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Communication and performance evaluation of 3-ary n-cubes onto network-on-chips

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Dear editor,

Network-on-chip (NoC) has the advantages of highly integrated, ultralow-power, low cost and small volume, and it has become one of the mainstreams of VLSI system design [1, 2]. Wirelength and layout area are parameters significantly affecting NoC by the limitation of chip area. Aiming at the bottleneck of big data applications, Chittamuru et al. [3] proposed a multi-core chip platform BiG-NoC, which uses a new type of dedicated photonic network on-chip (PNoC) structure. Gu et al. [4] proposed a novel optical network-on-chip architecture with superior performance and scalability. They introduced wavelength division multiplexing (WDM) into ONoC to make it have efficient wavelength allocation. Furthermore, Gu et al. [5] proposed an optical network-on-chip architecture based on time division multiplexing (TDM) and wavelength division multiplexing technology for solving the competition problem of ONoC. Yang et al. [6] proposed a ring-based passive ONoC structure TAONoC, which can be well adapted to the layout of on-chip multi-core processors.

Q_n^3 structure has the advantages of good symmetry, short network diameter and good scalability. Q_n^3 has been used in some parallel computers, such as the Blue Gene/L and Cray XT5 supercomputers [7]. Compared with mesh, the Q_n^3 has more physical links and shorter network diameter, and thus it has better performance than mesh. 3D packaging technology can shorten the communication distance between IP cores on chip, reduce average delay and power consumption. In this work, we study the communication and performance evaluation of the Q_n^3 onto 3D Network-on-Chip.

Topology mapping model. All graphs are referred to simple graphs in this study. For a graph G, let $V(G)$ and $E(G)$ denote the vertex set and edge set respectively and $P = (x_0, x_1, \ldots, x_{l-1})$ denote an (x, y)-path from vertex x to vertex y with length l. Let G_1 and G_2 be two graphs. If G_1 is isomorphic to G_2, we write $G_1 \cong G_2$. Let f be an injective mapping from $V(G_1)$ to $V(G_2)$. For a non-empty subset U, let $S = \{v \in V(G_2) | \text{there exists } u \in U, \text{ such that } v = f(u)\}$. Then we write $S = f(U)$ and $U = f^{-1}(S)$. Furthermore, let G' be a subgraph of G_1, and $E' = \{(u, v) \in E(G_2) | u, v \in f(V(G'))\}$. Then we write $f(G') = f(V(G'), E')$ and $G' = f^{-1}(f(V(G_1), E')$. In this study, we define an embedding of G_1 into G_2 as an injective mapping from $V(G_1)$ to $V(G_2)$.

There are two issues for edge isoperimetric problem of graph [8].

(i) For a subset $B \subseteq V(G)$, $\Theta_G(B) = \{(u, v) \in E(G) | u \in B, v \notin B\}$, and for every $m \in \{1, 2, \ldots, |V(G)|\}$, let $\Theta_G(m) = \min_{B \subseteq V(G), |A|=m} |\Theta_G(B)|$. Then the problem is to find $B \subseteq V(G)$ such that $|B| = m$ and $\Theta_G(m) = |\Theta_G(B)|$. Such subsets are called optimal with respect to (i).

(ii) For a subset $B \subseteq V(G)$, $I_G(B) = \{(u, v) \in E(G) | u, v \in B\}$, and for every $m \in \{1, 2, \ldots, |V(G)|\}$, let $I_G(m) = \max_{B \subseteq V(G), |B|=m} |I_G(B)|$. Then the problem is to find $B \subseteq V(G)$ such that $|B| = m$ and $I_G(m) = |I_G(B)|$. Such subsets are called optimal with respect to (ii).

Definition 1 ([8]). The 3-ary n-cube, denoted by Q_n^3 with $N = 3^n$ vertices. The vertex has the form $x = (x_{n-1} \cdots x_1 x_0)$, where $0 \leqslant x_i \leqslant 2$ for every $0 \leqslant i \leqslant n-1$. Two vertices $x = (x_{n-1} \cdots x_1 x_0)$ and $y = (y_{n-1} \cdots y_1 y_0)$ are adjacent if and only if there is an integer i with $0 \leqslant i \leqslant n-1$, which makes $x_i = y_i \pm 1 \pmod 3$ and $x_j = y_j$ for $j \in \{0, 1, 2, \ldots, n-1\} - \{i\}$.

Lemma 1 ([9]). Let G denote a regular graph and f is a mapping from graph G to graph H. Let S denote the edge cut of H, and then H can be partitioned into 2 components H_a and H_b after removing the edges of S. Let $G_a = f^{-1}(H_a)$ and $G_b = f^{-1}(H_b)$. S needs to meet the following issues:

(1) For $(x, y) \in (G_j)$, $j = 1, 2$, there is no edge for $P_f(x, y)$ in S.

(2) For $(x, y) \in E(G)$ with $x \in V(G_a)$ and $y \in V(G_b)$, there is only one edge for $P_f(x, y)$ in S.

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(3) Both $V(G_a)$ and $V(G_b)$ are optimal.

Thus $\mathrm{EC}_f(S)$ has the minimum value and $\mathrm{EC}_f(S) = \sum_{v\in V(G_a)} \deg(v) - 2|E(G_a)| = \sum_{v\in V(G_b)} \deg(v) - 2|E(G_b)|$.

Embedding 3-ary n-cube onto NoC. Let lex : $V(Q_n^3) \to \{1, 2, \ldots, 3^n\}$ denote a mapping function, where for any vertex $u = u_{n-1}u_{n-2}\cdots u_0$ in Q_n^3, $\mathrm{lex}(x) = \sum_{i=0}^{n-1} u_i 3^i + 1$, which is the decimal value of u.

Theorem 1 ([8]). L_i is an optimal set in Q_n^3 with $1 \leqslant i \leqslant 3^n$.

Lemma 2. Let $T \subseteq V(Q_n^3)$, $T = \{x \in V(Q_n^3)|f(x) \leqslant m\}$. Then $Q_n^3[S]$ is a maximum subgraph induced by m nodes.

Proof. Let A denote a class of m consecutive nodes in T. Let B denote a class of m non-consecutive nodes in S. Thus $B = \bigcup_{i=1}^{j} S_i$ with $j \geqslant 2$, these S_i are non-intersect and each S_i is a class of consecutive vertices such that $\sum_{i=1}^{j} |S_i| = k$. We claim that $|E(Q_n^3[Y])| \leqslant |E(Q_n^3[X])|$. We prove this claim by induction on n. For $n = 2$, we have $|E(Q_n^3[Y])| \leqslant |E(Q_n^3[X])|$. Assume that the claim is true for $n - 1$.

By Definition 1, Q_n^3 can be partitioned into $Q_{n-1}^3[0]$, $Q_{n-1}^3[1]$, $Q_{n-1}^3[2]$ by i-dimension. Let $E(Q_n^3[K_1 \wedge K_2])$ denote the set of edges in Q_n^3 with one side lies in K_1 and another side lies in K_2. Let K_1 and K_2 denote two non-intersect components induced by consecutive vertices $k_1 \in V(Q_{n-1}^3[i])$ and $k_2 \in V(Q_n^3\ Q_{n-1}^3[i])$, respectively, $k_1+k_2 = k$ with $k_1 \leqslant k_2$. Suppose $K_1 \subseteq Q_3^{n-1}(i), 0 \leqslant i \leqslant 2$. Let k_2 be the vertex number in K_2 that falls on $Q_n^3 \setminus Q_{n-1}^3(i)$. Then $|E(Q_n^3[K_1 \wedge K_2])| \leqslant k_2$. But $|E(Q_n^3[K_1])| \leqslant |E(Q_n^3[V_{k_1}])|$. Similarly $|E(Q_n^3[K_2])| \leqslant |E(Q_n^3[V_{k_2}])|$. This implies that $|E(Q_n^3[K_1 \cup K_2])| = |E(Q_n^3[K_1])| + |E(Q_n^3[K_2])| + |E(Q_n^3[K_1 \wedge K_2])| \leqslant |E(Q_n^3[V_{k_1}])| + |E(Q_n^3[V_{k_2}])| + k_2$. Then, we get $|E(Q_n^3[K_1 \cup K_2])| \leqslant |E(Q_n^3[K])|$. Then $|E(Q_n^3(\bigcup_{i=1}^{j} S_i)| \leqslant |E(Q_n^3[S])|$ where S is induced by $m - |S_n|$ consecutive vertices. Now, $|E[Q_n^3 \bigcup_{i=1}^{n} S_i]| = |E(Q_n^3[\bigcup_{i=1}^{n-1} S_i \cup S_n])| \leqslant |E(Q_n^3[S \cup S_n]| \leqslant |E(Q_n^3[X])|$. The lemma is thus proved.

Definition 2. A reverse incomplete 3-ary n-cube with i nodes is the subgraph induced by $\{\alpha(3^n - 1), \alpha(3^n - 2), \ldots, \alpha(3^n - i)\}$, which is denoted as IL_i, $1 \leqslant i \leqslant 3^n$.

Definition 3. Let $f : V(Q_n^3) \to V(C_{3^n})$ be a mapping. The vertices of C_{3^n} are labeled as $0, 1, \ldots, 3^n - 1$ in the clockwise direction. Thus for any vertex u in $V(Q_n^3)$, we have $f(u) = \mathrm{lex}(u)$.

Theorem 2. The minimum congestion of embedding Q_n^3 into a ring C_{3^n} is $2 \cdot 3^{n-1} - 1$.

Proof. The vertices in C_n are labeled from 1 to 3^{n-1}, which are the images of vertices $\{(0u_{n-2}\cdots u_0)\}$ in C_n. The vertices are labeled from $3^{n-1}+1$ to $2 \times 3^{n-1}$, which are the images of vertices of $\{(1u_{n-2}\cdots u_0)\}$ in C_n, and the vertices are labeled from $2 \times 3^{n-1} + 1$ to 3^n, which are the images of vertices $\{(2u_{n-2}\cdots u_0)C_n\}$. For $n \geqslant 2$, let $S_1 = \{(3^n, 1), (3^{n-1}, 3^{n-1} + 1)\}$, $S_2 = \{(3^{n-1}, 3^{n-1} + 1), (2 \times 3^{n-1}, 2\times 3^{n-1}+1)\}$, $S_3 = \{(2\times 3^{n-1}, 2\times 3^{n-1}+1), (3^n, 1)\}$, and then $C_{3^n} - S$ consists of three connected components C_1, C_2 and C_3. Since $f^{-1}(C_j) \cong Q_{n-1}^3$, $1 \leqslant j \leqslant 3$, then $f^{-1}(C_j)$ is the maximum induced subgraph of Q_n^3. Clearly, $\mathrm{EC}_f(3^n, 1) = \mathrm{EC}_f(3^{n-1}, 3^{n-1} + 1) = \mathrm{EC}_f(2 \times 3^{n-1}, 2 \times 3^{n-1} + 1) = 3^{n-1}$. Therefore, $\mathrm{EC}_f(S_j)$ is minimum. Then $V(G_{i1})$ is an optimal set. By Lemma 2, clearly, $\mathrm{EC}_f(Q_1^3, L_3) = 1$. Then congestion of embedding Q_n^3 into C_{3^n} under f is

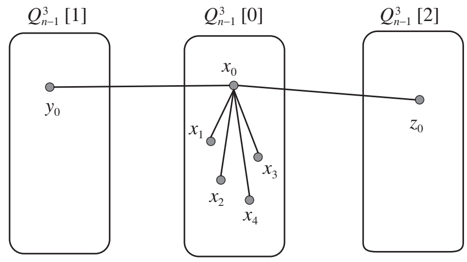

Figure 1 Adjacent vertices of x_0 in Q_n^3.

$$\begin{aligned}\mathrm{EC}(Q_n^3, C_{3^n}) &= \mathrm{EC}_f(Q_n^3, C_{3^n}) = \sum_{i=1}^{3^n} \mathrm{EC}_f(S_i)\\ &= \sum_{i=1}^{3^{n-1}} \mathrm{EC}_f(S_i) + 3^{n-1} + 3^{n-2}\\ &= 2 \cdot 3^{n-1} - 1.\end{aligned}$$

Theorem 3. The minimum wirelength from Q_n^3 to C_{3^n} under embedding f is

$$\mathrm{WL}(Q_n^3, C_{3^n}) = 3^n + \frac{1}{2}(3^{2n} - 3^{2n-1}) + \frac{1}{2}(3^{2n-2} - 3).$$

Proof. Let $G = Q_n^3$, $S_l = \{(l-1, i), (3^{n-1} + l - 1, 3^{n-1} + l)\}$, then $G_{l1} = f^{-1}(C_{l1})$ and $G_{l2} = f^{-1}(C_{l2})$. According to Theorem 1, $V(G_{l1})$ is an optimal set in Q_n^3. Therefore, $\mathrm{EC}_f(S_l)$ is minimum. By Lemma 2, $\bigcup_{i=1}^{3^l} G[S_l] \cong Q_l^3$. Therefore, the sum congestion of Q_n^3 into C_{3^n} is

$$\begin{aligned}\mathrm{WL}(Q_n^3, C_{3^n}) &= \mathrm{WL}_f(Q_n^3, C_{3^n}) = \sum_{i=1}^{3^n} \mathrm{EC}_f(S_i)\\ &= 3^n + \frac{1}{2}(3^{2n} - 3^{2n-1}) + \frac{1}{2}(3^{2n-2} - 3).\end{aligned}$$

Theorem 4. The cyclic bandwidth of embedding Q_n^3 into C_{3^n} under f is 3^{n-1}.

Proof. For any vertex $x_0 \in Q_{n-1}^3$, let its incident edges be (x_0, x_1), (x_0, x_2), (x_0, x_3), (x_0, x_4), (x_0, y_0), and (x_0, z_0), where x_1, x_2, x_3, $x_4 \in V(Q_{n-1}^3)$, $y_0 \in V(Q_{n-1}^3)$ and $z_0 \in V(Q_{n-1}^3)$ (see Figure 1). Clearly, $\max\{\mathrm{dist}(L_{3^n}, f(x), f(y))|x, y \in \{x_0, x_1, x_2, x_3, x_4, y_0, z_0\}\}$ $=\max\{|f(z_0) - f(x_0)|(x_0,\ z_0) \in Q_n^3\} = 3^{n-1}$. Therefore, the cyclic bandwidth of embedding Q_n^3 into C_{3^n} is 3^{n-1}.

Theorem 5. Let C_i be the induced cycle by the i-th vertices of $H = T(3^{n_1}, 3^{n_2})$, for $1 \leqslant i \leqslant n_2$. Let $f = \pi$, and then we have the sum of edge congestion $\sum_{i=1}^{3^{n_2}-1} \mathrm{EC}_f(C_i)$.

Proof. By the result of Lemma 3, the cyclic wirelength of embedding Q_n^3 into C_{3^n} under f is $\mathrm{WL}(Q_n^3, C_{3^n}) = 3^n + \frac{1}{2}(3^{2n} - 3^{2n-1}) + \frac{1}{2}(3^{2n-2} - 3)$. Therefore, for $1 \leqslant i \leqslant 3^{n_2} - 1$, the sum of edge congestion $\sum_{i=1}^{3^{n_2}-1} \mathrm{EC}_f(C_i)$ is

$$\begin{aligned}\sum_{i=1}^{3^{n_2}-1} \mathrm{EC}_f(C_i) &= \sum_{i=1}^{3^{n_2}-1} \lambda_G(j \cdot 3^{n_1})\\ &= 3^{n_1}\left(3^{n_2} + \frac{1}{2}(3^{2n_2} - 3^{2n_2-1}) + \frac{1}{2}(3^{2n_2-2} - 3)\right).\end{aligned}$$

Theorem 6. Embedding Q_n^3 onto the 3D torus $T(3^{n_1}, 3^{n_2}, 3^{n_3})$ under f has the minimum wirelength.

Proof. Let C_i denote a cycle induced by the X-axis vertices of $H = T(3^{n_1}, 3^{n_2}, 3^{n_3})$, for $1 \leqslant i \leqslant n_2$. Let R_j denote a cycle induced by the Y-axis vertices of $H = T(3^{n_1}, 3^{n_2}, 3^{n_3})$, for $1 \leqslant j \leqslant n_3$. Let W_k denote a cycle induced by the Z-axis vertices of $H = T(3^{n_1}, 3^{n_2}, 3^{n_3})$, for $1 \leqslant k \leqslant n_1$. By Lemma 3, the cyclic wirelength of embedding Q_n^3 into C_{3^n} under f is $\mathrm{WL}(Q_n^3, C_{3^n}) = 3^n + \frac{1}{2}(3^{2n} - 3^{2n-1}) + \frac{1}{2}(3^{2n-2} - 3)$. Then the minimum wirelength of embedding Q_n^3 into the torus $T(3^{n_1}, 3^{n_2}, 3^{n_3})$ is

$$\begin{aligned}
&\mathrm{WL}(Q_n^3, T(3^{n_1}, 3^{n_2}, 3^{n_3}))\\
&= \sum_{j=1}^{3^{n_1}} \mathrm{EC}_f(C_i) + \sum_{i=1}^{3^{n_2}} \mathrm{EC}_f(R_j) + \sum_{k=1}^{3^{n_3}} \mathrm{EC}_f(W_k)\\
&= \sum_{j=1}^{3^{n_1}} \delta_G(j \cdot 3^{n_2}) + \sum_{i=1}^{3^{n_2}} \delta_G(i \cdot 3^{n_1}) + \sum_{k=1}^{3^{n_3}} \delta_G(k \cdot 3^{n_3})\\
&= 3^{n_1} \left(3^{n_2} + \frac{1}{2}(3^{2n_2} - 3^{2n_2-1}) + \frac{1}{2}(3^{2n_2-2} - 3)\right)\\
&\quad + 3^{n_2} \left(3^{n_1} + \frac{1}{2}(3^{2n_1} - 3^{2n_2-1}) + \frac{1}{2}(3^{2n_1-2} - 3)\right)\\
&\quad + 3^{n_2} \left(3^{n_3} + \frac{1}{2}(3^{2n_3} - 3^{2n_3-1}) + \frac{1}{2}(3^{2n_3-2} - 3)\right).
\end{aligned}$$

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