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Nonlinear model predictive control for trajectory tracking of quadrotors using Lyapunov techniques

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Dear editor,

In recent years, the research on trajectory tracking and collision avoidance control of quadrotor unmanned aerial vehicles (UAVs) has gained considerable attention [1, 2]. Since quadrotor UAVs are used extensively, various autonomous commercial platforms, such as Parrot and DJI, have emerged and became preferable. Although such ready-to-fly platforms have autopilots that facilitate the control system developments, the commonly used Lyapunov-based control methods, such as backstepping control (BC) and sliding mode control (SMC), are subject to system constraints, e.g., the maximum allowed control signals for attitude stabilization. Although a modified BC method with input saturation has been developed [1], it fails to accommodate system constraints while achieving desired control performance.

Nonlinear model predictive control (NMPC) is capable of systematically handling system constraints. However, this method is quite limited because of the terminal constraint design for closed-loop system stability. It is difficult to find a locally continuous time-invariant state feedback controller for nonlinear quadrotor systems, or the feedback controller by linearization is difficult to be implemented in engineering. Although the NMPC strategy without terminal constraints can remove the requirement for such a local controller, large prediction horizons are normally required to ensure the closed-loop system stability [3], which significantly increases the computational complexity of solving the NMPC optimization problem.

Based on these considerations, this study proposes a novel NMPC-based integrated trajectory tracking (TT) and obstacle avoidance (OA) control algorithm. The auxiliary controller is developed by borrowing ideas from the Lyapunov's direct method and BC approach, based on which the stability constraint is designed. In addition, the OA can be achieved by incorporating a well-designed potential field-based cost term in the tracking cost function. Under this new regularized NMPC formulation, the integrated TT and OA control problem for commercial quadrotors in real-world working conditions is addressed.

Commercial quadrotor kinematics. The kinematic equations with yaw attitude description for translational motion is $\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{v}_b$, where $\boldsymbol{\eta} = [x, y, z, \psi]^{\mathrm{T}}$ is the position and yaw vector in the inertial frame, $\boldsymbol{v}_b = [u, v, w, r]^{\mathrm{T}}$ is the velocity and yaw rate vector in the body frame, and $\boldsymbol{R}(\psi)$ is the rotation matrix from the body frame to the inertial frame. The relationship between the velocity responses and the velocity reference commands in the forward u_x, sideward u_y, upward u_z directions and the yaw angular velocity around the z-axis u_ψ is approximated by $\dot{\boldsymbol{v}}_b = \boldsymbol{S}\boldsymbol{v}_b + \boldsymbol{F}\boldsymbol{u}$, where $\boldsymbol{u} = [u_x, u_y, u_z, u_\psi]^{\mathrm{T}}$ is the constrained control input, $\boldsymbol{S}$ and $\boldsymbol{F}$ are the model parameters. The nonlinear dynamics is established as $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$, where $\boldsymbol{x} = [\boldsymbol{\eta}^{\mathrm{T}}, {\boldsymbol{v}_b}^{\mathrm{T}}]^{\mathrm{T}}$.

NMPC-based integrated TT and OA control. Given a reference position trajectory $\boldsymbol{p}_r(t) = [x_r(t), y_r(t), z_r(t)]^{\mathrm{T}}$, we generate the reference system $\boldsymbol{x}_r(t) = [\boldsymbol{\eta}_r(t)^{\mathrm{T}}, \boldsymbol{v}_{br}(t)^{\mathrm{T}}]^{\mathrm{T}}$, which satisfies the kinematic equations. Each state of the quadrotor motion now has a unique reference.

To perform the switching behavior between OA and TT and maintain the continuous differentiability at the condition border, the continuous potential field function $J^{oa}(\boldsymbol{x}(t)) = \frac{\lambda}{1+e^{-k\cdot d(t)}}$ is imposed on the NMPC cost function, where $\lambda > 0$ is a weighting parameter, $d(t) = r_s^2 - (\boldsymbol{p}(t) - \boldsymbol{p}_o)^{\mathrm{T}}(\boldsymbol{p}(t) - \boldsymbol{p}_o)$, $k > 0$ determines the smoothness of the OA trajectory, and $\boldsymbol{p}(t) = [x(t), y(t), z(t)]^{\mathrm{T}}$ is the real position trajectory. $\boldsymbol{p}_o$ is the obstacle position. The closer distance between the obstacle and the quadrotor results in a higher potential value such that the OA is prioritized over TT. Notably, to achieve the control objective, the cost function is defined as $J(\boldsymbol{x}(t_k), \hat{\boldsymbol{u}}(s; t_k)) = \int_{t_k}^{t_k+T} ||\tilde{\boldsymbol{x}}(s; t_k)||^2_{\boldsymbol{Q}} + J^{oa}(\hat{\boldsymbol{x}}(s; t_k)) + ||\hat{\boldsymbol{u}}(s; t_k)||^2_{\boldsymbol{R}}\, \mathrm{d}s$, where $\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \boldsymbol{x}_r$ is the predicted error state, T is prediction horizon, and $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the positive-definite weighting matrices. The switching behavior from OA to TT may probably cause the instability. To address this issue, a Lyapunov-based contractive constraint is introduced as stability constraint. A new regularized NMPC optimization problem $\mathcal{P}(\boldsymbol{x})$ for the integrated TT and OA control at time t_k is formulated as follows:

$$\mathcal{P}(\boldsymbol{x}): \quad \min_{\hat{\boldsymbol{u}} \in M(\delta)} J(\boldsymbol{x}(t_k), \hat{\boldsymbol{u}}(s; t_k)) \tag{1a}$$

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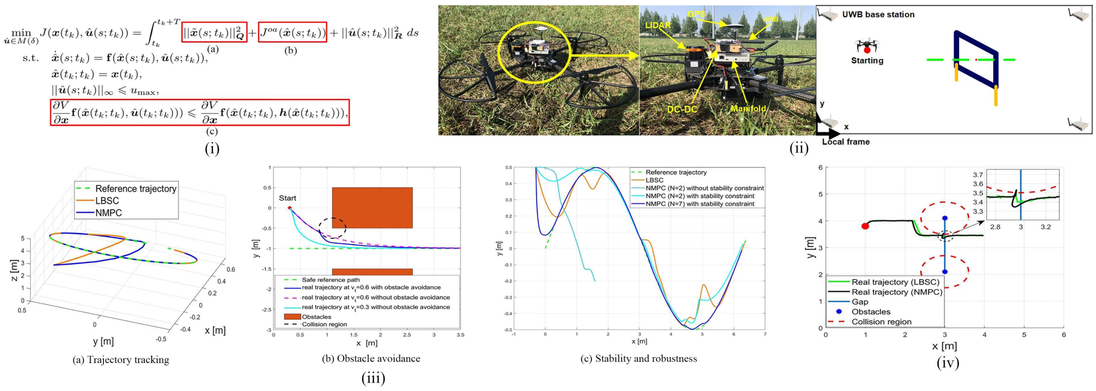

Figure 1 (Color online) The TT and OA control performance under the proposed nonlinear model predictive control algorithm. (i) NMPC formulation; (ii) experimental setup; (iii) control performance in the simulations; (iv) control performance in the experiment.

$$\text{s.t.}\quad \dot{\hat{\boldsymbol{x}}}(s;t_k)=\boldsymbol{f}(\hat{\boldsymbol{x}}(s;t_k),\ \hat{\boldsymbol{u}}(s;t_k)),\tag{1b}$$

$$\hat{\boldsymbol{x}}(t_k;t_k)=\boldsymbol{x}(t_k),\ \ ||\hat{\boldsymbol{u}}(s;t_k)||_\infty\leqslant u_{\max},\tag{1c}$$

$$\frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f}(\hat{\boldsymbol{x}}(t_k;t_k),\hat{\boldsymbol{u}}(t_k;t_k)) \leqslant \frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f}(\hat{\boldsymbol{x}}(t_k;t_k),\boldsymbol{h}(\hat{\boldsymbol{x}}(t_k;t_k))),\tag{1d}$$

where $s\in[t_k,t_k+T]$, $u_{\max}$ is the maximum amplitude of the input signal, $M(\delta)$ is the family of piece-wise constant functions with the sampling period δ, $\boldsymbol{h}(\boldsymbol{x})$ is the auxiliary Lyapunov-based nonlinear tracking control law, and $V(\boldsymbol{x})$ is the corresponding Lyapunov function. The presence of stability constraint (1d) shows that the NMPC controller automatically performs the best possible tracking control that respects the control input limitation owing to the online optimization procedure. The compatibility with sub-optimal solutions in $\mathcal{P}(\boldsymbol{x})$ introduces the flexibility between computational efficiency and control performance. For limited computational resources, acceptable tracking performance with low computational complexity can be realized. Figure 1 shows the performance of this control algorithm.

Auxiliary control law. Since the Lyapunov-based backstepping controller (LBSC) is extensively used in quadrotor systems, $\boldsymbol{h}(\boldsymbol{x})$ is constructed using the BC technique. The Lyapunov function is chosen as $V=\frac{1}{2}\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1+\frac{1}{2}\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{z}_2$, where $\boldsymbol{z}_1=\boldsymbol{\eta}_r-\boldsymbol{\eta}$ is the position error, and $\boldsymbol{z}_2=\dot{\boldsymbol{\eta}}_r-R(\psi)\boldsymbol{v}_b+\alpha_1\boldsymbol{z}_1$ is the velocity error. Then, the LBSC law can be inferred as follows: $\boldsymbol{h}(\boldsymbol{x})=-\boldsymbol{F}^{-1}\boldsymbol{S}\boldsymbol{v}_b+\boldsymbol{F}^{-1}\boldsymbol{R}^{\mathrm{T}}(\psi)\boldsymbol{\mu}$, where $\boldsymbol{\mu}=\ddot{\boldsymbol{\eta}}_r-\dot{\boldsymbol{R}}(\psi)\boldsymbol{v}_b+(\alpha_1+\alpha_2)\boldsymbol{z}_2+(1-\alpha_1^2)\boldsymbol{z}_1$, $\alpha_1>0$ and $\alpha_2>0$ are the LBSC parameters. The detailed expression of the stability constraint (1d) is illustrated in Appendix C.

Stability analysis. Unlike the traditional MPC, $\mathcal{P}(\boldsymbol{x})$ admits recursive feasibility if the auxiliary control law $\boldsymbol{h}(\boldsymbol{x})$ can be treated as a feasible solution under the condition that $||\boldsymbol{h}(\boldsymbol{x})||_\infty\leqslant u_{\max}$ can hold.

Assumption 1. The reference trajectory is smooth and bounded, satisfying the following: $|x_r(t)|\leqslant\bar{x}$, $|y_r(t)|\leqslant\bar{y}$, $|\dot{x}_r|\leqslant\bar{x}_1$, $|\dot{y}_r|\leqslant\bar{y}_1$, $|\ddot{x}_r|\leqslant\bar{x}_2$, $|\ddot{y}_r|\leqslant\bar{y}_2$.

Lemma 1. Suppose Assumption 1 holds. Then, the reference signal $\boldsymbol{\eta}_r$ and its first and second derivatives $\dot{\boldsymbol{\eta}}_r$, $\ddot{\boldsymbol{\eta}}_r$ are bounded, i.e., $||\boldsymbol{\eta}_r||_\infty\leqslant\bar{\eta}$, $||\dot{\boldsymbol{\eta}}_r||_\infty\leqslant\bar{\eta}_1$, $||\ddot{\boldsymbol{\eta}}_r||_\infty\leqslant\bar{\eta}_2$.

Theorem 1. Consider the auxiliary control law $\boldsymbol{h}(\boldsymbol{x})$. Given $\bar{f}=||F^{-1}||_\infty$, $\bar{s}=||S||_\infty$, if α_1 and α_2 satisfy

$$\sqrt{2}\cdot\bar{f}(\bar{s}\cdot l+\bar{\eta}_2+2\sqrt{2}\cdot l^2+m)\leqslant u_{\max},\tag{2}$$

where $l=\bar{\eta}_1+||\boldsymbol{z}_2(t_0)||_2+\alpha_1||\boldsymbol{z}_1(t_0)||_2$, $m=(\alpha_1+\alpha_2)||\boldsymbol{z}_2(t_0)||_2+(1-\alpha_1^2)||\boldsymbol{z}_1(t_0)||_2$, $\boldsymbol{z}_1(t_0)$ and $\boldsymbol{z}_2(t_0)$ are the position error and velocity error respectively at the initial time $t_0=0$, then $\mathcal{P}(\boldsymbol{x})$ admits recursive feasibility.

Remark 1. Theorem 1 restricts the selection range of the LBSC parameters. Once the parameters are determined, a region of attract (ROA) $\mathcal{S}_\Omega=\{\boldsymbol{x}\in\mathbb{R}^8|(2)\}$ for the solutions is given. For the sake of facilitating the exploration of the possible optimal solutions, it is desirable to make $\mathcal{S}_\Omega$ as large as possible to maximize the allowable operating region.

Theorem 2. Consider the auxiliary control law $\boldsymbol{h}(\boldsymbol{x})$ whose parameters satisfy (2). Then, the closed-loop system under the NMPC control action $\boldsymbol{u}$ derived from $\mathcal{P}(\boldsymbol{x})$ is asymptotically stable at the equilibrium $\tilde{\boldsymbol{x}}=\boldsymbol{0}$.

The proofs of Theorems 1 and 2 are given in Appendixes D and E. The simulation and experimental results are illustrated in Appendix F in detail.

Conclusion. This study proposes a novel NMPC algorithm for the TT and OA control of a commercial quadrotor. The OA can be achieved by designing the potential field function-based cost term with well-tuned parameters. By incorporating the stability constraint into the NMPC optimization problem, the control performance as well as the robustness can be effectively enhanced.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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