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# Constant modulus sequence set design with low weighted integrated sidelobe level in spectrally crowded environments

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Dear editor,

Radar waveform design with desired correlation properties has received considerable attention because of its performance improvement in the detection of a weak target [1, 2]. Moreover, owing to the increasing demand for access a wide radio frequency spectrum, coexistence among radar and telecommunication systems are currently instrumental in solving the spectrum-congestion problem [2]. Based on spectrum sensing in cognitive radar [3], several approaches have been developed [4, 5] to form radar signals with an appropriate frequency allocation, while minimizing its autocorrelation sidelobes. However, these studies have only focused on the single waveform design and did not consider the limitation of the available number of bits in the digital signal generators, likely resulting in performance deterioration.

This study focuses on a novel framework for establishing a discrete phase-modulated sequence set in terms of the weighted integrated sidelobe level (WISL) minimization, while controlling the interference energy produced on each shared frequency bandwidth. Additionally, to comply with current amplifier technology, the design also requires a constant modulus constraint. The resulting NP-hard optimization is solved using an iterative algorithm based on the inexact alternating direction penalty method (IADPM) framework that shows polynomial computational complexity. Numerical simulations are provided to demonstrate the effectiveness of the proposed algorithm.

*Problem formulation.* Let $\boldsymbol{s}_m \in \mathbb{C}^{N\times 1}$, $m \in \mathcal{M} = \{1\cdots M\}$ denote the $m$th orthogonal sequence, with $N$ representing the number of coded subpulses, i.e., $\boldsymbol{s}_m = \frac{1}{\sqrt{NM}}[\mathrm{e}^{\mathrm{j}\phi_m(1)}, \ldots, \mathrm{e}^{\mathrm{j}\phi_m(N)}]^{\mathrm{T}}$, where $\mathbb{C}^{N\times 1}$ denotes the set of $N$-dimensional vectors of complex numbers, $(\cdot)^{\mathrm{T}}$ denotes the transpose operation, and $\phi_m(n)$ denotes the $n$th phase of $\boldsymbol{s}_m$. The whole waveform matrix is defined as $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_M]$.

To suppress the corresponding correlation sidelobes of the designed sequences, the WISL can be expressed as $f(\boldsymbol{s}) = 2\sum_{m_1=1}^{M}\sum_{k=1-N}^{N-1}\gamma_k|r_{m_1m_1}(k)|^2 + 2\sum_{m_1=1}^{M}\sum_{m_2=m_1}^{M} w_k|r_{m_1m_2}(k)|^2$ [1], with $\boldsymbol{s} = \mathrm{vec}(\boldsymbol{S})$, where $r_{m_1m_2}(k)$ denotes the aperiodic crosscorrelation of $\boldsymbol{s}_{m1}$ and $\boldsymbol{s}_{m2}$ at the $k$th lag, and $|r_{m_1m_2}(k)|$ represents its modulus. $\{\gamma_k\}_{k=-N+1}^{N-1}$ and $\{w_k\}_{k=-N+1}^{N-1}$ denote nonnegative weights, and $\mathrm{vec}(\boldsymbol{S})$ represents the column vector obtained by stacking the $\boldsymbol{S}$ columns on top of each other.

To ensure spectral compatibility with $H$ licensed radiators, the waveform spectrum must be shaped out of a frequency band $\Omega_h = [f_1^h, f_2^h]$, $h \in \mathcal{H} = \{1, \ldots, H\}$, where $f_1^h$ and $f_2^h$ denote the lower and upper normalized frequencies of the $h$th cooperative radiators, respectively. The energy transmitted by the radar within the $h$th licensed bandwidth is given by $\sum_{m=1}^{M}\boldsymbol{s}_m^H\boldsymbol{R}_I^h\boldsymbol{s}_m$, where $(\cdot)^\dagger$ is the conjugate transpose operator. Moreover, the $(a, b)$th entry of $\boldsymbol{R}_I^h$ is $(\mathrm{e}^{\mathrm{j}2\pi f_2^h(a-b)} - \mathrm{e}^{\mathrm{j}2\pi f_1^h(a-b)})/(\mathrm{j}2\pi(a-b))$ for $a \neq b$, while $f_2^h - f_1^h$ for $a = b$ [6].

Denoting the amount of tolerated interference as $E_I$, the spectral constraint that the transmitted sequence set must comply with can be obtained as $\boldsymbol{s}^\dagger(\boldsymbol{I}_M \otimes \boldsymbol{R}_I)\boldsymbol{s} \leqslant E_I$, where $\boldsymbol{R}_I = \sum_{h=1}^{H} c_h\boldsymbol{R}_I^h$, with the coefficient $c_h$ corresponding to the $h$th radiator. $\boldsymbol{I}_M$ and $\otimes$ denote the $M\times M$-dimensional identity matrix and Kronecker product, respectively.

To this end, by imposing the constant modulus and discrete phase restrictions to ensure hardware compatibility [7], the optimization problem can be formulated as follows:

$$\mathcal{P}_0\begin{cases}\min\limits_{\boldsymbol{s}} \ f(\boldsymbol{s})\\ \text{s.t.} \ \ \boldsymbol{s}(n) = 1/\sqrt{NM}\mathrm{e}^{\mathrm{j}\phi_n}, \ \phi_n \in \Phi, \ n \in \mathcal{N},\\ \qquad \boldsymbol{s}^\dagger(\boldsymbol{I}_M \otimes \boldsymbol{R}_I)\boldsymbol{s} \leqslant E_I,\end{cases} \quad (1)$$

where $\boldsymbol{s}(n)$ is the $n$th element of vector $\boldsymbol{s}$ with phase $\phi_n$, $\mathcal{N} = \{1, 2, \ldots, NM\}$, and $\Phi = \frac{2\pi}{L}\{0, 1, \ldots, L-1\}$ is the set of the finite alphabet with cardinality $L$.

Actually, $\mathcal{P}_0$ is nonconvex and NP-hard owing to the presence of the discrete phase and constant modulus constraints.

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Some comments on $\mathcal{P}_0$ are presented in Appendix A. Next, an IADPM framework is derived to solve $\mathcal{P}_0$.

*IADPM framework for solving* $\mathcal{P}_0$. Introducing two auxiliary variables $\boldsymbol{x}$ and $\boldsymbol{y}$, an equivalent problem $\mathcal{P}_1$ can be derived using constraints $\boldsymbol{x} = \boldsymbol{s}$, $\arg \boldsymbol{x}(n) \in \Phi$, $|\boldsymbol{x}(n)| = 1/\sqrt{NM}$, $\boldsymbol{y}{=}\boldsymbol{s}$, $\boldsymbol{y}^{\dagger}(\boldsymbol{I}_M \otimes \boldsymbol{R}_I)\boldsymbol{y} \leqslant E_I$, and $|\boldsymbol{s}(n)| = 1/\sqrt{NM}$, $n \in \mathcal{N}$. The augmented Lagrangian of $\mathcal{P}_1$ is given by $L(\boldsymbol{s}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}_1, \boldsymbol{u}_2, \varrho_1, \varrho_2) = f(\boldsymbol{s}) + \Re\{\boldsymbol{u}_1^{\dagger}(\boldsymbol{x} - \boldsymbol{s})\} + \varrho_1/2\|\boldsymbol{x} - \boldsymbol{s}\|^2 + \Re\{\boldsymbol{u}_2^{\dagger}(\boldsymbol{y} - \boldsymbol{s})\} + \varrho_2/2\|\boldsymbol{y} - \boldsymbol{s}\|^2$, where $\boldsymbol{u}_p \in \mathbb{C}^{N\times 1}$ and $\varrho_p > 0$, $p \in \{1, 2\}$ are the multiplier vectors and penalty parameters, respectively. $||\cdot||$ denotes the Frobenius norm. $\Re(\boldsymbol{x}(n))$ and $\arg(\boldsymbol{x}(n))$ denote the real part and phase of $\boldsymbol{x}(n)$, respectively.

In each iteration, problem $\mathcal{P}_1$ can be split into three tractable subproblems, namely, $\mathcal{P}_{\boldsymbol{x}}^{(t)}$, $\mathcal{P}_{\boldsymbol{y}}^{(t)}$, and $\mathcal{P}_{\boldsymbol{s}}^{(t)}$, which are defined in Appendix B. The rule that iteratively solves $\mathcal{P}_{\boldsymbol{x}}^{(t)}$, $\mathcal{P}_{\boldsymbol{y}}^{(t)}$, and $\mathcal{P}_{\boldsymbol{s}}^{(t)}$ and adjusts $\boldsymbol{u}_p$, $\varrho_p > 0$, $p \in \{1, 2\}$ in each iteration is shown in Appendix B. The detailed procedures for addressing these three subproblems are provided in Appendixes C–E. Therein, the computational complexities of $\mathcal{P}_{\boldsymbol{x}}^{(t)}$, $\mathcal{P}_{\boldsymbol{y}}^{(t)}$, and $\mathcal{P}_{\boldsymbol{s}}^{(t)}$ are of the order $O(NM)$, $O(N^2M^2)$, and $O(2M(N-1)^3/3)$, respectively.

Some comments are provided in Appendix F regarding the proposed IADPM framework, and its convergence analysis is shown in Appendix G.

*Numerical results.* The performance of the proposed IADPM algorithm is assessed. Assume there are $H = 2$ licensed radiators with $c_1 = c_2 = 1$, and their normalized spectrum bands are $\Omega_1 = [0.47, 0.5]$, $\Omega_2 = [0.7, 0.71]$, respectively. To balance the joint design of correlation functions and spectral shapes, the weights at a specific time lag $k \in [1, K]$ are properly given by $\gamma_k = \max(0.1 \times (K - k + 1), 1)$ and $w_k = 1$. When $k \in [K + 1, N - 1]$, $\gamma_k = w_k = 1$.

The parameters of a sequence set are $M = 2$, $N = 64$, and $K = 16$. In Figures 1(a) and (b), under different allowed interference energies $E_I$, the WISL versus the computational time and iteration number are illustrated with $L = 16$, respectively. Each iteration curve fluctuates but can converge to a certain value, which numerically verifies the convergence performance of the IADPM framework. The autocorrelation function of the first sequence and crosscorrelation functions of the sequence set with different $E_I$ are shown in Figures 1(c) and (d), respectively. As expected, the obtained correlation functions show very low values at the relevant range cells. Figure 1(e) shows the energy spectral density (ESD) of the first sequence under different $E_I$, here, the stopbands are denoted in gray. This figure highlights the capability of the proposed technique to suitably control the amount of energy leaked to the shared frequency bands. The effect of the allowed interference energy $E_I$ on the WISL is illustrated in Figure 1(f) under different $L$. For each $L$, the algorithm is independently performed five times and the best trial is selected. The obtained WISL values decrease with weakening spectral constraint. The results confirm that larger cardinality $L$ usually leads to a lower WISL because more degrees of freedom are introduced for a fixed $E_I$.

*Conclusion.* This study has considered the sequence set design to obtain the desired correlation properties in spectrally crowded environments. The iteration algorithm based on the IADPM framework is proposed to minimize the WISL, and its effectiveness has been assessed using numerical simulations in terms of the WISL and ESD. The results have shown that the sequence set established using the proposed algorithm can obtain desired correlation properties as well as ensure coexistence with the overlayed licensed emitters.

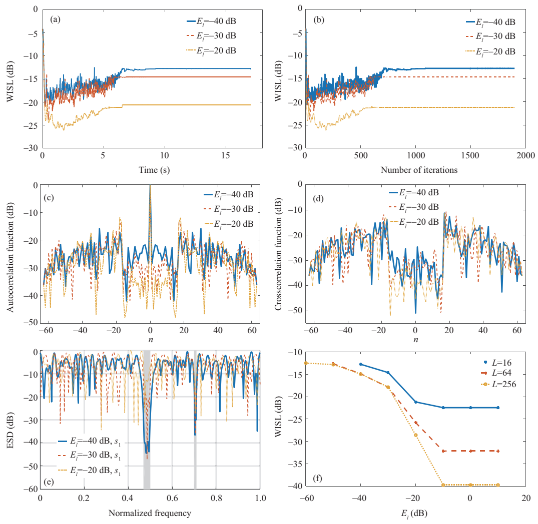


**Figure 1** (Color online) (a) and (b) WISL versus the computational time and iteration number for $L = 16$, respectively; (c) and (d) autocorrelation and crosscorrelation functions of the designed sequences for $L = 16$, respectively; (e) ESD of the first designed sequence for $L = 16$; (f) WISL versus $E_I$ with different cardinalities $L$.

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**Supporting information** Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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