SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

June 2022, Vol. 65 162203:1–162203:17
https://doi.org/10.1007/s11432-021-3375-5

Event-based sliding mode control under denial-of-service attacks

Yingxin TIAN[1,2], Xiang LI[1], Bo DONG[1], Yabin GAO[1*] & Ligang WU[1]

[1]*Department of Control Science and Engineering, Harbin Institute of Technology, Harbin* 150001*, China;*
[2]*School of Software, Harbin Institute of Technology, Harbin* 150001*, China*

Received 11 May 2021/Revised 24 July 2021/Accepted 8 October 2021/Published online 18 April 2022

Abstract This paper deals with the problem of estimator-based sliding mode control against denial-of-service (DoS) attacks and discrete events via a time-delay approach. A networked system is considered an uncertain dynamical system with matched and mismatched perturbations and exogenous disturbance in the network environment. A network-resource-aware event-triggering mechanism is designed with aperiodically releasing system measurements. Furthermore, to describe the DoS attack duration and inter-event time, a time-delay modeling approach considers the DOS attack duration and inter-event time as a "time delay" of the measurements between the sensor and controller over the network is proposed. Consequently, an interval-time-delay system with uncertainties is formulated. A state-observer-based sliding mode controller, by which the ideal sliding mode can be achieved, is proposed against the DoS attacks. The resulting sliding motion is proved to be robust and stable with an $\mathscr{L}_2$ gain performance. Finally, the effectiveness and applicability of the present sliding mode control are validated in a simulated pendulum system.

Keywords sliding mode control, time delay, uncertain system, state observer, event trigger

Citation Tian Y X, Li X, Dong B, et al. Event-based sliding mode control under denial-of-service attacks. Sci China Inf Sci, 2022, 65(6): 162203, https://doi.org/10.1007/s11432-021-3375-5

1 Introduction

It is widely accepted that the emergence of networks brought significant development to traditional control systems of point-to-point. The control loop forms a closed-loop control system through some communication networks, which produce the so-called networked control systems (NCSs) [1–3]. Since the 1950s, great changes have taken place in NCSs. The accelerated integration of computing, communication, and control has stimulated the interest of researchers from various disciplines in the emerging field of NCSs [4, 5]. Generally, an NCS is composed of plants, sensors, controllers, and actuators [4]. Sometimes, some filters or observers, as an estimator component, are also part of the components of the NCS. Among these components, the sensor unit is responsible for obtaining information, the controller unit provides commands or decisions, and the actuator unit executes the control commands or decisions. The estimator is sometimes used as the auxiliary of the controller or sensor unit to provide the update of the controller or sensor, and the communication network is responsible for exchanging information/data [6].

In signal transmission, compared with the analog implementation of signal, the advantage of NCS for signal (data) transmission through the digital communication channel is that it increases system flexibility and reliability, and reduces installation and maintenance cost. Consequently, significant studies focus on the application of NCS in industrial control, transportation, aerospace, and other fields. Additionally, there are outstanding achievements in applying complex control processes, such as advanced aircraft, robots, building intelligence, telemedicine, remote teaching, military command, and manufacturing. However, due to the actual operation characteristics of these control processes, there are some uncertainties in the operation of NCS, such as delay, multi-channel interference, congestion, timing disorder, data loss, and jitter. Even in the large-scale NCS, each system unit also faces complex uncertainties and constraints, such as anonymous attacks, failures, and limitations of communication networks. For

* Corresponding author (email: yabingao@hit.edu.cn)

© Science China Press and Springer-Verlag GmbH Germany, part of Springer Nature 2022

example, Liu et al. [7] investigated the state feedback controller design of nominal linear systems subject to denial-of-service (DoS) attacks and hybrid-triggering communication strategy. By considering that DoS attacks imposed by power-constrained pulse-width-modulated jammers are partially identified, Hu et al. [8] studied the event-based state feedback control of networked linear systems under the resilient event-triggering communication in a framework of a switched control system. Dong et al. [9] developed a robust adaptive quantized control of networked Markov jump systems with parameter uncertainty, matched uncertain nonlinearity, and disturbance.

Currently, there are many control theories and methods of the system with uncertainties. In addition to the proportion-integral-differential (PID) control commonly used in practice, there are some advanced control theories, such as fuzzy control, backstepping control, bang-bang control, sliding mode control (SMC), internal model control, model predictive control, prescribed performance control, nonlinear control, and some of their combination control. As a powerful and robust control methodology dealing with nonlinearity and uncertainties, i.e., matched system perturbations and exogenous disturbances, SMC has been successfully applied to dynamical systems with complex uncertainties [10–13] in practical applications, such as robots, underwater robots, aircraft and spacecraft, flexible space structures, motors, automotive engines, electronic systems, and fuel cell power systems. This successful application is due to its insensitivity to parameter uncertainty, strong robustness to external disturbances, good transient performance, and tracking ability [14, 15]. In SMC, a discontinuous control is utilized to force system trajectories onto a predesigned sliding surface in finite time and then toward the system equilibrium point asymptotically or boundedly. Thus, it possesses properties of fast response and excellent disturbance rejection [10, 16]. Benefiting from SMC for the networked systems, Liu et al. [17] dealt with the observer-based SMC problem of linear Itô stochastic time-delay systems with a logarithmic quantization between sensor and controller. Zhang et al. [18] solved the problem of SMC for linear NCSs with round trip delays by using current and previous measurements and inputs for reconstructing state variables. Wu et al. [19] studied the event-based SMC of networked stochastic systems based on state observer by using a time-delay approach. Then, the problem of the dissipativity-based resilient SMC under DoS attacks was addressed in [20, 21]. Notably, few techniques were developed to secure SMC against the DoS attacks when system states are unmeasurable in real NCSs. The main challenges dealing with such output feedback SMC against the DoS attacks are: how to construct a sliding surface that can be reached in finite time due to the intermittency of the DoS attacks, and how to analyze the stability of the SMC-based system in the complex network environment with mixed discrete events and network DoS attacks and delays.

Based on the discussions above, this paper deals with secure SMC of the NCS with discrete events and network DoS attacks via an output feedback approach. The network DoS attacks and delays between the sensor and controller units are formulated. The plant is considered an uncertain dynamical system with matched and mismatched perturbations and exogenous disturbance. The SMC designed in the paper is integral-type based on a state observer. The contributions of this work are summarized as follows.

(i) The discrete event at the sensor unit and the DoS attacks on the network between the sensor and the controller are modeled in a framework of an interval time-delay system, without using the constraints of the DoS frequency. Transmission delay in the network is also allowed. The discrete event is designed as an aperiodic event-triggering mechanism to regulate the stability of the closed-loop system subject to the DoS attacks and discrete events.

(ii) The networked system is fully considered with matched and mismatched system perturbations and external disturbances. The state observer is employed to estimate the unmeasurable states, and an integral-sliding-mode controller based on the observer state is designed to enforce the convergence of the estimates. Consequently, the ideal sliding mode can be ensured.

(iii) The stability of the resulting closed-loop system, including the resulting state estimation system, is analyzed by the Lyapunov-Krasovskii functional method. Then, sufficient conditions of the robustness and stability with an $\mathscr{L}_2$ gain performance are provided, by which the parameters of the controller and observer can be determined for the secure control of the networked uncertain dynamical system subject to the DoS attacks.

The remainder of this paper is organized as follows. The descriptions of the networked system with the DoS attacks and event-triggering mechanisms are shown in Section 2. Section 3 presents the main results for this work. Some discussions on the extensions of the presented secure control method are provided in Section 4. An application of the proposed method is given in Section 5. Finally, Section 6 presents the conclusion.

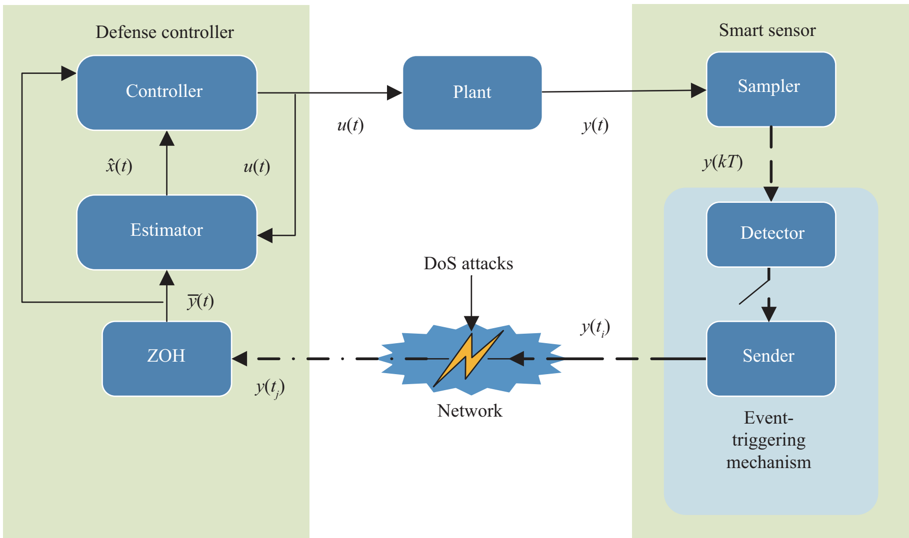

Figure 1 (Color online) An illustration of the proposed structure of the secure estimator-based SMC under DoS attacks and discrete events.

Notations. $\mathcal{L}_2 \in [0,+\infty)$ denotes the space of square-integrable vectors. The superscripts "T" and "-1" denote the matrix transpose and inverse, respectively. $[X]_{\mathrm{sym}}$ is used to denote $X + X^{\mathrm{T}}$ for simplicity. A block diagonal matrix is denoted by the shorthand $\mathrm{diag}\{X_1, X_2, \ldots, X_n\}$ with diagonal matrices $X_1, X_2, \ldots, X_n$.

2 System description and problem formulation

2.1 System description

Consider the following uncertain dynamical system:

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u(t) + v(x,t)) + f(x,t) + Ew(t), \\ y(t) = Cx(t) + Fw(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{d_{\mathrm{x}}}$, $u(t) \in \mathbb{R}^{d_{\mathrm{u}}}$, and $y(t) \in \mathbb{R}^{d_{\mathrm{y}}}$ are the vectors of system states, control inputs, and system measurement outputs, respectively. $v(x,t) \in \mathbb{R}^{d_{\mathrm{u}}}$ and $f(x,t) \in \mathbb{R}^{d_{\mathrm{x}}}$ are respectively the matched and mismatched system perturbations. $w(t) \in \mathbb{R}^{d_{\mathrm{w}}}$ denotes exogenous disturbance inputs belonging to $\mathcal{L}_2[0,+\infty)$. The matrices $A \in \mathbb{R}^{d_{\mathrm{x}} \times d_{\mathrm{x}}}$, $B \in \mathbb{R}^{d_{\mathrm{x}} \times d_{\mathrm{u}}}$, $C \in \mathbb{R}^{d_{\mathrm{y}} \times d_{\mathrm{x}}}$, $E \in \mathbb{R}^{d_{\mathrm{x}} \times d_{\mathrm{w}}}$, and $F \in \mathbb{R}^{d_{\mathrm{y}} \times d_{\mathrm{w}}}$ are known system parameters.

For the plant (1), when observer-based SMC strategy is applied integrating with the smart sensor, the structural diagram of the overall control system is depicted in Figure 1. In this paper, we consider the DoS attacks on the network between the sensor and the controller units. The smart sensor consisted of the sampler and the event trigger is used to send the measurement (data) aperiodically for the mitigation of the network communication. The network linking the sensor and the controller is considered with some DoS attacks interrupting the data transmission. In the defense controller unit, an estimator is used to estimate the system state and then to design a sliding mode controller, by taking the DoS attacks into account. The specific formulations of the smart sensor and the defense controller will be presented in Subsections 2.2–2.4.

2.2 Event-triggering mechanism

As for the measurement output $y(t)$, we employ a sampler to sample $y(t)$ with a sampling period T of which the sequence $\mathbb{T} \triangleq \{T_1, T_2, \ldots, T_\infty\}$ is allowed to be float, that is the sampler can aperiodically sample $y(t)$. Meanwhile, the k-th sampled measurement $y(kT_k)$ $(T \in \mathbb{T}, k = 1, 2, 3, \ldots)$ is set to be sent to the event detector of the event-triggering mechanism for calculating and detecting if the measurement $y(kT)$ violates the event-triggering condition formulated in (2). For simplicity, we write T_k as T for each sampling instant in the following context. We ignore the time to sample the measurement and calculate/detect the event.

$$\delta(k_i, l) \triangleq e_{\mathrm{y}}^{\mathrm{T}} e_{\mathrm{y}} - \rho y^{\mathrm{T}}(k_i T) y(k_i T) > 0, \tag{2}$$

where $e_{\mathrm{y}} \triangleq y((k_i + l)T) - y(k_i T)$. $0 < \rho < 1$ denotes a custom coefficient of triggering error tolerance. $y(k_i T)$ ($k_i \in \mathbb{N}$, $i = 1, 2, \ldots, \infty$) denotes the latest triggered measurement at the i-th triggering instant $k_i T$. $y((k_i + l)T)$ ($l = 1, 2, 3, \ldots$) denotes the current sampled/detected the measurement.

Hence, the next triggering time instant $k_{i+1}T$ of the event trigger can be determined by

$$k_{i+1}T = k_i T + \arg\min_{l \geqslant 1} \big\{\delta(k_i, l) \mid \delta(k_i, l) > 0\big\}. \tag{3}$$

Evidently, it holds that

$$\delta(k_i, l) \leqslant 0,\ \forall (k_i + l)T \in [k_i T, k_{i+1}T). \tag{4}$$

As a result, the sampled measurements will be released aperiodically to the next unit (the controller unit) over the network with certain network resources saved.

2.3 Network and DoS attacks

In this work, the network communication between the sensor and the controller units is considered in the whole control loop. When the triggered measurement $y(k_i T)$ is transmitted over the network to the controller unit, it may be confronted with data packet dropouts, transmission delay, and malicious attacks, particularly for large-scale wireless networks. This paper is focused on the technical development of the design of SMC against the DoS attacks. A DoS attack off and on interrupts the communication service for a segment of time. This attack leads to that triggered measurement cannot be transmitted and be received in time. It thus may detrimentally affect the system stability [22].

Without loss of generality, for the m-th DoS attack, i.e., the network is attacked initially at time instant a_m, let us define the attack interval with $\Gamma_m \triangleq \{a_m\} \cup [a_m + \mu_m)$, where μ_m denotes the length of the m-th DoS interval, and $m \in \mathbb{M}$ where $\mathbb{M} \triangleq \{1, 2, 3, \ldots, m_{\max}\}$ with $m_{\max}$ the maximum number of the attack segments. Let $\mathbb{A}$ denote the set of times at which a DoS attack is active. For simplicity, define $t_i \triangleq k_i T$. Then, the measurement $y'(t_i)$ received at the zero-order holder (ZOH) can be formulated as

$$y'(t_i) = \begin{cases} y(t_i), & t_i \notin \mathbb{A}, \\ \text{Null}, & t_i \in \mathbb{A}, \end{cases}$$

where "Null" denotes no data received at the ZOH.

2.4 Estimator and controller

The estimator is employed to estimate the state of the plant, by using the measurement transformed over the network. Considering the discrete event and the possible interrupt transmission caused by the DoS attack, we employ a ZOH to receive the discrete signal. It keeps outputting a continuous signal and updating the output when a new discrete signal $y'(t_i)$ is received. Let us define the output of the ZOH as $\bar{y}(t)$. Notably, $\bar{y}(t)$ should keep outputting the current signal if the newly received measurement $y'(t_i) = \text{Null}$. Then, the following Luenberger state observer used as the estimator is constructed

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(\bar{y}(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{5}$$

where $\hat{x}(t) \in \mathbb{R}^{d_{\mathrm{x}}}$ and $\hat{y}(t) \in \mathbb{R}^{d_{\mathrm{y}}}$ are respectively the state estimate vector and output estimate vector. The matrix $L \in \mathbb{R}^{d_{\mathrm{x}} \times d_{\mathrm{y}}}$ is the observation gain to be designed. The matrices $\{A, B, C\}$ are the parameters of the plant (1).

Our purpose in this paper is to design a sliding mode controller to stabilize the system (1). We may express the sliding mode controller

$$u(t) = u_{\mathrm{o}}(t) + u_{\mathrm{s}}(t) \tag{6}$$

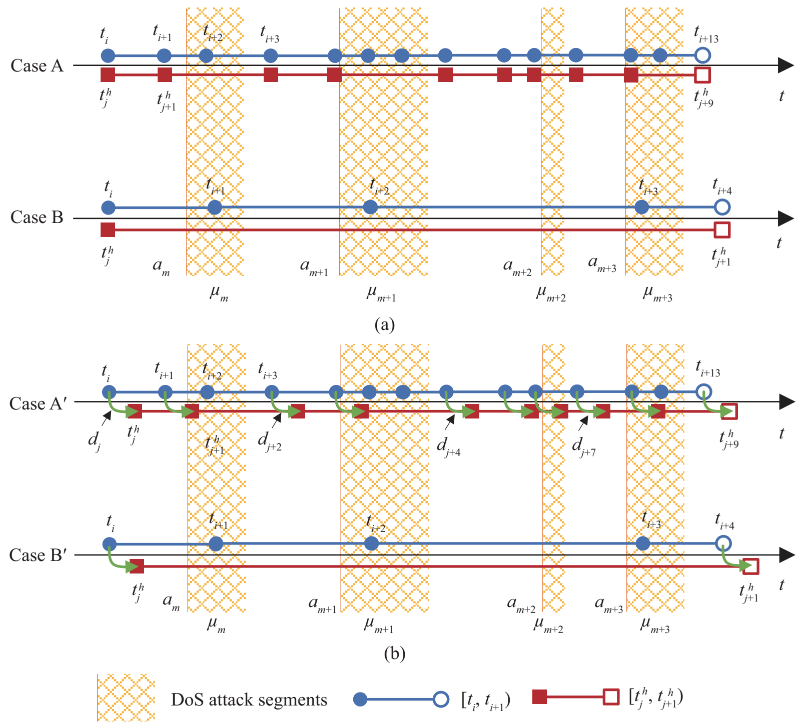

Figure 2 (Color online) Illustrations of the triggered instants t_i and the received instants t_j^{h} of the measurement $y(t_i)$ under the DoS attack. (a) Without the network transmission delay d_j; (b) with the network transmission delay d_j.

with two terms, that is the nominal term $u_{\mathrm{o}}(t)$ and the switching term $u_{\mathrm{s}}(t)$. The switching term $u_{\mathrm{s}}(t)$ acts on a guarantee of the finite-time reachability of sliding modes and the compensation to the matched perturbations. $u_{\mathrm{o}}(t)$ and $u_{\mathrm{s}}(t)$ are to be designed. Meanwhile, based on the state estimate $\hat{x}(t)$ the sliding variable $s(t) \in \mathbb{R}^{d_{\mathrm{u}}}$ is an integral-type one formulated as follows:

$$\begin{cases} s(t) \;=\; K\hat{x}(t) + z(t), \;\; z(0) = -K\hat{x}(0), \\ \dot{z}(t) \;=\; -K\big(A\hat{x}(t) + Bu_{\mathrm{o}}(t)\big), \end{cases} \tag{7}$$

where the matrix $K \in \mathbb{R}^{d_{\mathrm{u}} \times d_{\mathrm{x}}}$ is a given parameter of the (ideal) sliding surface $s(t) = 0$. $z(t) \in \mathbb{R}^{d_{\mathrm{u}}}$ is an intermediate variable denoting the integral term of the sliding function.

2.5 Formulation of time-delay model

In this paper, by using the idea of the "time delay", we formulate the measurement $y(\cdot)$ from the plant to the ZOH over the sampling, event-detecting, and interrupting (attacking).

For the discrete (triggered) measurement $y(t_i)$, it may suffer from the DoS attack segment before the ZOH receiving the triggered measurement $y(t_i)$. Therefore, the ZOH may not receive a triggered measurement every triggering time instant. Define t_j^{h} the (global) time instant of the j-th time ZOH receiving (updating) the signal from the network, and $j = 1, 2, 3, \ldots$. Figure 2 illustrates the instants when the measurement $y(t_i)$ was triggered at the event trigger and the instants when $y(t_i)$ was received at the ZOH.

One thing is certain that the time instant t_j^{h} will not locate in a DoS attack segment if there are no delays of the measurement transmission from the network to the ZOH, as illustrated in Figure 2. Hence, without losing generality and strictness, we consider there are some momentary transmission delay

$$d_j \;<\; \min_{k \in \mathbb{N}^+} \{T_k\},$$

when the ZOH receiving the measurement $y(t_i)$ at time instant t_j^{h}. The cases B and B′ what need to be noticed are that if lots of triggered instants in succession occur all in DoS attack segments, the ZOH will not receive new measurements for a long time. It leads to the feedback control losing its function and significance and thus the stability of the control system will no longer be guaranteed. To this end, we will require such an event-triggering condition fulfilling that the maximum

$$\Delta t_{\max} = \max_{i\in\mathbb{N}}\{\Delta t_i \triangleq t_{i+1} - t_i\}$$

of the inter-event time Δt_i (i.e., the interval length of two adjacent event-triggering instants) will be less than the minimum secure durations

$$\nu_{\min} = \min_{m\in\mathbb{M}}\{\nu_m \triangleq a_{m+1} - (a_m + \mu_m)\}.$$

In this requirement, the triggered instants t_i will not miss the secure durations and thus the cases B and B$'$ are avoided.

The following assumptions on the plant and the network DoS attack phenomenon are taken into account in this paper, for theoretical analysis.

Assumption 1. Consider the system in (1). The matrix B is of full column rank, i.e., rank$(B) = m$, and the pairs (A, B) and (A, C) are controllable and observable, respectively.

Assumption 2. Consider the system in (1). The matched uncertainty term $v(x, t)$ is bounded by $v(x, t) \leqslant \alpha$ with $\alpha > 0$. The mismatched uncertainty term $f(x, t)$ can be formulated with $f(x, t) = ML(t)Nx(t)$, where $M \in \mathbb{R}^{d_{\rm x}\times d_{\rm p}}$ and $N \in \mathbb{R}^{d_{\rm q}\times d_{\rm x}}$ are constant matrices, and $L(t) \in \mathbb{R}^{d_{\rm p}\times d_{\rm q}}$ is an unknown time-varying matrix satisfying Lebesgue measurable condition $L^{\rm T}(t)L(t) \leqslant I_q$, $\forall t \geqslant 0$.

Assumption 3. Consider the DoS attack in the network between the sensor and the controller units. For any $m \in \mathbb{M}$, there exist

(1) a maximum duration $\mu_{\max}$ of the DoS attack segments μ_m,

(2) and a minimum duration $\nu_{\min}$ of the secure segments $\nu_m \triangleq a_{m+1} - (a_m + \mu_m)$.

Assumption 4. The maximum sampling period $T_{\max} = \min_{k\in\mathbb{N}^+}\{T_k\}$ is fairly less than the minimum duration $\nu_{\min}$ of the secure segments between any two adjacent DoS attack time instants a_m and a_{m+1}, $m \in \mathbb{M}$.

Remark 1. In fact, due to the limited energy of DoS attacks and its own anonymity, the DoS attack duration cannot be very large. Assumptions 3 and 4 are reasonable and practical for a networked control system. It is also consistent with the assumptions and design in [22, 23]. Besides, we will develop the event-triggering condition (2) against the DoS attack segment, for avoiding too many triggering instants in succession occurring in the attack segments to update new measurements timely. Therefore, the event-triggering condition (2) should be mended and the next triggering time instant $k_{i+1}T$ of the event trigger is determined by

$$k_{i+1}T = k_iT + \arg\min_{l\geqslant 1}\left\{\delta(k_i, l) \mid \delta(k_i, l) > 0 \cup \Delta t_i \geqslant \nu_{\min}\right\} \cdot T. \tag{8}$$

Taking the requirement $\Delta t_{\max} < \nu_{\min}$ into account, we know that when a triggered measurement $y(t_i)$ newly arrives at the ZOH, the ZOH will "wait" a span of time including the corresponding transmission delay time and possible DoS attack duration. In addition, for the real-time system output $y(t)$, it will take the time of the sampling and repeatedly event-detections when a new triggered measurement $y(t_i)$ is generated. Then, for any $y(t_i)$ ($i \in \mathbb{N}^{+\cdot}$) arrives at the ZOH at the instant $t^{\rm h}_j$, it is not difficult to obtain the following conclusion.

• For the case of the network transmission without delay d_j,

$$t^{\rm h}_{j+1} - t^{\rm h}_j \underset{l^*\in\mathbb{N}^+}{=} t_{i+l^*} - t_i < 2\Delta t_{\max} + \mu_{\max}, \tag{9}$$

where t_{i+l^*} denotes the time instant of a new triggered measurement $y(t_{i+l^*})$ arriving at the ZOH at $t^{\rm h}_{j+1}$ without transmission delay.

• For the case of the network transmission with delay d_j,

$$t^{\rm h}_{j+1} - d_{j+1} - (t^{\rm h}_j - d_j) \underset{l^*\in\mathbb{N}^+}{=} t_{i+l^*} - t_i < 2\Delta t_{\max} + \mu_{\max}, \tag{10}$$

where t_{i+l^*} denotes the time instant of $y(t_{i+l^*})$ arriving at the ZOH at $t^{\rm h}_{j+1}$ with transmission delay d_{j+1}.

Apparently, the case of the network transmission without delay is a special case of the one with delay $d_j = 0$ for $j \in \mathbb{N}^+$.

Consequently, for any $t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}})$ $(j = 1, 2, 3, \ldots)$, we introduce the following auxiliary variable:

$$h(t) := t - t_j^{\mathrm{h}} = t - (t_i + d_j),\ t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}}), \tag{11}$$

which satisfies that

$$0 \leqslant h(t) < h_M \tag{12}$$

with $h_M = 2\Delta t_{\max} + \mu_{\max} + d_{\max} - d_{\min}$. Then, the triggered measurement $y(t_i)$ can be rewritten as

$$y(t_i) = y(t - \tilde{h}(t)),\ t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}}), \tag{13}$$

where $\tilde{h}(t) = h(t) + d_j \in [d_j, h_M + d_j) \subseteq [d_{\min}, h_M + d_{\max})$. Then, it holds that

$$\bar{y}(t) = y(t_i) = y(t - \tilde{h}(t)),\ \forall t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}}). \tag{14}$$

Considering the synchronicity of the plant and the controller, define $t_0^{\mathrm{h}} = 0$ and $\bar{y}(t_0^{\mathrm{h}}) = 0, t \in [t_0^{\mathrm{h}}, t_1^{\mathrm{h}})$. Then, the observer (5) can be further expressed as the following time-delay model:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t - \tilde{h}(t)) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t),\ t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}}). \end{cases} \tag{15}$$

In Section 3, we are ready to propose a sliding mode controller u, based on the observer model (15), for the uncertain system (1). Because part of the state is not measurable, in this paper, our design principle is to employ the observer to obtain the state estimates, and then use the state estimates to construct a sliding mode controller, for achieving stability of the system with the network DoS attacks. Definition 1 is for stability analysis of the resulting SMC system.

Definition 1. The nominal system (1) with $v(t) \equiv 0$, $f(t) \equiv 0$, and $w(t) \equiv 0$ is said to be stable if there exists $\beta > 0$ fulfilling $\|x(t)\|^2 < \beta$ for $t > 0$. Furthermore, if $\lim_{t\to\infty} \|x(t)\|^2 = 0$ for all initial $x(0)$, then the nominal system (1) is said to be asymptotically stable. Additionally, if the nominal system in (1) is asymptotically stable, the uncertain system (1) with internal control $u(t)$, external disturbance $w(t)$, and all admissible uncertainties $\{v(t), f(t)\}$ described in Assumption 2, is said to be robustly stable.

3 Theoretical results

3.1 Controller design

By using the state estimates $\hat{x}(t)$, the following sliding mode controller is constructed as $u(t) = u_{\mathrm{o}}(t) + u_{\mathrm{s}}(t)$, with

$$u_{\mathrm{o}}(t) = G\hat{x}(t) - \eta(t), \tag{16}$$

$$u_{\mathrm{s}}(t) = -(KB)^{-1}\|KL(\bar{y}(t) - \hat{y}(t))\|\mathrm{sign}(s(t)) - \kappa(KB)^{-1}|s|^{\lambda}\mathrm{sign}(s(t)), \tag{17}$$

where $\eta(t) \triangleq (KB)^{-1}KL(\bar{y}(t) - \hat{y}(t))$, $\kappa > 0$, and $0 < \lambda < 1$. Matrix $G \in \mathbb{R}^{d_{\mathrm{u}} \times d_{\mathrm{x}}}$ is a controller parameter to be designed.

As a result, based on the observer (15), the following sliding motion dynamics can be yielded under the ideal sliding mode $s(t) \equiv \dot{s}(t) \equiv 0$ in the light of $\dot{s}(t) = 0$ and $s(0) = 0$:

$$\dot{\hat{x}}(t) = (A + BK)\hat{x}(t) + HL\big(y(t - \tilde{h}(t)) - \hat{y}(t)\big) \tag{18}$$

with $H \triangleq I - B(KB)^{-1}K$. Considering the design objective of the convergence of the system state estimation error $\tilde{x}(t) \triangleq x(t) - \hat{x}(t)$ and the system state $x(t)$, define $\xi(t) \triangleq [\tilde{x}^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$. Then, the following dynamics of the system augmented with the plant and the observer in the sliding mode can be extended as follows. For $t \in [t_j^{\mathrm{h}}, t_{j+1}^{\mathrm{h}})$,

$$\dot{\xi}(t) = (\mathcal{A} + \tilde{\mathcal{L}}(t))\xi(t) + \mathcal{H}\xi(t - \tilde{h}(t)) + \mathcal{E}\tilde{w}(t), \tag{19}$$

where $w(t) = w(0)$ and $\xi(t) = \xi(0)$ for $t \in [-h_M - d_{\max}, 0)$, $\tilde{w}(t) \triangleq [w^{\mathrm{T}}(t)\ v^{\mathrm{T}}(t)\ w^{\mathrm{T}}(t - \tilde{h}(t))]^{\mathrm{T}}$, and

$$\mathcal{A} \triangleq \begin{bmatrix} A & -A + HLC \\ \mathbf{0}_{d_x \times d_x} & A + BG \end{bmatrix},\ \mathcal{E} \triangleq \begin{bmatrix} E & B & -HLF \\ \mathbf{0}_{d_x \times d_w} & \mathbf{0}_{d_x \times d_u} & HLF \end{bmatrix},\ \mathcal{H} \triangleq \begin{bmatrix} -HLC & \mathbf{0}_{d_x \times d_x} \\ HLC & \mathbf{0}_{d_x \times d_x} \end{bmatrix},$$
$$\mathcal{M} \triangleq \begin{bmatrix} M^{\mathrm{T}} & \mathbf{0}_{d_p \times d_x} \end{bmatrix}^{\mathrm{T}},\ \ \mathcal{N} \triangleq \begin{bmatrix} N & \mathbf{0}_{d_q \times d_x} \end{bmatrix},\ \tilde{\mathcal{L}}(t) \triangleq \mathcal{M}L(t)\mathcal{N}.$$

Remark 2. Since the system (19) is augmented with the dynamics of the plant and the observer is derived in the sliding mode, the finite-time reachability of the ideal sliding mode $s(t) = 0$ should be guaranteed. Then, some conditions of the stability of the augmented system (19) can be explored for the stability of the whole closed-loop system, including the convergence of the resulting sliding motion (18) which is relative to the observer dynamics.

3.2 Reachability analysis of the sliding mode

Theorem 1. Consider the sliding variable (7) and the SMC law (6). The sliding mode $s(t) = 0$ can be achieved in finite-time under the SMC law (6) with the controller terms in (16) and (17).

Proof. Consider the following Lyapunov function $V_s(s(t)) = 0.5s^{\mathrm{T}}(t)s(t)$. Recalling the dynamics of $\hat{x}(t)$ in (15) or (5), we have

$$\begin{aligned} \dot{V}_s(s(t)) &= s^{\mathrm{T}}(t)\dot{s}(t) \\ &= s^{\mathrm{T}}(t)\big[K\big(A\hat{x}(t) + Bu(t) + L(\bar{y}(t) - \hat{y}(t))\big) - K\big(A\hat{x}(t) + Bu_o(t)\big)\big] \\ &= -\|KL(\bar{y}(t) - \hat{y}(t))\|\|s(t)\|_1 + s^{\mathrm{T}}(t)KL(\bar{y}(t) - \hat{y}(t)) - \kappa|s|^{\lambda}\mathrm{sign}(s(t)) \\ &\leqslant -\kappa|s|^{\lambda}\mathrm{sign}(s(t)), \end{aligned}$$

which can be concluded that $V_s(s(t)) = 0$, that is $s(t) = 0$, can be realized within finite time.

From the results of proof above we known that, the term $-(KB)^{-1}\|KL(\bar{y}(t) - \hat{y}(t))\|\mathrm{sign}(s(t))$ in $u_s(t)$ acts on the compensation for the observation error $\bar{y}(t) - \hat{y}(t)$ which is influenced by some factors including the DoS attack duration. Meanwhile, one can apply some other reaching functions instead of $-\kappa|s|^{\lambda}\mathrm{sign}(s(t))$ in $u_s(t)$, for a good reachability performance including the fast reaching and the chattering weakening.

3.3 Stability analysis of the sliding motion

The augmented system (19) is a kind of time-varying delay system with interval time delay $d_{\min} \leqslant \tilde{h}(t) < h_M + d_{\max}$, as discussed in (13). In this paper, we apply the Lyapunov-Krasovskii functional method to explore the stability of this time-delay system. Let $h_d = h_M + d_{\max}$ in the following context, for simplicity.

Theorem 2. Consider the uncertain time-varying delay system (19). For some scalars $h_d > d_{\min} > 0$, the uncertain system (19) is robustly stable with an $\mathscr{L}_2$ gain less than or equal to γ if there exist a scalar $\lambda > 0$, positive definite matrices $Q_i \in \mathbb{R}^{d_x \times d_x}$, $R_i \in \mathbb{R}^{d_x \times d_x}$, matrices $S_{jk} \in \mathbb{R}^{2d_x \times 2d_x}$, and $S_{j5} \in \mathbb{R}^{(2d_w + d_v) \times 2d_x}$ ($i = 1, 2, \dots, 6$ and $j, k = 1, 2, \dots, 4$) such that

$$\bar{\Phi} + [\Psi]_{\mathrm{sym}} + \bar{\Upsilon}_1^{\mathrm{T}}(d_{\min}\mathcal{Q}_4 + \bar{h}\mathcal{Q}_5 + h_d\mathcal{Q}_6)\bar{\Upsilon}_1 + \mathcal{S} < 0, \tag{20}$$

where $\mathcal{S} \triangleq +S_1\mathcal{Q}_4^{-1}S_1^{\mathrm{T}} + S_2\mathcal{Q}_5^{-1}S_2^{\mathrm{T}} + S_3\mathcal{Q}_5^{-1}S_3^{\mathrm{T}} + S_4\mathcal{Q}_6^{-1}S_4^{\mathrm{T}}$ and

$$\bar{\Phi} \triangleq \begin{bmatrix} \bar{\Phi}_0 & \mathbf{0} & \mathcal{Q}_1\mathcal{H} & \mathbf{0} & \mathcal{Q}_1\mathcal{E} \\ * & -\mathcal{Q}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathcal{Q}_3 & \mathbf{0} \\ * & * & * & * & -\gamma^2 I \end{bmatrix},\ \mathcal{C} \triangleq \begin{bmatrix} \bar{C} & \bar{C} \\ \bar{C} & \bar{C} \end{bmatrix},\ \bar{\Phi}_0 \triangleq \big[\mathcal{Q}_1(\mathcal{A} + \tilde{\mathcal{L}}(t))\big]_{\mathrm{sym}} + \mathcal{C} + \mathcal{Q}_2 + \mathcal{Q}_3,$$

$$\begin{aligned} &\Psi \triangleq \begin{bmatrix} (S_1 + S_4) & -(S_1 - S_2) & -(S_2 - S_3) & -(S_3 + S_4) & \mathbf{0}_{d_w \times d_w} \end{bmatrix},\ \ S_i \triangleq \begin{bmatrix} S_{i1}^{\mathrm{T}} & S_{i2}^{\mathrm{T}} & S_{i3}^{\mathrm{T}} & S_{i4}^{\mathrm{T}} & S_{i5}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &\bar{\Upsilon}_1 \triangleq \begin{bmatrix} \mathcal{A} + \tilde{\mathcal{L}}(t) & \mathbf{0} & \mathcal{H} & \mathbf{0} & \mathcal{E} \end{bmatrix},\ \ \bar{C} \triangleq C^{\mathrm{T}}C,\ \ \mathcal{Q}_i \triangleq \mathrm{diag}\{Q_i, R_i\}, \\ &\Lambda_1 \triangleq \mathrm{diag}\big\{-\mathcal{Q}_4, -\mathcal{Q}_5, -\mathcal{Q}_6\big\},\ \ \Lambda_2 \triangleq \mathrm{diag}\big\{-\mathcal{Q}_4, -\mathcal{Q}_5, -\mathcal{Q}_5, -\mathcal{Q}_6\big\}. \end{aligned} \tag{21}$$

Proof. Choose the Lyapunov functional candidate:

$$V(t)=\sum_{i=1}^{6}V_i(t),\tag{22}$$

where

$$\begin{aligned}&V_1(t)=\xi^{\mathrm T}(t)\mathcal{Q}_1\xi(t),\quad V_2(t)=\int_{t-d_{\min}}^{t}\xi^{\mathrm T}(s)\mathcal{Q}_2\xi(s)\mathrm{d}s,\\&V_3(t)=\int_{t-h_{\mathrm d}}^{t}\xi^{\mathrm T}(s)\mathcal{Q}_3\xi(s)\mathrm{d}s,\quad V_4(t)=d_{\min}\int_{-d_{\min}}^{0}\int_{t+v}^{t}\dot\xi^{\mathrm T}(s)\mathcal{Q}_4\dot\xi(s)\mathrm{d}s\mathrm{d}v,\\&V_5(t)=\bar h\int_{-h_{\mathrm d}}^{-d_{\min}}\int_{t+v}^{t}\dot\xi^{\mathrm T}(s)\mathcal{Q}_5\dot\xi(s)\mathrm{d}s\mathrm{d}v,\quad V_6(t)=h_{\mathrm d}\int_{-h_{\mathrm d}}^{0}\int_{t+v}^{t}\dot\xi^{\mathrm T}(s)\mathcal{Q}_6\dot\xi(s)\mathrm{d}s\mathrm{d}v,\end{aligned}$$

where $\bar h\triangleq h_{\mathrm d}-d_{\min}$. Consider that $\dot{\tilde h}(t)=1,\forall t\in[t_j^{\mathrm h},t_{j+1}^{\mathrm h})$. Then, by calculating the derivatives of $V_i(t)$, we obtain that

$$\begin{aligned}&\dot V_1=2\xi^{\mathrm T}(t)\mathcal{Q}_1\dot\xi(t),\quad \dot V_2=\xi^{\mathrm T}(t)\mathcal{Q}_2\xi(t)-\xi^{\mathrm T}(t-d_{\min})\mathcal{Q}_2\xi(t-d_{\min}),\\&\dot V_3=\xi^{\mathrm T}(t)\mathcal{Q}_3\xi(t)-\xi^{\mathrm T}(t-h_{\mathrm d})\mathcal{Q}_3\xi(t-h_{\mathrm d}),\\&\dot V_4=d_{\min}^2\dot\xi^{\mathrm T}(t)\mathcal{Q}_4\dot\xi(t)-d_{\min}\int_{t-d_{\min}}^{t}\dot\xi^{\mathrm T}(v)\mathcal{Q}_4\dot\xi(v)\mathrm{d}v,\\&\dot V_5=\bar h^2\dot\xi^{\mathrm T}(t)\mathcal{Q}_5\dot\xi(t)-\bar h\int_{t-h_{\mathrm d}}^{t-d_{\min}}\dot\xi^{\mathrm T}(v)\mathcal{Q}_5\dot\xi(v)\mathrm{d}v,\\&\dot V_6=h_{\mathrm d}^2\dot\xi^{\mathrm T}(t)\mathcal{Q}_6\dot\xi(t)-h_{\mathrm d}\int_{t-h_{\mathrm d}}^{t}\dot\xi^{\mathrm T}(v)\mathcal{Q}_6\dot\xi(v)\mathrm{d}v.\end{aligned}$$

Consider the slack matrices: $S_i\triangleq[S_{i1}^{\mathrm T}\ S_{i2}^{\mathrm T}\ S_{i3}^{\mathrm T}\ S_{i4}^{\mathrm T}\ S_{i5}^{\mathrm T}]^{\mathrm T}$ $(i=1,2,3,4)$. By combining them with the Leibniz-Newton formula $\xi(t-d_{\min})-\xi(t)=\int_{t-d_{\min}}^{t}\dot\xi(s)\mathrm{d}s$, we can get the following identities:

$$\begin{aligned}&0=2\zeta^{\mathrm T}(t)S_1\left(\xi(t)-\xi(t-d_{\min})-\int_{t-d_{\min}}^{t}\dot\xi(s)\mathrm{d}s\right),\\&0=2\zeta^{\mathrm T}(t)S_2\left(\xi(t-d_{\min})-\xi(t-\tilde h(t))-\xi_{t-\tilde h(t)}^{t-d_{\min}}\dot\xi(v)\mathrm{d}v\right),\\&0=2\zeta^{\mathrm T}(t)S_3\left(\xi(t-\tilde h(t))-\xi(t-h_{\mathrm d})-\int_{t-h_{\mathrm d}}^{t-\tilde h(t)}\dot\xi(v)\mathrm{d}v\right),\\&0=2\zeta^{\mathrm T}(t)S_4\left(\xi(t)-\xi(t-h_{\mathrm d})-\int_{t-h_{\mathrm d}}^{t}\dot\xi(v)\mathrm{d}v\right),\end{aligned}\tag{23}$$

where $\zeta(t)=[\xi^{\mathrm T}(t)\ \xi^{\mathrm T}(t-d_{\min})\ \xi^{\mathrm T}(t-\tilde h(t))\ \xi^{\mathrm T}(t-h_{\mathrm d})\ \tilde w^{\mathrm T}(t)]^{\mathrm T}$.

Before further, let us introduce the following lemma [24]: For any vectors a, b and appropriate dimension matrix $X>0$, it holds that $a^{\mathrm T}b+b^{\mathrm T}a\leqslant a^{\mathrm T}X^{-1}a+b^{\mathrm T}Xb$. Furthermore, consider the following lemma from [25] that: For a matrix $0<X\in\mathbb{R}^{n\times n}$, and a continuous vector function $b(t)$ in $[t_1,t_2]\to\mathbb{R}^n$, it holds that

$$\left(\int_{t_1}^{t_2}b^{\mathrm T}(s)\mathrm{d}s\right)X\left(\int_{t_1}^{t_2}b(s)\mathrm{d}s\right)\leqslant(t_2-t_1)\int_{t_1}^{t_2}b^{\mathrm T}(s)Xb(s)\mathrm{d}s.$$

Then, by considering that

$$\zeta^{\mathrm T}(t)S_1\int_{t-d_{\min}}^{t}\dot\xi(s)\mathrm{d}s=\int_{t-d_{\min}}^{t}\underbrace{\zeta^{\mathrm T}(t)S_1}_{a^{\mathrm T}}\underbrace{\dot\xi(s)}_{b}\mathrm{d}s,$$

one can derive that

$$-2\zeta^{\mathrm T}(t)S_1\int_{t-d_{\min}}^{t}\dot\xi(s)\mathrm{d}s\leqslant\zeta^{\mathrm T}(t)S_1\mathcal{Q}_4^{-1}S_1^{\mathrm T}\zeta(t)+d_{\min}\int_{t-d_{\min}}^{t}\dot\xi^{\mathrm T}(v)\mathcal{Q}_4\dot\xi(v)\mathrm{d}v,\tag{24}$$

$$-2\zeta^{\mathrm{T}}(t)S_2\int_{t-\tilde{h}(t)}^{t-d_{\min}}\dot{\xi}(s)\mathrm{d}s\leqslant\zeta^{\mathrm{T}}(t)S_2\mathcal{Q}_5^{-1}S_2^{\mathrm{T}}\zeta(t)+h_1\int_{t-\tilde{h}(t)}^{t-d_{\min}}\dot{\xi}^{\mathrm{T}}(v)\mathcal{Q}_5\dot{\xi}(v)\mathrm{d}v, \tag{25}$$

$$-2\zeta^{\mathrm{T}}(t)S_3\int_{t-h_{\mathrm{d}}}^{t-\tilde{h}(t)}\dot{\xi}(s)\mathrm{d}s\leqslant\zeta^{\mathrm{T}}(t)S_3\mathcal{Q}_5^{-1}S_3^{\mathrm{T}}\zeta(t)+h_2\int_{t-\tilde{h}(t)}^{t-d_{\min}}\dot{\xi}^{\mathrm{T}}(v)\mathcal{Q}_5\dot{\xi}(v)\mathrm{d}v, \tag{26}$$

$$-2\zeta^{\mathrm{T}}(t)S_4\int_{t-h_{\mathrm{d}}}^{t}\dot{\xi}(v)\mathrm{d}s\leqslant\zeta^{\mathrm{T}}(t)S_4\mathcal{Q}_6^{-1}S_4^{\mathrm{T}}\zeta(t)+h_{\mathrm{d}}\int_{t-h_{\mathrm{d}}}^{t}\dot{\xi}^{\mathrm{T}}(v)\mathcal{Q}_6\dot{\xi}(v)\mathrm{d}v, \tag{27}$$

where $h_1\triangleq\tilde{h}(t)-d_{\min}$, $h_2\triangleq h_{\mathrm{d}}-\tilde{h}(t)$ and $h_1+h_2=\bar{h}$.

Firstly, for the nominal time-delay system (19) with $\tilde{w}(t)\equiv 0$, recalling $\tilde{h}(t)-d_{\min}<h_{\mathrm{d}}-d_{\min}=\bar{h}$ and $h_{\mathrm{d}}-\tilde{h}(t)\leqslant h_{\mathrm{d}}-d_{\min}=\bar{h}$, from the calculated results of $\dot{V}(t)$ and (23)–(27), we can directly obtain that $\dot{V}(t)<0$ for any $\xi(t)\neq 0$, according to the condition (20). This means that the asymptotic stability of the system (19) can be guaranteed.

Secondly, for the time-delay system (19), let us consider a finite-gain dissipativity [26, 27] to evaluate the system performance. Suppose γ is a given positive real number. The time-delay system (19) is said to have an $\mathscr{L}_2$ gain less than or equal to γ if

$$\int_0^{t_N^{\mathrm{h}}}y^{\mathrm{T}}(t)y(t)\mathrm{d}t\leqslant\int_0^{t_N^{\mathrm{h}}}\gamma^2w^{\mathrm{T}}(t)w(t)\mathrm{d}t$$

for all $t\in[t_j^{\mathrm{h}},t_{j+1}^{\mathrm{h}})$ and all $\tilde{w}(t)\in\mathcal{L}_2[0,+\infty)$ with the output $y(t)$ resulting by $\tilde{w}(t)$ from initial state $\xi(t_j^{\mathrm{h}})$. Then, considering the uncertain time-delay system (19) with the disturbance $\tilde{w}(t)\in\mathcal{L}_2[0,\infty)$, we further derive that

$$\begin{aligned}&\dot{V}(t)+y^{\mathrm{T}}(t)y(t)-\gamma^2\tilde{w}^{\mathrm{T}}(t)\tilde{w}(t)\\&<\xi^{\mathrm{T}}(t)\left[\bar{\Phi}+\bar{\Upsilon}_1^{\mathrm{T}}(d_{\min}^2\mathcal{Q}_4+\bar{h}^2\mathcal{Q}_5+h_{\mathrm{d}}^2\mathcal{Q}_6)\bar{\Upsilon}_1+[\Psi]_{\mathrm{sym}}+\mathcal{S}\right]\xi(t).\end{aligned} \tag{28}$$

According to the condition (20), it holds that for $\xi(t)\neq 0$,

$$\dot{V}(t)+y^{\mathrm{T}}(t)y(t)-\gamma^2\tilde{w}^{\mathrm{T}}(t)\tilde{w}(t)<0,$$

from which by integrating the left-hand side terms from t_j^{h} to t_{j+1}^{h}, it yields

$$\int_{t_j^{\mathrm{h}}}^{t_{j+1}^{\mathrm{h}}}\left[y^{\mathrm{T}}(t)y(t)\mathrm{d}t-\gamma^2w^{\mathrm{T}}(t)w(t)\right]\mathrm{d}t<V(t_j^{\mathrm{h}})-V(t_{j+1}^{\mathrm{h}}).$$

Furthermore, for $j=0,1,2,3,\ldots$, that is $t\in\bigcup_{j=0}^{N}[t_j^{\mathrm{h}},t_{j+1}^{\mathrm{h}})\subset[0,\infty)$, we can further obtain

$$\begin{aligned}\int_{t_0^{\mathrm{h}}}^{t_N^{\mathrm{h}}}\left[y^{\mathrm{T}}(t)y(t)\mathrm{d}t-\gamma^2w^{\mathrm{T}}(t)w(t)\right]\mathrm{d}t&=\sum_{j=0}^{N}\int_{t_j^{\mathrm{h}}}^{t_{j+1}^{\mathrm{h}}}\left[y^{\mathrm{T}}(t)y(t)\mathrm{d}t-\gamma^2w^{\mathrm{T}}(t)w(t)\right]\mathrm{d}t\\&<\sum_{j=0}^{N}V(t_j^{\mathrm{h}})-V(t_{j+1}^{\mathrm{h}})=V(t_0^{\mathrm{h}})-V(t_N^{\mathrm{h}}).\end{aligned}$$

Under the zero initial condition $V(t_0^{\mathrm{h}})=V(0)=0$, and $V(t_N^{\mathrm{h}})\geqslant 0$, it thus can be concluded that

$$\int_0^{t_N^{\mathrm{h}}}y^{\mathrm{T}}(t)y(t)\mathrm{d}t\leqslant\int_0^{t_N^{\mathrm{h}}}\gamma^2\tilde{w}^{\mathrm{T}}(t)\tilde{w}(t)\mathrm{d}t.$$

Therefore, according to Definition 1, the uncertain time-delay system (19) is robustly stable with an $\mathscr{L}_2$ gain less than or equal to γ, with the system parameters fulfilling the condition (20). This completes the proof.

The condition in Theorem 2 embeds the time-varying matrix $\tilde{\mathcal{L}}(t)$, which includes the unknown system perturbation $f(x,(t))$. We develop this condition to another condition without time-varying terms for the verifiability of the stability condition for the resulting system (19).

Corollary 1. Consider the uncertain time-varying delay system (19). For some scalars $h_{\rm d} > d_{\min} > 0$, the uncertain system (19) is robustly stable with an $\mathscr{L}_2$ gain less than or equal to γ if there exist a scalar $\lambda > 0$, positive definite matrices $Q_i \in \mathbb{R}^{\rm d_x \times d_x}$, $R_i \in \mathbb{R}^{d_{\rm x} \times d_{\rm x}}$, matrices $S_{jk} \in \mathbb{R}^{2d_{\rm x} \times 2d_{\rm x}}$, and $S_{j5} \in \mathbb{R}^{(2d_{\rm w}+d_{\rm v}) \times 2d_{\rm x}}$ ($i = 1, 2, \ldots, 6$ and $j, k = 1, 2, \ldots, 4$) such that

$$\Xi \triangleq \begin{bmatrix} \Phi + [\Psi]_{\rm sym} & \Pi_1 & \Pi_2 & \Pi_3^{\rm T} \\ * & \Lambda_1 & \mathbf{0} & \Pi_4^{\rm T} \\ * & * & \Lambda_2 & \mathbf{0} \\ * & * & * & -\lambda I \end{bmatrix} < 0, \tag{29}$$

where

$$\Phi \triangleq \begin{bmatrix} \Phi_0 & \mathbf{0} & \mathcal{Q}_1\mathcal{H} & \mathbf{0} & \mathcal{Q}_1\mathcal{E} \\ * & -\mathcal{Q}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathcal{Q}_3 & \mathbf{0} \\ * & * & * & * & -\gamma^2 I \end{bmatrix}, \quad \Phi_0 \triangleq [\mathcal{Q}_1\mathcal{A}]_{\rm sym} + \mathcal{Q}_2 + \mathcal{Q}_3 + \mathcal{C} + \lambda\mathcal{N}^{\rm T}\mathcal{N},$$

$$\begin{aligned} &\Pi_1 \triangleq \begin{bmatrix} d_{\min}\Upsilon_1^{\rm T}\mathcal{Q}_4 & \bar{h}\Upsilon_1^{\rm T}\mathcal{Q}_5 & h_{\rm d}\Upsilon_1^{\rm T}\mathcal{Q}_6 \end{bmatrix}, \ \Pi_2 \triangleq \begin{bmatrix} S_1 & S_2 & S_3 & S_4 \end{bmatrix}, \ \Upsilon_1 \triangleq \begin{bmatrix} \mathcal{A} & \mathbf{0} & \mathcal{H} & \mathbf{0} & \mathcal{E} \end{bmatrix}, \\ &\Pi_3 \triangleq \begin{bmatrix} \mathcal{M}^{\rm T}\mathcal{Q}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \Pi_4 \triangleq \begin{bmatrix} d_{\min}\mathcal{M}^{\rm T}\mathcal{Q}_4 & \bar{h}\mathcal{M}^{\rm T}\mathcal{Q}_5 & h_{\rm d}\mathcal{M}^{\rm T}\mathcal{Q}_6 \end{bmatrix}. \end{aligned} \tag{30}$$

Proof. To get the same conclusion, we may prove that the condition (20) can be derived from the condition (29). By using the Schur complement to $\Xi < 0$ in (29), it can be obtained that $\tilde{\Xi} + \lambda\bar{N}^{\rm T}\bar{N} + \lambda^{-1}\Pi_5^{\rm T}\Pi_5 < 0$, where

$$\tilde{\Xi} \triangleq \begin{bmatrix} \tilde{\Phi} + [\Psi]_{\rm sym} & \Pi_1 & \Pi_2 \\ * & \Lambda_1 & \mathbf{0} \\ * & * & \Lambda_2 \end{bmatrix}, \quad \tilde{\Phi} \triangleq \begin{bmatrix} \tilde{\Phi}_0 & \mathbf{0} & \mathcal{Q}_1\mathcal{H} & \mathbf{0} & \mathcal{Q}_1\mathcal{E} \\ * & -\mathcal{Q}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\mathcal{Q}_3 & \mathbf{0} \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned} &\bar{N} \triangleq \begin{bmatrix} \mathcal{N} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \tilde{\Phi}_0 \triangleq \left[\mathcal{Q}_1\mathcal{A}\right]_{\rm sym} + \mathcal{Q}_2 + \mathcal{Q}_3, \\ &\Pi_5 \triangleq \begin{bmatrix} \Pi_{51} & d_{\min}\mathcal{M}^{\rm T}\mathcal{Q}_4 & \bar{h}\mathcal{M}^{\rm T}\mathcal{Q}_5 & h_{\rm d}\mathcal{M}^{\rm T}\mathcal{Q}_6 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \Pi_{51} \triangleq \begin{bmatrix} \mathcal{M}^{\rm T}\mathcal{Q}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \end{aligned}$$

Furthermore, in the light of Proposition 2.1 in [28], considering $\tilde{\Xi} + \lambda\bar{N}^{\rm T}\bar{N} + \lambda^{-1}\Pi_5^{\rm T}\Pi_5 < 0$, one can obtain $\bar{\Xi} < 0$, where

$$\bar{\Xi} \triangleq \begin{bmatrix} \bar{\Phi} + [\Psi]_{\rm sym} & \bar{\Pi}_1 & \Pi_2 \\ * & \Lambda_1 & \mathbf{0} \\ * & * & \Lambda_2 \end{bmatrix}, \quad \bar{\Pi}_1 \triangleq \begin{bmatrix} d_{\min}\bar{\Upsilon}_1^{\rm T}\mathcal{Q}_4 & \bar{h}\bar{\Upsilon}_1^{\rm T}\mathcal{Q}_5 & h_{\rm d}\bar{\Upsilon}_1^{\rm T}\mathcal{Q}_6 \end{bmatrix},$$

and $\bar{\Phi}$ and $\bar{\Upsilon}_1$ are defined in (21). Then, using the Schur complement to $\bar{\Xi} < 0$, we obtain

$$\bar{\Phi} + [\Psi]_{\rm sym} + \bar{\Upsilon}_1^{\rm T}(d_{\min}^2 Q_4 + \bar{h}^2 Q_5 + h_{\rm d}^2 Q_6)\bar{\Upsilon}_1 + \mathcal{S} < 0,$$

which meets the condition (20) in Theorem 2. The proof is completed.

Remark 3. Theorem 2 and Corollary 1 provide the stability criterion of closed-loop system in sliding mode state. In fact, the stability criterion also gives the stability condition of the sliding motion, that is dynamics of the observer state $\hat{x}$. Moreover, since the initial $s(0) = 0$ and the derivative $\dot{s}(t) = 0, \forall t \geqslant 0$, it guarantees that the system trajectories will always move towards the system origin from the initial condition $s(0) = 0$.

Remark 4. These stability criteria which are delay-dependent ones depend on the bounds $d_{\min}$ and $h_{\rm d}$ of the time delay. Since $h_{\rm d} = 2\Delta t_{\max} + \mu_{\max} + 2d_{\max} - d_{\min}$, large intervals of the sampling, inter-event time, DoS attack duration and transmission delay may affect the stability of the control system. Moreover, the sampling period T also affects the event-triggering instants because the event detecting period is designed in accordance with T. Besides, for the triggering error tolerance $0 < \rho < 1$, a small ρ implies a faster triggering, which may reduce the value of the delay. Considering the allowed maximums of the DoS attack durations and transmission delays, we may apply the event-triggering condition (8) for releasing the measurements.

Based on the results designed and analyzed above, we summarize the algorithm of the presented time-delay-based SMC against DoS attacks and discrete events as Algorithm 1.

Algorithm 1 Time-delay-based estimation and SMC against DoS attacks and discrete events

Require: Complete parameters of a physical system (1).

Ensure: Secure control of the system (1) with DoS attacks on the network between the controller and the sensor.

1: Initialize $k = 0$, sampling sequence $\mathbb{T} \triangleq \{T_1, T_2, \ldots, T_\infty\}$, triggering error tolerance $0 < \rho < 1$ of event-triggering condition (8), sliding surface matrix K (7), $h_M = 2\Delta t_{\max} + \mu_{\max} + d_{\max} - d_{\min}$, $h_{\rm d} = h_M + d_{\max}$, initial condition $x(0) = \hat{x}(0)$;

2: Solve the conditions (29) in Corollary 1 for the parameter $\{G, L\}$ of the controller and the observer;

3: Update the controller according to the SMC law: $u(t) = u_{\rm o}(t) + u_{\rm s}(t)$ as given in (16), (17), and (7);

4: Update the state estimator under the updated controller: $\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(\bar{y}(t) - \hat{y}(t))$ (5);

5: Update the system (1) under the updated controller;

6: **return** $x(t)$ and $\hat{x}(t)$.

4 Discussions on the extension of the presented secure control method

In fact, the designed SMC law is based on the integral-type model with the sliding surface $s(t) = s(0) = 0$. Therefore, the switching term can be removed from the SMC law and some modifications on the estimator can be made to design a continuous control law to secure control against the DoS attacks. Under the same assumptions, let us consider the plant (1). The following sliding mode observer is employed to estimate the system state.

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - \left[(KB)^{-1}\|KL(\bar{y}(t) - \hat{y}(t))\| + \lambda I_{d_{\rm u}}\right]\mathrm{sign}(s(t)) + L(\bar{y}(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{31}$$

where λ is a positive scalar. The controller can be thus modified as

$$u(t) = G\hat{x}(t). \tag{32}$$

By using the same sliding variable $s(t) \in \mathbb{R}^{d_{\rm u}}$ in (7), the finite-time reachability of the sliding surface $s(t) = 0$ can be achieved.

Theorem 3. Consider the observer-based sliding variable (7) and control law (32). The sliding surface $s(t) = 0$ can be achieved in finite-time under the control law (32).

Proof. The result can be easily obtained by considering the following Lyapunov function $V_{\rm s}(s(t)) = 0.5s^{\rm T}(t)s(t)$, and thus the proof is omitted.

Hence, in the ideal sliding mode, the resulting augmented system (19) consisting of the control system and the estimation system is updated as

$$\dot{\xi}(t) = (\mathcal{A} + \tilde{\mathcal{L}}(t))\xi(t) + \mathcal{H}\xi(t - \tilde{h}(t)) + \mathcal{E}\tilde{w}(t), \tag{33}$$

where $w(t) = w(0)$ and $\xi(t) = \xi(0)$ for $t \in [-h_M - d_{\max}, 0)$, $\tilde{w}(t) \triangleq [w^{\rm T}(t)\ v^{\rm T}(t)\ w^{\rm T}(t - \tilde{h}(t))]^{\rm T}$, $\mathcal{M}$, $\mathcal{N}$, and $\tilde{\mathcal{L}}$ are defined in (19), and

$$\mathcal{A} :\triangleq \begin{bmatrix} A & -A + LC \\ \mathbf{0}_{d_{\rm x}\times d_{\rm x}} & A + BG \end{bmatrix}, \quad \mathcal{E} :\triangleq \begin{bmatrix} E & B & -LF \\ \mathbf{0}_{d_{\rm x}\times d_{\rm w}} & \mathbf{0}_{d_{\rm x}\times d_{\rm u}} & LF \end{bmatrix}, \quad \mathcal{H} :\triangleq \begin{bmatrix} -LC & \mathbf{0}_{d_{\rm x}\times d_{\rm x}} \\ LC & \mathbf{0}_{d_{\rm x}\times d_{\rm x}} \end{bmatrix}.$$

By following Theorem 2 and Corollary 1, the following conditions are directly obtained to judge the stability of the control system and the estimation system.

Corollary 2. Consider the uncertain time-varying delay system (33). For some scalars $h_{\rm d} > d_{\min} > 0$, the uncertain system (33) is robustly stable with an $\mathscr{L}_2$ gain less than or equal to γ if there exist a scalar $\lambda > 0$, positive definite matrices $Q_i \in \mathbb{R}^{\rm d_x \times d_x}$, $R_i \in \mathbb{R}^{d_{\rm x} \times d_{\rm x}}$, matrices $S_{jk} \in \mathbb{R}^{2d_{\rm x} \times 2d_{\rm x}}$, and $S_{j5} \in \mathbb{R}^{(2d_{\rm w}+d_{\rm v}) \times 2d_{\rm x}}$ $(i = 1, 2, \ldots, 6$ and $j, k = 1, 2, \ldots, 4)$ such that the matrix inequality in (29) holds.

Remark 5. Since the control law includes no switching term, this sliding-mode-observer-based control can be extended to the plant with the output $y(t) = Cx(t) + Du(t) + Fw(t)$. Some nonlinearities in the system dynamics can be also allowed. The switching term in the sliding mode observer can be modified to compensate for the nonlinearities. In this DoS attack case, the switching term plays a role to compensate for the error caused by the DoS attacks and the external disturbances.

Remark 6. The presented event-triggering mechanism (2) is a common static one used in NCSs. Recently, some advanced event-triggering conditions for improving the control and communication performance have been reported in the literature. Readers can refer to [29, 30] for details. Since the errors induced by the advanced event-triggering conditions can be described by the time-delay model, the secure control design method can be integrated with the advanced event-triggering conditions for some advanced event-based secure control in NCSs.

5 Application to networked pendulum systems

Let us consider a networked system of an inverted pendulum with a cart [31] to show the effectiveness of the presented secure control method. By taking the mass of the pendulum $m = 0.535$ kg, the length of the pendulum $l = 0.365$ m, the mass of the cart $M = 3.2$ kg, the gravitational acceleration $g = 9.807$ m/s^2, the cart position coordinate x_1, the pendulum angle from vertical x_3, the state vector $x = [x_1\ \dot{x}_1\ x_3\ \dot{x}_3] = [x_1\ x_2\ x_3\ x_4]$, and the control input force u, a linearized system dynamics of the inverted pendulum system is obtained below.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-(I+ml^2)b}{I(M+m)+Mml^2} & \dfrac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-mlb}{I(M+m)+Mml^2} & \dfrac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{I+ml}{I(M+m)+Mml^2} \\ 0 \\ \dfrac{ml}{I(M+m)+Mml^2} \end{bmatrix} u,$$

where $y = \left[\begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{smallmatrix}\right] x$ is considered the measurable system output. In the simulations, consider the external disturbance $w = 0.1(\text{rand} - 0.5)$ with $E = [0.01\ 0\ 0.01\ 0]^{\rm T}$ and $F = [0.01\ 0]^{\rm T}$, the system perturbations $v(t) = 0.1\sin(u(t))$ and $f(t) = ML(t)Nx(t)$ with $M = [0.1\ 0.1\ 0\ 0]^{\rm T}$, $N = [0.1\ 0.1\ 0\ 0]$, and $L(t) = \sin(x_1(t) + x_3(t))$.

For the simulated networked pendulum system, we use a fixed sampling period of the sampler as $T = 0.02$ s. Assume that the transmission delay d_j on the j-th network transmission is in $[0.004, 0.016]$ s with $d_{\min} = 0.004$ s and $d_{\max} = 0.016$ s, and the constraints of the DoS attack duration $\mu_{\max} = 0.8$ s, $\nu_{\min} = 0.1$ s. Then, according to the conditions in Corollary 1, we use the following parameters of the controller and observer in the simulations to show the response of the controlled system:

$$L = \begin{bmatrix} 27.7832 & -0.0098 \\ 160.5792 & -119.7819 \\ 4.4431 & 35.0350 \\ -93.6083 & 268.5555 \end{bmatrix}, \quad \begin{aligned} G &= \begin{bmatrix} 12.2506\ 13.1018\ -80.1023\ -15.7809 \end{bmatrix}, \\ K &= \begin{bmatrix} 0.10\ -0.50\ 0.20\ -0.30 \end{bmatrix}, \end{aligned}$$

and the $\mathscr{L}_2$ gain performance $\gamma_{\min} = 0.0023$. By using the event-triggering condition coefficient $\rho = 0.05$ and the initial condition $x(0) = \hat{x}(0) = [\pi/3\ \ 0\ \ \pi/4\ \ 0]^{\rm T}$, a DoS attack sequence is shown in Figure 3(a), and the responses of the networked pendulum system are in Figures 4 and 5. The inner-event intervals are depicted in Figure 3(b) based on the event-triggering mechanism. Figures 4(a) and (b) show the real measurements at the sampler $y(kT)$, the event sender $y(t_i)$ and the ZOH $\bar{y}(t)$ of the system output $y(t)$, at different time instants. The control input force is depicted in Figure 4(c), under which the states of the pendulum and its estimation states are shown in Figures 5(a) and (b), respectively. Besides, the

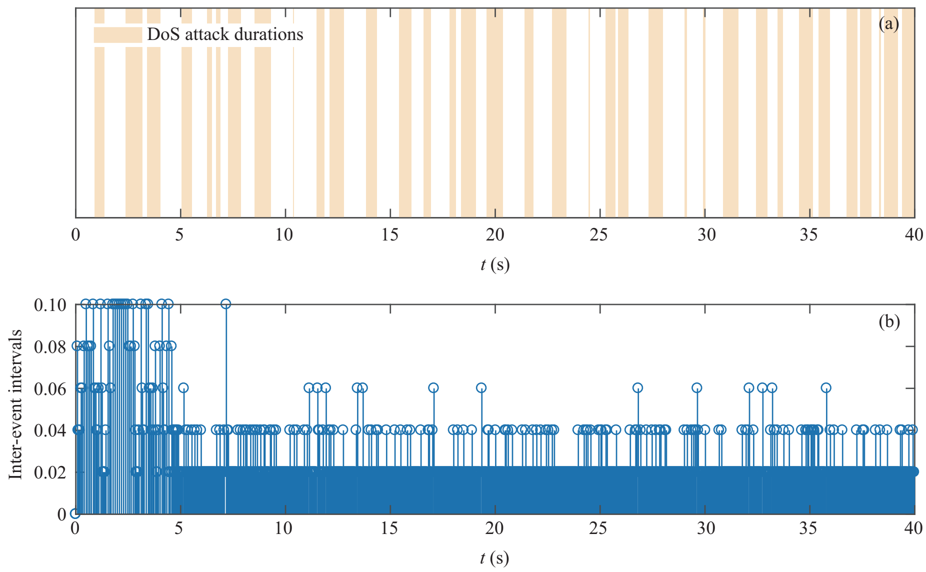

Figure 3 (Color online) Illustrations of (a) the DoS attack durations and (b) the event-triggering instants.

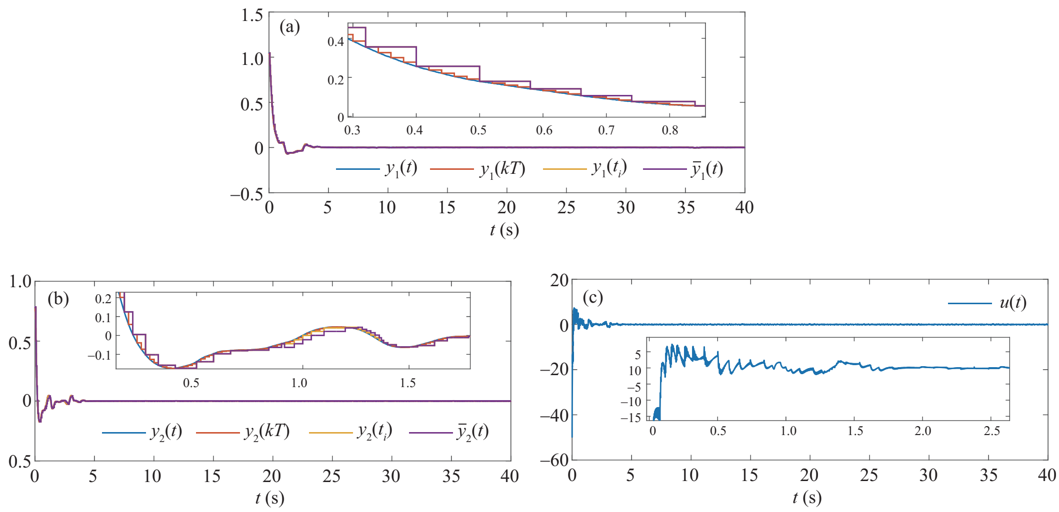

Figure 4 (Color online) (a) and (b) are the measurements at the samplers $y_1(kT)$ and $y_2(kT)$, the event senders $y_1(t_i)$ and $y_2(t_i)$, and the ZOHs $\bar{y}_1(t)$ and $\bar{y}_2(t)$ of the system outputs $y_1(t)$ and $y_2(t)$, at different time instants, while (c) is the control input.

estimation errors of the states are shown in Figure 5(c). Obviously, the simulation results illustrate that the proposed estimator-based SMC is effective to secure control against the DoS attacks.

Additionally, let us check different DoS attack durations on the network affect the stability of the networked pendulum system, by using several sets of the constraints of the DoS attack durations, that is

- Set 1: No DoS attacks $\mu_{\max} = 0$,
- Set 2: DoS attack duration constraints $\mu_{\max} = 1.0$ s, $\nu_{\min} = 0.1$ s,
- Set 3: DoS attack duration constraints $\mu_{\max} = 3.0$ s, $\nu_{\min} = 0.1$ s,
- Set 4: DoS attack duration constraints $\mu_{\max} = 3.0$ s, $\nu_{\min} = 1.0$ s.

Then, the corresponding results are obtained as shown in Figures 6–9, from which one can observe that the DoS attacks (with larger values of the DoS attack duration $\mu_{\max}$) degrade the system stability (more serious). Figures 6–9 show the trajectories of the plant states $x(t)$, the observer states $\hat{x}(t)$ and their errors $\tilde{x}(t)$, under Sets 1–4, respectively. Besides, from Figures 8 and 9, it can be concluded that a large $\nu_{\min}$ can mitigate the impact of the DoS attacks on the system stability since more triggered measurements can be transformed from the sensor to the controller.

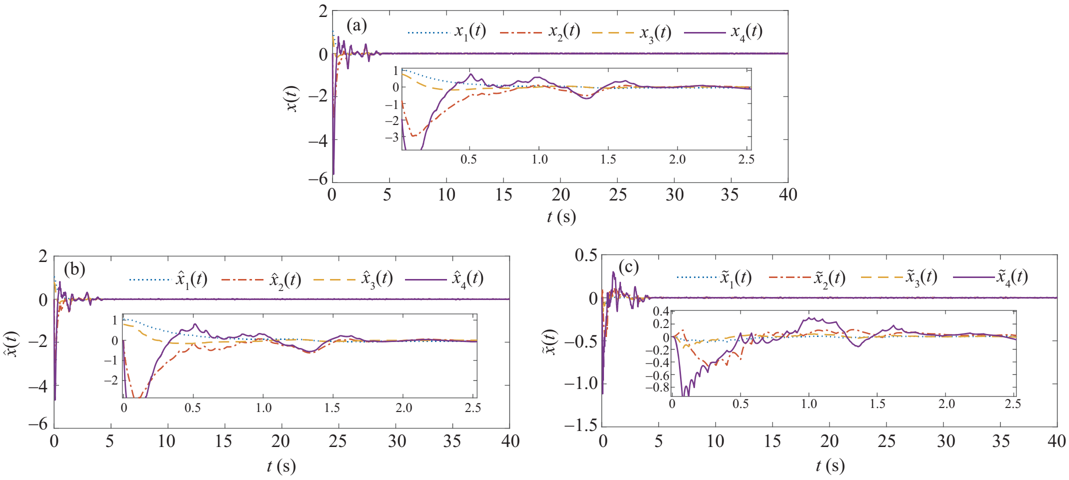

Figure 5 (Color online) (a) Trajectories of the plant states $x_i(t)$, (b) the observer states $\hat{x}_i(t)$, and (c) their errors $\tilde{x}_i(t)$ ($i = 1, 2, 3, 4$).

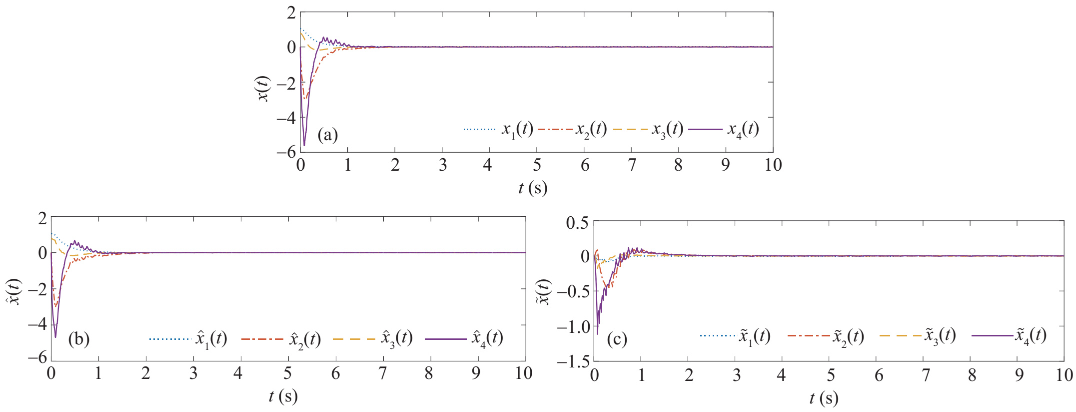

Figure 6 (Color online) (a) Trajectories of the plant states $x(t)$, (b) the observer states $\hat{x}(t)$, and (c) their errors $\tilde{x}(t)$, under Set 1.

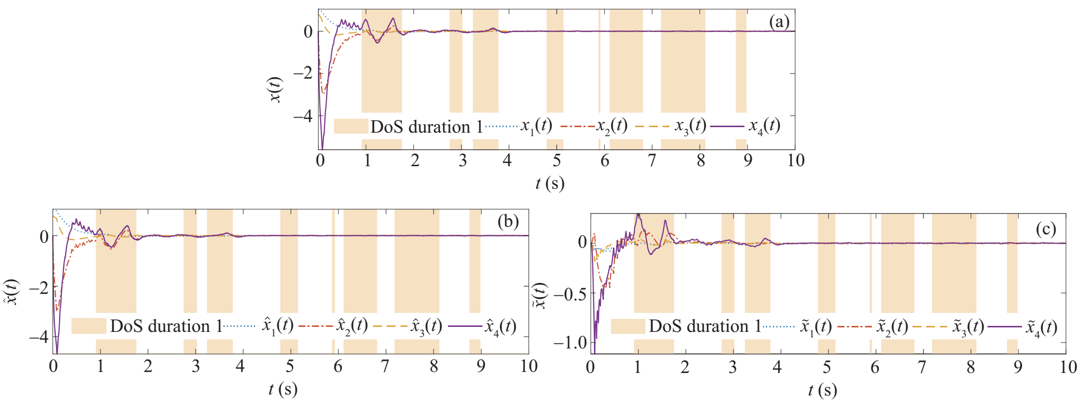

Figure 7 (Color online) (a) Trajectories of the plant states $x(t)$, (b) the observer states $\hat{x}(t)$, and (c) their errors $\tilde{x}(t)$, under Set 2.

6 Conclusion

This paper solved the problem of SMC against the DoS attacks and discrete events, for a class of uncertain dynamical systems in the network environment. The time-delay modeling approach was proposed to describe the DoS attack duration and inter-event time as the "time delay". The event-triggering mechanism was integrated into a smart sensor aperiodically releasing the measurement to the network with

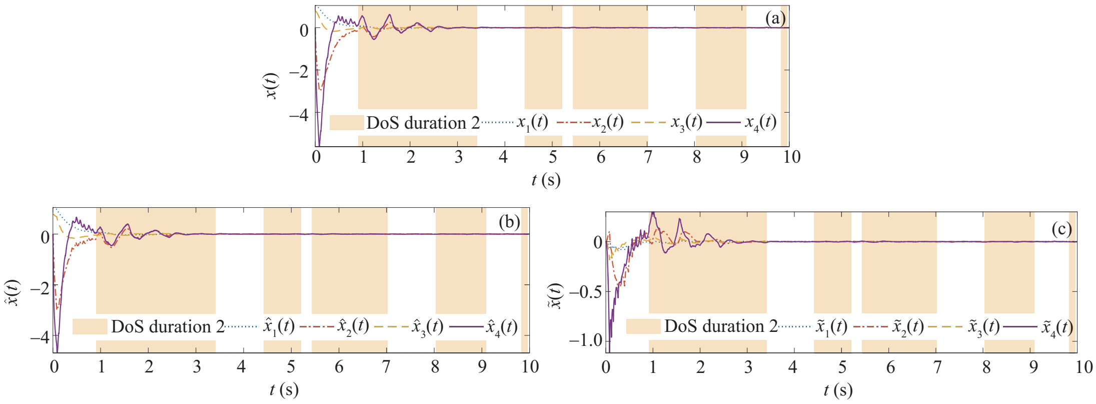

Figure 8 (Color online) (a) Trajectories of the plant states $x(t)$, (b) the observer states $\hat{x}(t)$, and (c) their errors $\tilde{x}(t)$, under Set 3.

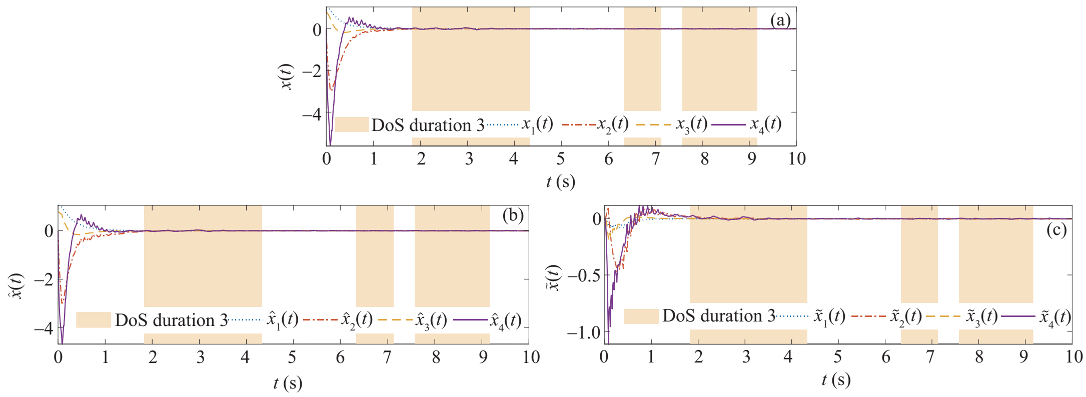

Figure 9 (Color online) (a) Trajectories of the plant states $x(t)$, (b) the observer states $\hat{x}(t)$, and (c) their errors $\tilde{x}(t)$, under Set 4.

certain network resources saved. Then, an interval-time-delay system with uncertainties was represented. A state observer was employed with the ZOH, and then an estimator-based controller was designed by which the ideal sliding mode can be achieved. Moreover, the resulting sliding motion was proved to be robust and stable with an $\mathscr{L}_2$ gain performance. The present SMC was finally validated by some simulations. As a matter of fact, one can develop the present time-delay-based modeling and control to nonlinear and stochastic systems in terms of the DoS attacks.

Acknowledgements This work was supported in part by National Key R&D Program of China (Grant No. 2019YFB1312001), National Natural Science Foundation of China (Grant Nos. 62033005, 62103118, 62022030), China Postdoctoral Science Foundation (Grant Nos. 2021T140160, 2021M700037), and Natural Science Foundation of Heilongjiang Province (Grant No. ZD2021F001).

References

1 Zhang W, Branicky M S, Phillips S M. Stability of networked control systems. IEEE Control Syst, 2001, 21: 84–99
2 Bemporad A, Heemels M, Johansson M, et al. Networked Control Systems. Berlin: Springer, 2010
3 Farivar F, Haghighi M S, Jolfaei A, et al. On the security of networked control systems in smart vehicle and its adaptive cruise control. IEEE Trans Intell Transp Syst, 2021, 22: 3824–3831
4 Zhang X M, Han Q L, Ge X, et al. Networked control systems: a survey of trends and techniques. IEEE CAA J Autom Sin, 2019, 7: 1–17
5 Yang L, Gao Y B, Feng Z G, et al. Event-based output feedback control of periodic systems: a piecewise impulsive method. Int J Robust Nonlinear Control, 2021, 31: 7569–7586
6 Li L Y, Cong P J, Cao K, et al. Feedback control of real-time EtherCAT networks for reliability enhancement in CPS. In: Proceedings of Design, Automation & Test in Europe Conference & Exhibition (DATE), 2018. 688–693
7 Liu J L, Wu Z G, Yue D, et al. Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks. IEEE Trans Syst Man Cybern Syst, 2021, 51: 943–953
8 Hu S L, Yue D, Xie X P, et al. Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks. IEEE Trans Cybern, 2019, 49: 4271–4281
9 Dong S L, Chen G R, Liu M Q, et al. Robust adaptive H_∞ control for networked uncertain semi-Markov jump nonlinear systems with input quantization. Sci China Inf Sci, 2022, 65: 189201
10 Edwards C, Colet E F, Fridman L. Advances in Variable Structure and Sliding Mode Control. Berlin: Springer, 2006
11 Utkin V, Poznyak A, Orlov Y, et al. Conventional and high order sliding mode control. J Franklin Inst, 2020, 357: 10244–10261

12 Huang X, Zhai D, Dong J X. Adaptive integral sliding-mode control strategy of data-driven cyber-physical systems against a class of actuator attacks. IET Control Theory Appl, 2018, 12: 1440–1447
13 Gao Y B, Liu J X, Sun G H, et al. Fault deviation estimation and integral sliding mode control design for Lipschitz nonlinear systems. Syst Control Lett, 2019, 123: 8–15
14 Sun G H, Wu L G, Kuang Z A, et al. Practical tracking control of linear motor via fractional-order sliding mode. Automatica, 2018, 94: 221–235
15 Liu J X, Gao Y B, Su X J, et al. Disturbance-observer-based control for air management of PEM fuel cell systems via sliding mode technique. IEEE Trans Control Syst Technol, 2019, 27: 1129–1138
16 Utkin V I. Sliding Modes in Control and Optimization. Berlin: Springer, 2013
17 Liu M, Sun G H. Observer-based sliding mode control for Itô stochastic time-delay systems with limited capacity channel. J Franklin Inst, 2012, 349: 1602–1616
18 Zhang J H, Lam J, Xia Y Q. Output feedback sliding mode control under networked environment. Int J Syst Sci, 2013, 44: 750–759
19 Wu L G, Gao Y B, Liu J X, et al. Event-triggered sliding mode control of stochastic systems via output feedback. Automatica, 2017, 82: 79–92
20 Ma R J, Shi P, Wu L G. Dissipativity-based sliding-mode control of cyber-physical systems under denial-of-service attacks. IEEE Trans Cybern, 2021, 51: 2306–2318
21 Guo L N, Zheng B C, Liu X G. Event-triggered sliding-mode control of linear uncertain system under periodic DoS attacks. In: Proceedings of Chinese Control And Decision Conference (CCDC), 2020. 1080–1085
22 Dolk V S, Tesi P, de Persis C, et al. Event-triggered control systems under denial-of-service attacks. IEEE Trans Control Netw Syst, 2017, 4: 93–105
23 de Persis C, Tesi P. Input-to-state stabilizing control under denial-of-service. IEEE Trans Autom Control, 2015, 60: 2930–2944
24 Xie L, de Souza C E. Robust $\mathcal{H}_\infty$ control for linear systems with norm-bounded time-varying uncertainty. IEEE Trans Autom Control, 1992, 37: 1188–1191
25 Gu K Q, Chen J, Kharitonov V L. Stability of Time Delay Systems. Berlin: Springer, 2003
26 Hill D, Moylan P. Connections between finite-gain and asymptotic stability. IEEE Trans Autom Control, 1980, 25: 931–936
27 Lin W, Byrnes C I. H_∞-control of discrete-time nonlinear systems. IEEE Trans Autom Control, 1996, 41: 494–510
28 Li X, de Souza C E. Criteria for robust stability and stabilization of uncertain linear systems with state delay. Automatica, 1997, 33: 1657–1662
29 Ge X H, Han Q L, Ding L, et al. Dynamic event-triggered distributed coordination control and its applications: a survey of trends and techniques. IEEE Trans Syst Man Cybern Syst, 2020, 50: 3112–3125
30 Ge X, Han Q L, Zhang X M, et al. Dynamic event-triggered control and estimation: a survey. Int J Autom Comput, 2021, 18: 857–886
31 Zhang J H, Feng G. Event-driven observer-based output feedback control for linear systems. Automatica, 2014, 50: 1852–1859