-1.481772341	-6.624952125			

Figure B7 The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$, and the landscape of $f(x_1,x_2)$ for F49



F50:

$$\begin{cases} f_1(x_1,x_2) = 10\times(-20\exp\left(-0.2\sqrt{\frac{1}{2}\sum_{i=1}^{2}(x_i+3)^2}\right) - \exp(\frac{1}{2}\sum_{i=1}^{2}\cos(\pi\times(x_i+3))) + 10 + \exp(1)) = 0 \\ f_2(x_1,x_2) = 100\times(-0.0001(\left|(\sin(x_1+2)\sin(x_2-2)\exp(|100 - \frac{\sqrt{(x_1+2)^2+(x_2-2)^2}}{\pi}|)\right| + 1)^{0.1} + 1.8) = 0 \end{cases} \tag{B8}$$

where $x_i \in [-10,10], i=1,2$. It has 4 roots as shown in Table B8. The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ are illustrated in the left part of Fig.B8. The landscape of $f(x_1,x_2)$ is illustrated in the right part of Fig.B8.

Table B8 The roots of F50

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
7.19E-07	-4.695376769	1.140668856	2.09E-07	-1.760694807	0.248429147
2.40E-08	-2.217108154	0.506000948	4.74E-07	-0.091506947	-0.723760586

F51:

$$\begin{cases} f_1(x_1,x_2) = 10\times(-20\exp\left(-0.2\sqrt{\frac{1}{2}\sum_{i=1}^{2}(x_i+3)^2}\right) - \exp(\frac{1}{2}\sum_{i=1}^{2}\cos(\pi\times(x_i+3))) + 10 + \exp(1)) = 0 \\ f_2(x_1,x_2) = 100\times(0.05 - \frac{1+\cos\left(2\sqrt{(x_1-1)^2+(x_2+2)^2}\right)}{0.5((x_1-1)^2+(x_2+2)^2)+2}) = 0 \end{cases} \tag{B9}$$

where $x_i \in [-10,10], i=1,2$. It has 18 roots as shown in Table B9. The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ are illustrated in the left part of Fig.B9. The landscape of $f(x_1,x_2)$ is illustrated in the right part of Fig.B9.

F52:

$$\begin{cases} f_1(x_1,x_2) = 10\times(-20\exp\left(-0.2\sqrt{\frac{1}{2}\sum_{i=1}^{2}(x_i+3)^2}\right) - \exp(\frac{1}{2}\sum_{i=1}^{2}\cos(\pi\times(x_i+3))) + 10 + \exp(1)) = 0 \\ f_2(x_1,x_2) = 2(0.1((x_1+3)^2+(x_2+6)^2) - 10\cos(\pi(x_1+3)) - 10\cos(\pi(x_2+6)) - 5) = 0 \end{cases} \tag{B10}$$

where $x_i \in [-10,10], i=1,2$. It has 44 roots as shown in Table B10. The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ are illustrated in the left part of Fig.B10. The landscape of $f(x_1,x_2)$ is illustrated in the right part of Fig.B10.

Figure B8 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F50

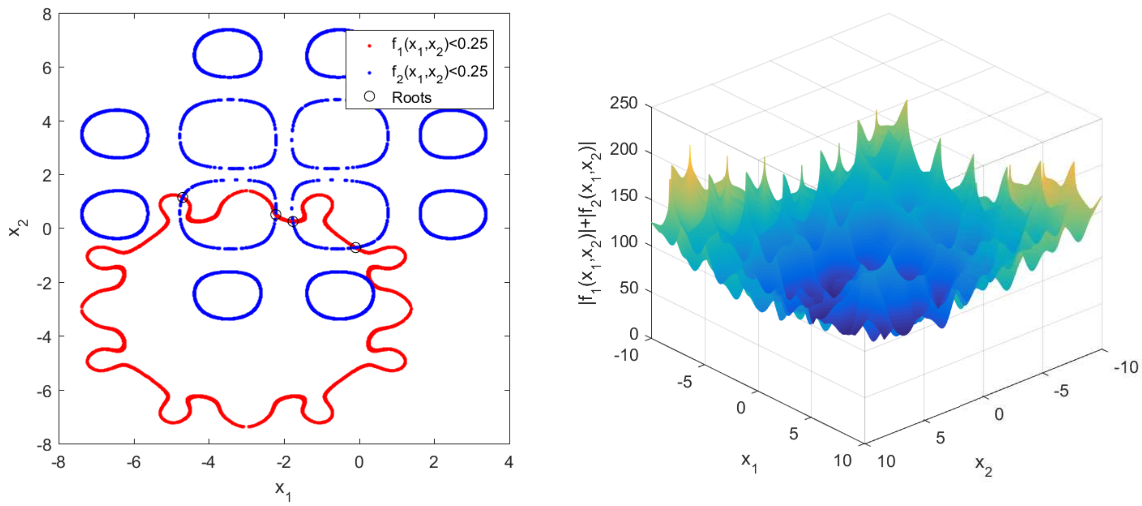


Table B9 The roots of F51

$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	x_1	x_2
1.13E-07	-5.776723497	-0.608682201	4.39E-07	-3.302215846	1.286279997	1.33E-06	-0.245744612	-0.588036446
4.85E-07	-5.295823031	0.867465593	1.46E-07	-2.257619631	0.601747819	1.16E-07	0.285631854	-0.912928543
3.16E-07	-5.15277984	1.162759367	2.86E-07	-2.14163522	-6.408948349	2.72E-08	0.468682169	-3.80643906
1.44E-07	-4.440507967	-6.273244635	6.98E-07	-1.501884173	-6.800965839	2.25E-06	0.739018473	-0.72566298
6.61E-08	-3.837804099	0.429886614	3.57E-07	-0.790659585	-7.109034709	2.71E-06	0.885614191	-0.704252276
1.26E-07	-3.735873978	0.623012593	6.81E-07	-0.653887161	-5.826987671	2.76E-06	1.309869024	-3.263339774

Figure B9 The roots of $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$, and the landscape of $f(x_1, x_2)$ for F51

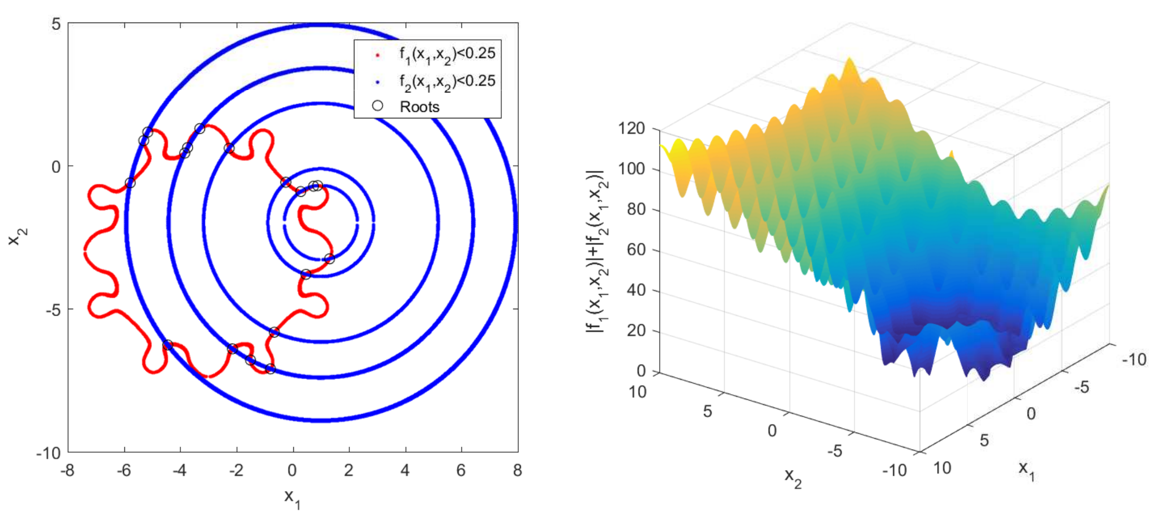
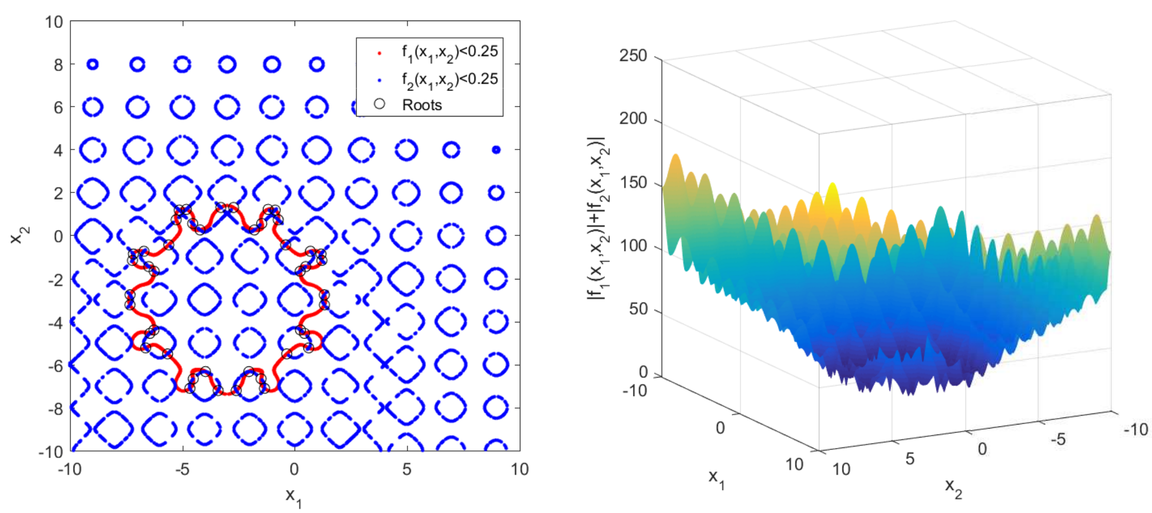


Table B10 The roots of F52

$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2	$f(x_1,x_2)$	x_1	x_2
7.57E-06	-7.327857668	-3.229681334	4.79E-06	-4.659300826	-7.110867006	4.97E-07	-0.865313647	1.175803911
1.88E-05	-7.323792863	-2.762325888	2.16E-06	-4.536318855	0.551481519	5.27E-07	-0.729246007	0.727490916
1.06E-06	-7.215463678	-1.16535432	1.26E-07	-4.509086478	-6.667151243	2.43E-07	-0.399768593	-0.440310908
1.97E-06	-7.176229592	-0.865945296	1.01E-08	-4.206525714	0.252178025	9.17E-07	-0.348736812	-5.509620638
5.99E-06	-6.702309889	-0.737877085	1.07E-07	-3.965839721	-6.331612288	1.58E-08	0.247956812	-1.668415144
1.73E-06	-6.62642934	-5.230344677	1.07E-05	-3.40207529	-7.21188764	8.28E-08	0.25865243	-4.39647561
2.56E-07	-6.494045712	-1.451329573	5.46E-07	-3.28418415	1.297593571	1.42E-07	0.438956153	-4.554453718
7.48E-07	-6.438956203	-4.554453764	3.75E-07	-2.715815854	1.297593572	1.93E-07	0.494045743	-1.451329544
1.03E-06	-6.258652307	-4.396475515	1.94E-06	-2.597924868	-7.211887388	1.25E-06	0.626429385	-5.230344605
4.14E-08	-6.247956814	-1.668415143	6.53E-08	-2.034160303	-6.331612292	2.08E-07	0.702309758	-0.737876938
9.14E-06	-5.65126335	-5.509620796	7.45E-07	-1.79347436	0.252177967	4.21E-07	1.176229616	-0.865945264
4.37E-07	-5.600231396	-0.440310917	2.74E-06	-1.490913373	-6.667151412	1.30E-06	1.215463683	-1.165354325
1.04E-07	-5.270754004	0.727490906	3.57E-06	-1.463680739	0.551481908	6.66E-06	1.323792683	-2.762326126
4.79E-07	-5.134686353	1.175803911	2.30E-06	-1.340699097	-7.110866802	2.23E-06	1.32785759	-3.229681231
1.84E-06	-4.81923501	1.210030554	2.60E-06	-1.180764893	1.210030464			

Figure B10 The roots of $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$, and the landscape of $f(x_1,x_2)$ for F52



References

1 Liao Z W, Gong W Y, Yan X S, et al. Solving nonlinear equations system with dynamic repulsion-based evolutionary algorithms. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, (99):1-12

2 Gao W F, Yen G G, and Liu S Y. A dual-population differential evolution with coevolution for constrained optimization. IEEE Transactions on Cybernetics, 2015, 45(5):1108-1121.

3 García S, Molina D, Lozano M, et al. A study on the use of non-parametric tests for analyzing the evolutionary algorithms behaviour: a case study on the cec 2005 special session on real parameter optimization. Journal of Heuristics, 2009, 15(6):617