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Exponential stabilization of an ODE system with Euler-Bernoulli beam actuator dynamics

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Dear editor,

Recently, finite-dimensional systems with infinite-dimensional actuator or sensor dynamics have drawn the attention of researchers worldwide. Many real-world systems can be modeled as ordinary differential equation (ODE)-partial differential equation (PDE) cascade systems, such as traffic flows, chemical reactor processes, heat exchangers, and oil well drilling. In this study, we consider the problem of compensating the actuator dynamics modeled by the Euler-Bernoulli beam (EBB) equation.

Compared with existing results, such as those in [1–3], the considered problem is more challenging because the controls act only on the beam equation. As stated in [4], PDE backstepping is hard to apply to an EBB except for the special case where the EBB equation with a proper boundary can be converted into a Schrödinger equation [5]. Therefore, conventional PDE backstepping that was used in [6–8] cannot be directly used for beam dynamics compensation. Herein, we find a novel transformation to compensate for the EBB dynamics. In contrast to the conventional PDE backstepping method, the kernel of the proposed method always satisfies an ODE, which is much easier than PDE and is analytically solvable. The problem is described by the system:

$$\begin{cases} \dot{X}(t) = AX(t) + B(w_x(0,t), w(0,t))^{\mathrm{T}}, \\ w_{tt}(x,t) + w_{xxxx}(x,t) = 0, \\ w_{xxx}(0,t) = 0,\ w_{xx}(0,t) = 0, \\ w_{xx}(1,t) = u_1(t),\ w_{xxx}(1,t) = u_2(t), \end{cases} \tag{1}$$

where $x \in (0,1)$, $t > 0$, $A \in \mathbb{R}^{n\times n}$, u_1 and u_2 are the two scalar control inputs for the entire system, and $B = [B_1\ B_2] \in \mathbb{R}^{n\times 2}$ represents the interconnection. This model has wide practical engineering applications, such as in helicopter rotor blades, space aircraft, space structures, and turbine blades. The system (1) can describe the stabilization problem of a finite-dimensional system through a communication medium such as a flexible robotic arm. The controls act on one end of the robotic arm and the other end is connected to the control plant. Thanks to the boundary conditions, the connection at the left end $x = 0$ can move freely. This configuration is more in line with practice.

Our objective is to stabilize the cascade system exponentially in the state space $\mathbb{R}^n \times H^2(0,1)\times L^2(0,1)$ via feedback controls u_1 and u_2. To this end, we propose the following transformation:

$$\begin{aligned} \tilde{X}(t) = X(t) + H_3 w(1,t) + H_4 w_x(1,t) \\ + \int_0^1 H_1(x) w(x,t) + H_2(x) w_t(x,t)\mathrm{d}x, \end{aligned} \tag{2}$$

where $H_1,\ H_2 : [0,1] \to \mathbb{R}^n$ are vector-valued functions and $H_3,\ H_4 \in \mathbb{R}^n$ are vectors. All of these are unknown and will be determined later. Finding the derivative of $\tilde{X}(t)$ along the control plant (1) and selecting H_1 and H_2 specially such that

$$\begin{cases} H_{2xxxx} + AH_1 = 0, \quad H_1 - AH_2 = 0, \\ H_{2xxx}(1) = c_2 H_2(1) + AH_3, \\ H_{2xx}(1) = -c_1 H_{2x}(1) - AH_4, \end{cases} \tag{3}$$

we have

$$\begin{aligned} \dot{\tilde{X}}(t) = A\tilde{X}(t) - H_2(1)[u_2(t) - c_2 w(1,t)] \\ + H_{2x}(1)[u_1(t) + c_1 w_x(1,t)] + H_3 w_t(1,t) \\ + H_4 w_{xt}(1,t) + [B_1 + H_{2xx}(0)]w_x(0,t) \\ + [B_2 - H_{2xxx}(0)]w(0,t). \end{aligned} \tag{4}$$

Furthermore, assume that

$$\begin{cases} H_3 = k_2 H_2(1),\ H_4 = k_1 H_{2x}(1), \\ B_1 + H_{2xx}(0) = 0,\ B_2 - H_{2xxx}(0) = 0. \end{cases} \tag{5}$$

Eq. (4) is thus

$$\begin{aligned} \dot{\tilde{X}}(t) = A\tilde{X}(t) - H_2(1)[u_2(t) - c_2 w(1,t) - k_2 w_t(1,t)] \\ + H_{2x}(1)[u_1(t) + c_1 w_x(1,t) + k_1 w_{xt}(1,t)]. \end{aligned} \tag{6}$$

Thus, the controllers can be designed by stabilizing the $\tilde{X}$-system as

$$\begin{cases} u_1(t) = K_1^{\mathrm{T}}\tilde{X}(t) - c_1 w_x(1,t) - k_1 w_{xt}(1,t), \\ u_2(t) = K_2^{\mathrm{T}}\tilde{X}(t) + c_2 w(1,t) + k_2 w_t(1,t), \end{cases} \tag{7}$$

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where $c_i,\ k_i > 0,\ i = 1, 2$ and $K_1,\ K_2$ are column vectors such that the matrix $A + H_{2x}(1)K_1^{\mathrm{T}} - H_2(1)K_2^{\mathrm{T}}$ is a Hurwitz matrix. It follows from Lemma 1 in Appendix A that system (3) and (5) admits a unique solution. Consequently, we obtain the following system:

$$\begin{cases} \dot{\tilde{X}}(t) = [A + H_{2x}(1)K_1^{\mathrm{T}} - H_2(1)K_2^{\mathrm{T}}]\tilde{X}(t), \\ w_{tt}(x,t) + w_{xxxx}(x,t) = 0, \\ w_{xx}(0,t) = 0, \quad w_{xxx}(0,t) = 0, \\ w_{xx}(1,t) = u_1(t), \quad w_{xxx}(1,t) = u_2(t), \\ u_1, u_2 \text{ are given by (7)}, \end{cases} \tag{8}$$

which is a cascade system of two exponentially stable systems (see [9]). In Lemma 4 of Appendix B, we show that this cascade system is exponentially stable in $\mathbb{R}^n \times H^2(0,1) \times L^2(0,1)$. In terms of the transformation (2), we obtain the control laws:

$$\begin{cases} u_1(t) = K_1^{\mathrm{T}}\Big[X(t) + \int_0^1 H_1(x)w(x,t)\mathrm{d}x \\ \quad + \int_0^1 H_2(x)w_t(x,t)\mathrm{d}x + H_3 w(1,t) \\ \quad + H_4 w_x(1,t)\Big] - c_1 w_x(1,t) - k_1 w_{xt}(1,t), \\ u_2(t) = c_2 w(1,t) + k_2 w_t(1,t) + K_2^{\mathrm{T}}\Big[X(t) \\ \quad + \int_0^1 H_1(x)w(x,t)\mathrm{d}x + H_3 w(1,t) \\ \quad + \int_0^1 H_2(x)w_t(x,t)\mathrm{d}x + H_4 w_x(1,t)\Big]. \end{cases} \tag{9}$$

Theorem 1. Let $c_i,\ k_i > 0,\ i = 1, 2,\ A \in \mathbb{R}^{n\times n}$ and $B = [B_1\ B_2] \in \mathbb{R}^{n\times 2}$. Suppose that $H_j,\ j = 1, \ldots, 4$ satisfy (3) and (5). Suppose that the pair (A, B) is controllable and $\sigma(A) \subset \{\lambda \in \mathbb{C} \mid \mathrm{Re}\lambda \geqslant 0\}$. Then, there exists a $[K_1\ K_2] \in \mathbb{R}^{n\times 2}$ such that the matrix $A + H_{2x}(1)K_1^{\mathrm{T}} - H_2(1)K_2^{\mathrm{T}}$ is Hurwitz matrix. Moreover, for any initial state $(X(0), w(\cdot,0), w_t(\cdot,0)) \in \mathbb{R}^n \times H^2(0,1) \times L^2(0,1)$, the closed-loop system (1) and (9) admits a unique solution $(X, w, w_t) \in C([0,\infty); \mathbb{R}^n \times H^2(0,1) \times L^2(0,1))$ that decays to zero exponentially in $\mathbb{R}^n \times H^2(0,1) \times L^2(0,1)$ as time t goes to infinity.

The proof of Theorem 1 and numerical simulations are provided in Appendixes B and D, respectively.

Conclusion. In this study, a new approach is proposed for the stabilization of a finite-dimensional system with PDE actuator dynamics by stabilizing an ODE-EBB cascade system. Different from the conventional PDE backstepping method, the kernel function of the proposed method is governed by an ODE, which is typically much simpler than a PDE. As a result, we obtain the feedback laws explicitly. We will apply the new approach to compensations for wave dynamics and time delay dynamics in future studies.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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