

# Beam alignment for millimeter wave multiuser MIMO systems using sparse-graph codes

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Dear editor,

To achieve millimeter wave (mmWave) beam alignment, a class of beam scanning and searching schemes have been extensively studied [1–3]. Recently, to address the problems of high sample complexity in traditional algorithms, some adaptive beam scanning approaches employed the hierarchical beamforming codebook to reduce the training time at the cost of frequent feedback [2]. Then, to eliminate the feedback link, a random beam alignment algorithm has been proposed using the pseudo-random spreading codes [3]. However, the random beam alignment algorithm needs pseudo-noise sequences with sufficient length to ensure the good correlation properties of different beams. In addition to these limitations, most existing algorithms require either a separate pilot sequence per user or long beam scanning time when considering mmWave multiuser uplinking systems.

To solve the above problems, we propose a novel class of beam alignment algorithms based on sparse-graph coding theory in this study. First, we analyze the uplink mmWave beam training structure. Based on the analysis, the mmWave multiuser beam alignment problem is transformed into a sparse-graph design and detection problem. Second, we propose a beam alignment algorithm framework based on sparse-graph coding and decoding. Further, we derive a theoretical bound to choose the optimal parameters when designing the sparse-graph coding matrix. Finally, we propose two beam alignment algorithms to detect the beam index in different settings. Simulation results confirmed that our beam algorithms outperformed the traditional beam-training methods.

*Proposed uplink beam-training scheme.* In this study, we consider a typical uplink mmWave multiuser MIMO system, where the base station (BS) communicates with  $K$  users (UEs) simultaneously. Suppose that BS is equipped with  $N_R$  antennas and  $N_{RF}$  radio frequency (RF) chains, and the  $k$ -th UE has  $M_T$  antennas and  $M_{RF}$  RF chains. Then,

the channel associated with the  $k$ -th UE is given by [4]

$$\mathbf{H}_k = \sum_{l=1}^{L_k} \alpha_{l,k} \mathbf{a}_{BS}(\theta_{l,k}) \mathbf{a}_{UE}^H(\varphi_{l,k}), \quad (1)$$

where  $(\theta_{l,k}, \varphi_{l,k}, \alpha_{l,k})$  denote the AoA, AoD, complex gain of the  $l$ -th path of the  $k$ -th UE, respectively. The multipath complex gain can be modeled as Rice fading given by [5, 6]

$$\alpha_{l,k} \sim \sqrt{\rho_{l,k}} \left( \sqrt{\frac{\eta_{l,k}}{\eta_{l,k} + 1}} + \sqrt{\frac{1}{\eta_{l,k} + 1}} \tilde{\alpha}_{l,k} \right), \quad (2)$$

where  $\sqrt{\rho_{l,k}}$  is the overall multipath complex gain strength,  $\eta_{l,k}$  denotes the ratio between the line of sight (LOS) component and the non-line of sight (NLOS) components, and  $\tilde{\alpha}_{l,k} \sim \mathcal{CN}(0, 1)$  denotes the complex Gaussian random variable. In addition, the uniform linear array is used by the BS and the UEs.

In the proposed scheme, the BS can locally customize its own beamforming codebook, then combining matrix  $\mathbf{w}(t) = \mathbf{W}_{RF}(t) \mathbf{W}_{BB}(t) = \mathbf{F}_{BS} \mathbf{v}(t)$ , where  $\mathbf{W}_{RF}(t) \in \mathbb{C}^{N_R \times N_{RF}}$ ,  $\mathbf{W}_{BB,k}(t) \in \mathbb{C}^{N_{RF} \times N_{RF}}$ ,  $\mathbf{F}_{BS} \in \mathbb{C}^{N_R \times N_R}$ , and  $\mathbf{v}(t) \in \mathbb{C}^{N_R \times 1}$  are the RF precoder matrix, digital precoder matrix, discrete Fourier transform (DFT) matrix, and index vector of the quantized angle, respectively. Particularly, the nonzero value in the beam selection vector  $\mathbf{v}(t)$  indicates which direction should be formed. Without loss of generality, we set the nonzero value to one. On the UE side, the beamforming matrix is given by  $\mathbf{f}_k(t) = \mathbf{F}_{RF,k}(t) \mathbf{F}_{BB,k}(t) = \mathbf{F}_{MS} \boldsymbol{\psi}_k(t)$ , where  $\mathbf{F}_{RF,k}(t) \in \mathbb{C}^{M_T \times M_{RF}}$ ,  $\mathbf{F}_{BB,k}(t) \in \mathbb{C}^{M_{RF} \times M_{RF}}$ ,  $\mathbf{F}_{MS} \in \mathbb{C}^{M_T \times M_T}$ , and  $\boldsymbol{\psi}_k(t) \in \mathbb{C}^{M_T \times 1}$  denote the RF precoder matrix, digital precoder, DFT matrix, and coding vector, respectively. Notably, the mmWave channel can be given by a beamspace representation expression. Thus, the

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received signal can be rewritten as follows:

$$\begin{aligned}
 \mathbf{r}(t) &= \mathbf{w}^H(t) \sum_{k=1}^K \mathbf{H}_k \mathbf{f}_k(t) + \mathbf{n}(t) \\
 &= \mathbf{v}^H(t) \mathbf{F}_{\text{BS}}^H \sum_{k=1}^K \mathbf{F}_{\text{BS}} \widehat{\mathbf{H}}_k \mathbf{F}_{\text{MS}}^H \mathbf{F}_{\text{MS}} \boldsymbol{\psi}_k(t) + \mathbf{n}(t) \\
 &= \mathbf{v}^H(t) \sum_{k=1}^K \widehat{\mathbf{H}}_k \boldsymbol{\psi}_k(t) + \mathbf{n}(t), \tag{3}
 \end{aligned}$$

where  $\widehat{\mathbf{H}}_k$  is the virtual angle domain index. Further, considering that the BS can separate the received signal and the nonzero value in the selection vector  $\mathbf{v}(t)$  is one, the received signal of the  $i$ -th RF chain can be expressed as follows:

$$r_i(t) = \boldsymbol{\psi}^T(t) \widehat{\mathbf{h}} + n(t), \tag{4}$$

where  $\widehat{\mathbf{h}} = [\widehat{h}_1^T, \widehat{h}_2^T, \dots, \widehat{h}_K^T]^T \in \mathbb{C}^{KM_T \times 1}$ ,  $\widehat{\mathbf{h}}_k$  denotes the selected row of  $\widehat{\mathbf{H}}_k$ , and then  $\boldsymbol{\psi}^T(t) = [\boldsymbol{\psi}_1^T(t), \boldsymbol{\psi}_2^T(t), \dots, \boldsymbol{\psi}_K^T(t)] \in \mathbb{C}^{1 \times KM_T}$ .

In addition, by collecting  $T$  received pilot signals at the BS, we obtain the following:

$$\mathbf{r}_i = \boldsymbol{\psi} \widehat{\mathbf{h}} + \mathbf{n}, \tag{5}$$

where  $\boldsymbol{\psi} = [\boldsymbol{\psi}^T(1), \boldsymbol{\psi}^T(2), \dots, \boldsymbol{\psi}^T(T)] \in \mathbb{C}^{T \times KM_T}$ . Notably, the nonzero elements in vector  $\boldsymbol{\psi}_k^T(t)$  denote the selected beam indexes; therefore,  $\boldsymbol{\psi}$  is the measurement matrix. In particular, since we process the signals of each receiving link separately, we denote  $\mathbf{r}_i = \mathbf{r}$  for simplicity whenever no ambiguity arises.

*Proposed beam alignment scheme using sparse-graph codes.* For the purposes of later analysis, we first define  $T = NM$ . Then, we divide the measurement matrix  $\boldsymbol{\psi} \in \mathbb{C}^{NM \times KM_T}$  into two parts: sparse coding matrix  $\mathbf{G} \in \mathbb{C}^{M \times MT}$  and bin detection matrix  $\mathbf{S} \in \mathbb{C}^{N \times MT}$ . Consequently, the measurement matrix  $\boldsymbol{\psi}$  is expressed as follows:

$$\boldsymbol{\psi} = \mathbf{G} \odot \mathbf{S}, \tag{6}$$

where  $\odot$  denotes the row-tensor operator. Mathematically, we rewrite the measurement matrix  $\boldsymbol{\psi}$  as follows:

$$\boldsymbol{\psi} = [\mathbf{G}_1 \otimes \mathbf{S}_1 \cdots \mathbf{G}_{M_T} \otimes \mathbf{S}_{M_T}], \tag{7}$$

where  $\mathbf{G}_i$  and  $\mathbf{S}_i$  denote the  $i$ -th column of the matrix  $\mathbf{G}$  and the matrix  $\mathbf{S}$ , respectively, and  $\otimes$  denotes the Kronecker product.

Inspired by the traditional LDPC coding method [7], we construct a regular bipartite graph  $\Gamma^{M_T}(R, b)$  with  $M_T$  left nodes and  $R$  right nodes for the sparse coding matrix, where each left node is connected to  $b$  right nodes at random [8]. Here, we divide the observation detections into three categories: zero-ton, single-ton, and multi-ton. Detailed descriptions can be found in Appendix A.

The function of the matrix  $\mathbf{S}$  is to effectively distinguish the types of right nodes. Then, if the receiver gets oracle information about the types of right nodes, similar to the message passing algorithm [7], a peeling decoder can be used to peel off all single-ton in the bipartite graph. In addition, by iteratively repeating the peeling off process, all edges can be removed from the graph. Finally, we perform a probability analysis of the proposed peeling decoder, over a random selection from the regular graph ensemble  $\mathfrak{R}^K(F, m)$ . In this

ensemble, the  $m$  detections are divided into  $d$  stages, with each left node randomly connected to one right node per stage. The set  $F$  is defined as  $F = \{f_1, \dots, f_d\}$ , where the number of right nodes in the  $i$ -th stage is  $f_i$ . In particular,  $f_i = \mu K + O(1)$ , for all  $i$  and some redundancy parameter  $\mu$ . Further, using the density evolution analysis in [8], the proposed peeling decoder can recover  $K$  sparse  $\widehat{\mathbf{h}}$  with a probability, given by Theorem 1.

**Theorem 1.** If the proposed peeling decoder over a random graph from the ensemble  $\mathfrak{R}^K(F, m)$  satisfies the stages of  $d \geq 3$  and  $f_i = \mu K + O(1)$  with the constant  $\mu$  being chosen from Appendix B, it can successfully recover  $K$  nonzero entries with probability  $1 - O(1/m)$ .

The proof is given in Appendix C.

Thus, based on the same sparse coding matrix  $\mathbf{G}$ , we proposed two beam alignment algorithms by designing different detection matrices  $\mathbf{S}$ : one for the noiseless case in Appendix D and the other for the noisy environment in Appendix E.

*Experiment.* The experiment results of the proposed algorithms can be seen in Appendix F.

*Conclusion.* In this study, we propose several beam alignment algorithm designs for mmWave multiuser MIMO systems using sparse-graph codes. Using the peeling off method and the density evolution analysis method, the proposed algorithm can approach a theoretical bound under a noiseless setting and obtain the reduction of pilot overhead under a noise setting.

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**Supporting information** Appendixes A–F. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

## References

- Liu C, Li M, Hanly S V, et al. Millimeter-wave small cells: base station discovery, beam alignment, and system design challenges. *IEEE Wireless Commun*, 2018, 25: 40–46
- Noh S, Zoltowski M D, Love D J. Multi-resolution codebook and adaptive beamforming sequence design for millimeter wave beam alignment. *IEEE Trans Wireless Commun*, 2017, 16: 5689–5701
- Chiu S E, Ronquillo N, Javidi T. Active learning and CSI acquisition for mmWave initial alignment. *IEEE J Sel Areas Commun*, 2019, 37: 2474–2489
- Alkhateeb A, Ayach O E, Leus G, et al. Channel estimation and hybrid precoding for millimeter wave cellular systems. *IEEE J Sel Top Signal Process*, 2014, 8: 831–846
- Song X, Haghighatshoar S, Caire G. Efficient beam alignment for millimeter wave single-carrier systems with hybrid MIMO transceivers. *IEEE Trans Wireless Commun*, 2019, 18: 1518–1533
- Lioliou P, Viberg M, Matthaiou M. Bayesian approach to channel estimation for AF MIMO relaying systems. *IEEE J Sel Areas Commun*, 2012, 30: 1440–1451
- Richardson T J, Urbanke R L. The capacity of low-density parity-check codes under message-passing decoding. *IEEE Trans Inform Theor*, 2001, 47: 599–618
- Luby M G, Mitzenmacher M, Shokrollahi M A, et al. Efficient erasure correcting codes. *IEEE Trans Inform Theor*, 2001, 47: 569–584