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Blind adaptive identification and equalization using bias-compensated NLMS methods

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Abstract In this paper, two new blind adaptive identification and equalization algorithms based on second-order statistics are proposed. We consider a practical case where the noise statistics of each transmission channel is unknown. Resorting to the technique of antennas array, a single-input double-output channel can be obtained. We further convert the problem of blind identification into an errors-in-variables (EIV) parameter estimation problem, then we apply the normalized least-mean squares (NLMS) algorithms to tackle the problem. To improve the performance of the NLMS algorithms, we also develop a variable step-size NLMS (VSS-NLMS) algorithm that ensures the stability of the algorithm and faster convergence speed at the beginning of the iterations process. Under various practical scenarios, noise affects transmission channels; it is necessary to estimate the variance and remove the bias. By modifying the cost function, we present a bias-compensated NLMS (BC-NLMS) algorithm and a bias-compensated NLMS algorithm with variable step-size (BC-VSS-NLMS) to eliminate the bias. The proposed algorithms estimate the variances of the noise online, and therefore, the noise-induced bias can be removed. The estimate of the channel characteristics is available for equalization. Simulation results are presented to demonstrate the performance of the proposed algorithms.

Keywords blind adaptive identification, equalization, normalized least mean squares algorithm, bias compensation, errors-in-variables

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1 Introduction

Equalization technology is an effective method to tackle the problem that wireless communication systems suffer from channel distortion due to inter-symbol interference (ISI) and noise [1]. Training-based methods restrict the transmission efficiency of the communication channel due to its high dependence on training sequence or prior information about the channel, which may not be available in practical cases, e.g., acoustic dereverberation, wireless communications, and time delay estimation [2]. By contrast, without requiring training or prior knowledge of channels, blind equalization, which only uses the output signals to equalize the channel, was proposed by Sato in the 1970s [3].

Approaches that employ high-order statistics (HOS) are first used to estimate the channel and calculate the equalizer [4–6], of which the constant-modulus algorithm (CMA) [7–11] is a typical one. It is known that HOS-based methods can effectively eliminate the influence of Gaussian noise. However, HOS methods experience a slow rate of convergence and high computation complexity [9, 12]. Exploiting the structure of the single-input multiple-output channel with the techniques of sensor array or time oversampling, second-order statistics (SOS) methods have been developed [13–18]. Batch-based SOS approaches requiring the computation of the eigendecomposition of channel matrix still have high computation complexity and unable to track time-varying channels [19, 20].

To overcome the abovementioned problem, adaptive methods have been proposed [21–23]. Ref. [23] developed adaptive approaches implemented using the recursive least squares (RLS) algorithm and least mean squares (LMS) algorithm to track time-varying channels. Among the adaptive filtering algorithms,

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LMS and RLS algorithms are two typical algorithms. The RLS algorithm is suitable when a fast rate of convergence is required but suffers from a complex computation. Unlike the RLS algorithm, the computational cost of the LMS algorithm is much lower, and it is widely used in many industrial applications. However, the LMS algorithm suffers from a gradient noise amplification problem when the input power is large [24]. This signal-dependent instability can be overcome by using a normalized LMS (NLMS) algorithm. As there is a conflict between the rate of convergence and steady-state error in stochastic gradient algorithm, it is significant to choose an appropriate step size to ensure steady-state mean square error approaches to the least mean square error. Many variable step-size adaptive algorithms have been proposed to solve this tradeoff issue by changing the step size dynamically [25–29], where the key idea is to use a large step-size initially for a fast rate of convergence and reduce the step size gradually to a small value for an improved performance at the steady-state. The NLMS algorithm with power normalization in [30] dynamically changed the step size by estimating the power of the input sequence iteratively, which effectively reduced the computational complexity of the NLMS algorithm.

To further improve convergence accuracy of traditional adaptive algorithms under the condition of noise environment, Refs. [31–36] presented a series of bias-compensated adaptive methods of parameter estimation of distributed network. The bias-compensated-based algorithms can effectively eliminate the noise-induced bias and be implemented with good estimation performance under noisy conditions. In [36], a bias-compensated NLMS (BC-NLMS) algorithm was developed. Compared with the traditional NLMS algorithm, BC-NLMS had a better performance of convergence in distributed networks. Ref. [37] applied a bias-compensated RLS algorithm to the field of blind identification, which can well-estimate the channel characters under a noisy environment and has a significant improvement in accuracy at the cost of high computational complexity due to matrix inversion in the RLS algorithm.

In this paper, we propose a BC-NLMS blind adaptive equalization algorithm to tackle the problem of blind equalization with additive noise. In addition, a BC-NLMS with a variable step-size blind adaptive equalization algorithm is developed. By receiving signals with two antennas, we obtain a single-input double-output system. Further, we convert the system to a model in which the input and output signals are both corrupted by additive noise. Therefore, the problem of blind equalization turns into a problem of parameter estimation in errors-in-variables (EIV) model [38]; then, the second ordered adaptive algorithms can be applied to tackle the problem of blind equalization. By modifying the cost function, we propose a BC-NLMS algorithm for blind equalization. An on-line noise variance estimation method is also developed, and the noise-induced bias in the NLMS algorithm can be estimated and removed. We also developed the variable step-size NLMS algorithm (VSS-NLMS) proposed in [30] to further improve the stability of the algorithm and applied a bias compensated algorithm to improve the convergence accuracy. Simulation results demonstrate that the proposed algorithms performed well in blind equalization.

The rest of this paper is organized as follows. In Section 2, the problem of blind identification is formulated. In Section 3, a BC-NLMS blind identification algorithm is proposed. A BC-NLMS with variable step-size blind adaptive identification algorithm is developed in Section 4. Section 5 presents an application of the proposed algorithms to blind channel equalization. Some simulations are presented to demonstrate the effectiveness of the proposed algorithms in Section 6. Finally, conclusion is drawn in Section 7.

Notation. In this paper, we use boldface letters to denote vectors and matrices. The normal font is used for scalars. The superscript H, and T denote the Hermitian and matrix transpose operations, respectively. Besides, $(\cdot)^*$ denotes the complex conjugate transpose. $\mathrm{E}(\cdot)$ stands for the expectation operator, and $(\hat{\cdot})$ stands for an estimator.

2 Problem statement

Consider a discrete-time communication system obtained by technique of sensor array or time oversampling as follows:

$$y_i(k) = \sum_{n=0}^{M-1} h_i(k)s(n-k) + v_i(k), \quad i = 0, 1, \ldots, N, \tag{1}$$

where $s(k)$ denotes the transmitted symbol, $h(k)$ denotes the impulse response of the continuous-time channel, and $v(k)$ is the additive noise. $y(t)$ denotes the output. M is the order of the N subchannels.

The objective of blind channel equalization is to recover the input sequence $s(k)$ from the output $y(k)$.

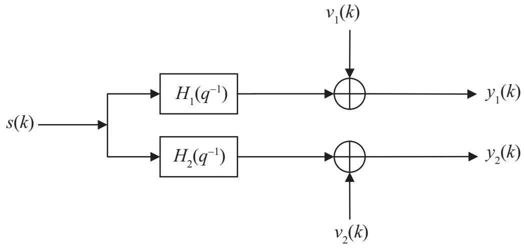

Figure 1 Single-input double-output system.

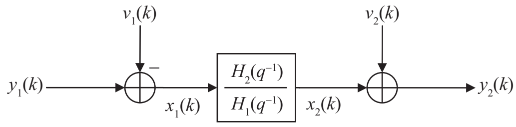

Figure 2 EIV model.

By receiving signals with two antennas, a single-input double-output system can be obtained as shown in Figure 1.

The model for each transmitted symbol $s(k)$ can be expressed as

$$\begin{aligned} y_1(k) =& H_1(q^{-1})s(k) + v_1(k) = x_1(k) + v_1(k), \\ y_2(k) =& H_2(q^{-1})s(k) + v_2(k) = x_2(k) + v_2(k), \end{aligned} \tag{2}$$

where $H_1(q^{-1})$ and $H_2(q^{-1})$ are transfer functions of the two channels, which are coprime polynomials. q^{-1} denotes a unit delay operator, $x_1(k)$ and $x_2(k)$ are the noise-free measurements. $v_1(k)$ and $v_2(k)$ are assumed to be uncorrelated zero-mean white processes with unknown variance σ_1^2 and σ_2^2 respectively and they are independent of $s(k)$. As the signals are received by means of two antennas, different noises are added to each channel, i.e., $\sigma_1^2 \neq \sigma_2^2$.

Then we have

$$\begin{aligned} H_1(q^{-1}) &= h_{1,0} + h_{1,1}q^{-1} + \cdots + h_{1,L}q^{-L}, \\ H_2(q^{-1}) &= h_{2,0} + h_{2,1}q^{-1} + \cdots + h_{2,L}q^{-L}, \end{aligned} \tag{3}$$

where h is the system parameter to be estimated and $L \geqslant 0$ is the channel order, which is known.

From (2), we have

$$\begin{aligned} y_1(k) - v_1(k) &= H_1(q^{-1})s(k), \\ y_2(k) - v_2(k) &= H_2(q^{-1})s(k). \end{aligned} \tag{4}$$

Then

$$\begin{aligned} H_2(q^{-1})(y_1(k) - v_1(k)) &= H_2(q^{-1})H_1(q^{-1})s(k) \\ &= H_1(q^{-1})H_2(q^{-1})s(k) \\ &= H_1(q^{-1})(y_2(k) - v_2(k)). \end{aligned} \tag{5}$$

Therefore, the single-input double-output system can be equivalent to an EIV model as shown in Figure 2.

According to (5), we can write

$$H_1(q^{-1})y_2(k) = H_2(q^{-1})y_1(k) + H_1(q^{-1})v_2(k) - H_2(q^{-1})v_1(k). \tag{6}$$

Eq. (6) can be expressed as

$$\begin{aligned} & h_{1,0}y_2(k) + h_{1,1}y_2(k-1) + \cdots + h_{1,L}y_2(k-L) \\ & = h_{2,0}y_1(k) + h_{2,1}y_1(k-1) + \cdots + h_{2,L}y_1(k-L) + H_1(q^{-1})v_2(k) - H_2(q^{-1})v_1(k). \end{aligned} \tag{7}$$

Define

$$\boldsymbol{\theta} = \frac{1}{h_{1,0}}[h_{1,1}, \ldots, h_{1,L}, h_{2,0}, h_{2,1}, \ldots, h_{2,L}]^{\mathrm{H}}, \tag{8}$$

$$\boldsymbol{\phi}_{y,k} = [-y_2(k-1), -y_2(k-2), \ldots, -y_2(k-L), y_1(k), y_1(k-1), \ldots, y_1(k-L)]^{\mathrm{T}}, \tag{9}$$

$$\boldsymbol{\phi}_{x,k} = [-x_2(k-1), \ldots, -x_2(k-L), x_1(k), x_1(k-1), \ldots, x_1(k-L)]^{\mathrm{T}}, \tag{10}$$

$$\boldsymbol{\phi}_{v,k} = [-v_2(k-1), \ldots, -v_2(k-L), v_1(k), v_1(k-1), \ldots, v_1(k-L)]^{\mathrm{T}}. \tag{11}$$

Then we have

$$\boldsymbol{\phi}_{y,k} = \boldsymbol{\phi}_{x,k} + \boldsymbol{\phi}_{v,k}. \tag{12}$$

According to (6)–(8), we have

$$y_2(k) = \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\phi}_{y,k} + n(k) \tag{13}$$

with

$$n(k) = v_2(k) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\phi}_{v,k}. \tag{14}$$

Similarly, we have

$$x_2(k) = \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{\phi}_{x,k}. \tag{15}$$

Eq. (8) shows that once the $\boldsymbol{\theta}$ are estimated, the characteristics of the transmission channels can be obtained.

3 Blind adaptive identification based on bias-compensated NLMS method

We first consider the noise-free outputs, i.e., $\boldsymbol{\phi}_{v,k} = 0$. The system can estimate the channel parameter $\boldsymbol{\theta}$ by minimizing the following normalized mean-square error cost function:

$$J_x(\boldsymbol{\theta}) = \frac{\mathrm{E}\left|x_2(k) - \boldsymbol{\phi}_{x,k}^{\mathrm{H}}\boldsymbol{\theta}_k\right|^2}{\epsilon + \boldsymbol{\phi}_{x,k}^{\mathrm{H}}\boldsymbol{\phi}_{x,k}}, \tag{16}$$

where ϵ is regularization parameter. Then we can obtain the optimal solution as

$$\boldsymbol{\theta}_o = \boldsymbol{R}_x^{-1}\boldsymbol{r}_{x_2x}, \tag{17}$$

where $\boldsymbol{R}_x = \mathrm{E}[\boldsymbol{\phi}_{x,k}\boldsymbol{\phi}_{x,k}^{\mathrm{H}}]$ and $\boldsymbol{r}_{x_2x} = \mathrm{E}[\boldsymbol{\phi}_{x,k}^{\mathrm{H}}x_2(k)]$.

As $x_2(k)$ and $\boldsymbol{\phi}_{x,k}$ are not available, we replace them by $y_2(k)$ and $\boldsymbol{\phi}_{y,k}$ respectively, i.e., $\boldsymbol{\phi}_{v,k} \neq 0$. The cost function can be written as

$$J_y(\boldsymbol{\theta}) = \frac{\mathrm{E}\left|y_2(k) - \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\theta}_k\right|^2}{\epsilon + \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}}. \tag{18}$$

Then the optimal solution is obtained as

$$\boldsymbol{\theta}_b = \boldsymbol{R}_y^{-1}\boldsymbol{r}_{y_2y}, \tag{19}$$

where $\boldsymbol{R}_y = \mathrm{E}[\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}]$ and $\boldsymbol{r}_{y_2y} = \mathrm{E}[\boldsymbol{\phi}_{y,k}^{\mathrm{H}}y_2(k)]$.

Since the noise vector $\boldsymbol{\phi}_{v,k}$ is independent of $\boldsymbol{\phi}_{x,k}$ and $y_2(k)$, then we have

$$\boldsymbol{R}_y = \boldsymbol{R}_x + \boldsymbol{R}_v, \tag{20}$$

$$\boldsymbol{r}_{y_2y} = \boldsymbol{r}_{x_2x}, \tag{21}$$

where

$$\boldsymbol{R}_v = \mathrm{E}\left[\boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\phi}_{v,k}\right] = \begin{bmatrix} \sigma_2^2\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_1^2\boldsymbol{I} \end{bmatrix}. \tag{22}$$

Then the optimal solution in (19) can be expressed as

$$\boldsymbol{\theta}_b = (\boldsymbol{R}_x + \boldsymbol{R}_v)^{-1}\boldsymbol{r}_{y_2y}. \tag{23}$$

As $\boldsymbol{\theta}_b \neq \boldsymbol{\theta}_o$, $\boldsymbol{\theta}_b$ is biased and the bias is dependent on the input noise.

In order to obtain the unbiased optimal solution (17), we search for a cost function whose gradient vector is identical to that of (16). So we modify the cost function as

$$J(\boldsymbol{\theta}) = \frac{\mathrm{E}\left|y_2(k) - \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\theta}_k\right|^2 - \boldsymbol{\theta}_k^{\mathrm{H}}\boldsymbol{R}_v\boldsymbol{\theta}_k}{\epsilon + \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}}. \tag{24}$$

Then the corresponding optimal solution can be obtained as

$$\boldsymbol{\theta} = (\boldsymbol{R}_y - \boldsymbol{R}_v)^{-1}\boldsymbol{r}_{y_2y}. \tag{25}$$

By substituting (20) and (21) into (25), we have

$$\boldsymbol{\theta} = \boldsymbol{R}_x^{-1}\boldsymbol{r}_{y_2x} = \boldsymbol{\theta}_o. \tag{26}$$

Eq. (26) shows that we can arrive at unbiased parameter estimate by modifying the cost function under noisy condition. Consequently, we use the stochastic gradient descent method to minimize the cost function (24) iteratively,

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} - \frac{\mu}{2}\nabla J(\boldsymbol{\theta}_{k-1}), \tag{27}$$

where the non-negative parameter μ is step-size, and $\nabla J(\boldsymbol{\theta}_k)$ denotes the gradient vector of $J(\boldsymbol{\theta}_k)$.

Then we have

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \frac{\mu}{\epsilon + \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}}\left[\boldsymbol{r}_{y_2y} - \boldsymbol{R}_y\boldsymbol{\theta}_{k-1} + \boldsymbol{R}_v\boldsymbol{\theta}_{k-1}\right]. \tag{28}$$

As the moments $\boldsymbol{R}_y$ and $\boldsymbol{r}_{y_2y}$ are usually unavailable, in stochastic gradient descent method we replace them by their instantaneous approximations that $\boldsymbol{R}_y = \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}$ and $\boldsymbol{r}_{y_2y} = \boldsymbol{\phi}_{y,k}^{\mathrm{H}}y_2(k)$. Then we obtain the BC-NLMS algorithm as

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \frac{\mu}{\epsilon + \boldsymbol{\phi}_{x,k}^{\mathrm{H}}\boldsymbol{\phi}_{x,k}}\left[\boldsymbol{\phi}_{y,k}\left(y_2(k) - \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_{k-1}\right) + \begin{bmatrix}\sigma_2^2\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \sigma_1^2\boldsymbol{I}\end{bmatrix}\hat{\boldsymbol{\theta}}_{k-1}\right]. \tag{29}$$

As the variances of the noises σ_1^2 and σ_2^2 are unknown, it is necessary to estimate them.

The traditional NLMS update equation can be described as

$$\hat{\boldsymbol{\theta}}_k' = \hat{\boldsymbol{\theta}}_{k-1}' + \frac{\mu}{\epsilon + \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}}\left[\boldsymbol{\phi}_{y,k}\left(y_2(k) - \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_{k-1}'\right)\right], \tag{30}$$

where $\boldsymbol{\theta}_k'$ indicates the estimate parameter based on NLMS algorithm without bias-compensation.

Consider the property of the least-mean-square error, we have

$$e_1(k) = y_2(k) - \boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_k'. \tag{31}$$

Define the auto-correlation function of the least-mean-square error as

$$f_1(k) = \mathrm{E}\left[e_1^2(k)\right]. \tag{32}$$

According to the principle of orthogonality, it is deduced that the regression vector $\boldsymbol{\phi}_{y,k}$ and the least-mean-square error $e_1(k)$ are uncorrelated as

$$\mathrm{E}[\boldsymbol{\phi}_{y,k}e_1(k)] = 0. \tag{33}$$

Substituting (13), (31) and (33) into (32), we have

$$f_1(k) = \mathrm{E}\left[n(k)y_2(k) - n(k)\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_k'\right]. \tag{34}$$

Since

$$\mathrm{E}\left[n(k)y_2(k)\right] = \mathrm{E}\left[\left(v_2(k) - \boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\theta}\right)\left(\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\theta} + n(k)\right)\right] = \mathrm{E}\left[v_2^2(k)\right] = \sigma_2^2, \tag{35}$$

$$\mathrm{E}\left[n(k)\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\right]=\mathrm{E}\left[\left(v_2(k)-\boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\theta}\right)\left(\boldsymbol{\phi}_{x,k}^{\mathrm{H}}\boldsymbol{\theta}+\boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\theta}\right)\right]=-\boldsymbol{\theta}^{\mathrm{H}}\begin{bmatrix}\sigma_2^2\boldsymbol{I} & \mathbf{0}\\ \mathbf{0} & \sigma_1^2\boldsymbol{I}\end{bmatrix}. \tag{36}$$

So Eq. (34) can be written as

$$f_1(k)=\sigma_2^2\left(1+\boldsymbol{\theta}_1^{\mathrm{H}}\hat{\boldsymbol{\theta}}_1'\right)+\sigma_1^2\boldsymbol{\theta}_2^{\mathrm{H}}\hat{\boldsymbol{\theta}}_2', \tag{37}$$

where $\boldsymbol{\theta}=[\boldsymbol{\theta}_1^{\mathrm{H}}\ \boldsymbol{\theta}_2^{\mathrm{H}}]^{\mathrm{H}}$ and $\hat{\boldsymbol{\theta}}'=[\hat{\boldsymbol{\theta}}_1'^{\mathrm{H}}\ \hat{\boldsymbol{\theta}}_2'^{\mathrm{H}}]^{\mathrm{H}}$.

Here we introduce a backward output estimate method. We first introduce an auxiliary parameter $\boldsymbol{\beta}$ as follows:

$$\hat{\boldsymbol{\beta}}_k=\hat{\boldsymbol{\beta}}_{k-1}+\frac{\mu}{\epsilon+\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\boldsymbol{\phi}_{y,k}}\left[\boldsymbol{\phi}_{y,k}\left(y_2(k-L-1)-\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_{k-1}\right)\right], \tag{38}$$

and the backward output prediction error $e_2(k)$ is defined as

$$e_2(k)=y_2(k-L-1)-\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_k. \tag{39}$$

The cross-correlation function between the least-mean-square error and prediction error is defined as

$$f_2(k)=\mathrm{E}\left[e_1(k)e_2(k)\right]. \tag{40}$$

By the similar orthogonal property as (33), we have

$$\mathrm{E}\left[\boldsymbol{\phi}_{y,k}e_2(k)\right]=0. \tag{41}$$

Using (13), (39), and (41) into (40), we have

$$f_2(k)=\mathrm{E}\left[n(k)y_2(k-L-1)-n(k)\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_k\right]. \tag{42}$$

Since

$$\begin{aligned}\mathrm{E}\left[n(k)y_2(k-L-1)\right]&=\mathrm{E}\left[\left(v_2(k)-\boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\theta}\right)\left(\boldsymbol{\phi}_{y,k-L-1}^{\mathrm{H}}\boldsymbol{\theta}+n(k-L-1)\right)\right]\\&=\mathrm{E}\left[v_2(k)v_2(k-L-1)\right]-\mathrm{E}\left[\boldsymbol{\phi}_{v,k}^{\mathrm{H}}\boldsymbol{\theta}v_2(k-L-1)\right]\\&=0.\end{aligned} \tag{43}$$

Using (36), we have

$$f_2(k)=\sigma_2^2\boldsymbol{\theta}_1^{\mathrm{H}}\hat{\boldsymbol{\beta}}_1'+\sigma_1^2\boldsymbol{\theta}_2^{\mathrm{H}}\hat{\boldsymbol{\beta}}_2', \tag{44}$$

where $\boldsymbol{\beta}=[\boldsymbol{\beta}_1^{\mathrm{H}}\ \boldsymbol{\beta}_2^{\mathrm{H}}]^{\mathrm{H}}$.

Combine (37) and (44), the unknown noise variances σ_1^2 and σ_2^2 can be estimated by

$$\begin{bmatrix}f_1(k)\\ f_2(k)\end{bmatrix}=\begin{bmatrix}1+\boldsymbol{\theta}_1^{\mathrm{H}}\hat{\boldsymbol{\theta}}_1' & \boldsymbol{\theta}_2^{\mathrm{H}}\hat{\boldsymbol{\theta}}_2'\\ \boldsymbol{\theta}_1^{\mathrm{H}}\hat{\boldsymbol{\beta}}_1 & \boldsymbol{\theta}_2^{\mathrm{H}}\hat{\boldsymbol{\beta}}_2\end{bmatrix}\begin{bmatrix}\sigma_2^2\\ \sigma_1^2\end{bmatrix}, \tag{45}$$

where $\boldsymbol{\theta}_1^{\mathrm{H}}$ and $\boldsymbol{\theta}_2^{\mathrm{H}}$ can be replaced by $\hat{\boldsymbol{\theta}}_{1,k-1}^{\mathrm{H}}$ and $\hat{\boldsymbol{\theta}}_{2,k-1}^{\mathrm{H}}$, respectively. Meanwhile, if the values of $f_1(k)$ and $f_2(k)$ are available, the unknown noise variances σ_1^2 and σ_2^2 can be estimated by

$$\begin{bmatrix}\hat{\sigma}_2^2\\ \hat{\sigma}_1^2\end{bmatrix}=\begin{bmatrix}1+\hat{\boldsymbol{\theta}}_{1,k-1}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_1' & \hat{\boldsymbol{\theta}}_{2,k-1}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_2'\\ \hat{\boldsymbol{\theta}}_{1,k-1}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\theta}}_{2,k-1}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_2\end{bmatrix}^{-1}\begin{bmatrix}\hat{f}_1(k)\\ \hat{f}_2(k)\end{bmatrix}. \tag{46}$$

According to the definition of $f_1(k)$ and $f_2(k)$, $\hat{f}_1(k)$ and $\hat{f}_2(k)$ can be computed respectively as

$$\hat{f}_1(k)=\lambda\hat{f}_1(k-1)+(1-\lambda)|e_1(k)|^2, \tag{47}$$

$$\hat{f}_2(k)=\lambda\hat{f}_2(k-1)+(1-\lambda)e_1(k)e_2(k). \tag{48}$$

Therefore, the BC-NLMS algorithm can be summarized as Algorithm 1.

Algorithm 1 Bias-compensated NLMS algorithm

1: Initialization
$\hat{\boldsymbol{\theta}}_0 = 0,\ \hat{\boldsymbol{\theta}}'_0 = 0,\ \hat{\boldsymbol{\beta}}_0 = 0,\ \hat{f}_1(0),\ \hat{f}_2(0) = 0;$
2: Traditional NLMS algorithm for blind identification
$\hat{\boldsymbol{\theta}}'_k = \hat{\boldsymbol{\theta}}'_{k-1} + \frac{\mu}{\epsilon+\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}}[\boldsymbol{\phi}_{y,k}(y_2(k) - \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\hat{\boldsymbol{\theta}}'_{k-1})];$
3: Estimation of $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$
$\hat{\boldsymbol{\beta}}_k = \hat{\boldsymbol{\beta}}_{k-1} + \frac{\mu}{\epsilon+\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}}[\boldsymbol{\phi}_{y,k}(y_2(k-L-1) - \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\hat{\boldsymbol{\beta}}_{k-1})],$
$\hat{f}_1(k) = \lambda\hat{f}_1(k-1) + (1-\lambda)|e_1(k)|^2,$
$\hat{f}_2(k) = \lambda\hat{f}_2(i-1) + (1-\lambda)e_1(k)e_2(k),$
$\begin{bmatrix}\hat{\sigma}_2^2\\ \hat{\sigma}_1^2\end{bmatrix} = \begin{bmatrix}1+\hat{\boldsymbol{\theta}}^{\mathrm{H}}_{1,k-1}\hat{\boldsymbol{\theta}}'_1 & \hat{\boldsymbol{\theta}}^{\mathrm{H}}_{2,k-1}\hat{\boldsymbol{\theta}}'_2\\ \hat{\boldsymbol{\theta}}^{\mathrm{H}}_{1,k-1}\hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\theta}}^{\mathrm{H}}_{2,k-1}\hat{\boldsymbol{\beta}}_2\end{bmatrix}^{-1}\begin{bmatrix}\hat{f}_1(k)\\ \hat{f}_2(k)\end{bmatrix};$
4: Bias compensation update
$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \frac{\mu}{\epsilon+\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}}\left[\boldsymbol{\phi}_{y,k}(y_2(k) - \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\hat{\boldsymbol{\theta}}_{k-1}) + \begin{bmatrix}\hat{\sigma}_2^2\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \hat{\sigma}_1^2\boldsymbol{I}\end{bmatrix}\hat{\boldsymbol{\theta}}_{k-1}\right];$
5: Repeat from **2–4** to update the estimation results until convergence.

4 Blind adaptive identification based on bias-compensated variable step-size NLMS method

The NLMS algorithm is popular for its low computational power, but still suffers from the contradiction between convergence speed and estimation accuracy. Therefore we consider the variable step-size mechanism to introduce dynamic adjustment step parameters in the adaptive blind identification algorithm.

Ref. [25] described a power normalized NLMS algorithm, which reduced computational complexity of the NLMS algorithm. We now apply the algorithm to our model as a variable step-size method and further develop the algorithm by bias-compensated technique.

We scale both the numerator and denominator of the term $\mu/(\epsilon + \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k})$ in (30) by N, and the NLMS algorithm can be written as

$$\hat{\boldsymbol{\theta}}'_k = \hat{\boldsymbol{\theta}}'_{k-1} + \frac{\mu/N}{\epsilon/N + \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}/N}\left[\boldsymbol{\phi}_{y,k}\big(y_2(k) - \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\hat{\boldsymbol{\theta}}'_{k-1}\big)\right] \tag{49}$$

$$= \hat{\boldsymbol{\theta}}'_{k-1} + \frac{\mu'}{\epsilon' + \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}/N}\left[\boldsymbol{\phi}_{y,k}\big(y_2(k) - \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\hat{\boldsymbol{\theta}}'_{k-1}\big)\right], \tag{50}$$

where μ' and ϵ' are smaller step-size and regularization parameter, respectively. As

$$\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k} = y_2^2(k-1) + y_2^2(k-2) + \cdots + y_2^2(k-L) + y_1^2(k) + y_1^2(k-1) + \cdots + y_1^2(k-L), \tag{51}$$

a careful inspection of the equation reveals that $\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}$ contains two part and each of them can be calculated respectively as

$$y_2^2(k-1) + y_2^2(k-2) + \cdots + y_2^2(k-L) = (L-1)\overline{y_2^2}(k), \tag{52}$$

$$y_1^2(k) + y_1^2(k-1) + \cdots + y_1^2(k-L) = L\overline{y_1^2}(k). \tag{53}$$

$\overline{y_1^2}(k)$ and $\overline{y_2^2}(k)$ can be calculated by iteration respectively as

$$p_1(k) = \alpha p_1(k-1) + (1-\alpha)|y_1(k)|^2 = \overline{y_1^2}(k), \tag{54}$$

$$p_2(k) = \alpha p_2(k-1) + (1-\alpha)|y_2(k-1)|^2 = \overline{y_2^2}(k). \tag{55}$$

Then $\boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}/N$ can be further calculated as

$$p(k) = Lp_1(k-1) + (L-1)p_2(k-1) = \boldsymbol{\phi}^{\mathrm{H}}_{y,k}\boldsymbol{\phi}_{y,k}/N. \tag{56}$$

As the value of $p(k)$ is small in the initial stage of iteration, the novel NLMS algorithm will gain large step size, while gain smaller step size with the increasing of iterations. The step-size variation makes it possible for the VSS-NLMS algorithm to converge faster and to a lower steady-state error than in the fixed step-size case.

Besides, another property of the further developed algorithm is its computational simplicity. For NLMS algorithm, each iteration of recursion requires $24L+16$ real multiplications, $24L+12$ real additions and one real division for general complex-valued data, while $6L+4$ real multiplications, $6L+2$ real additions and one real division for real-valued data. On the other hand, the developed variable step-size NLMS method requires $16L+18$ real multiplications, $16L+22$ real additions and one real division for general complex-valued data, while $4L+11$ real multiplications, $4L+8$ real additions and one real division for real-valued data per iteration. In summary, for real-valued data, the developed variable step-size NLMS method reduce the computational simplicity when $L>4$, while $L>2$ for general complex-valued data.

Then, we further develop the modified NLMS algorithm with power normalization to a bias compensate algorithm. Similar with the BC-NLMS algorithm, the BC-VSS-NLMS algorithm can be summarized as Algorithm 2.

Algorithm 2 Bias-compensated NLMS algorithm with variable step-size

1: Initialization
$p_1(0)=|y_1(L)|^2$, $p_2(0)=|y_2(L)|^2$, $p(0)=0$, $\hat{\boldsymbol{\theta}}_0=0$, $\hat{\boldsymbol{\theta}}'_0=0$, $\hat{\boldsymbol{\beta}}_0=0$, $\hat{f}_1(0)=0$, and $\hat{f}_2(0)$=0;

2: Update $p(i)$
$p_1(k)=\alpha p_1(k-1)+(1-\alpha)|y_1(k)|^2$,
$p_2(k)=\alpha p_2(k-1)+(1-\alpha)|y_2(k-1)|^2$,
$p(k)=Lp_1(k-1)+(L-1)p_2(k-1)$;

3: The VSS-NLMS algorithm for blind identification
$\hat{\boldsymbol{\theta}}'_k=\hat{\boldsymbol{\theta}}'_{k-1}+\frac{\mu}{\epsilon+p(k)}[\boldsymbol{\phi}_{y,k}(y_2(k)-\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}'_{k-1})]$;

4: Estimation of $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$
$\hat{\boldsymbol{\beta}}_k=\hat{\boldsymbol{\beta}}_{k-1}+\frac{\mu}{\epsilon+p(k)}[\boldsymbol{\phi}_{y,k}(y_2(k-L-1)-\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_{k-1})]$,
$\hat{f}_1(k)=\lambda\hat{f}_1(k-1)+(1-\lambda)|e_1(k)|^2$,
$\hat{f}_2(k)=\lambda\hat{f}_2(i-1)+(1-\lambda)e_1(k)e_2(k)$,
$\begin{bmatrix}\hat{\sigma}_2^2\\ \hat{\sigma}_1^2\end{bmatrix}=\begin{bmatrix}1+\hat{\boldsymbol{\theta}}_{1,k-1}^{\mathrm{H}}\hat{\boldsymbol{\theta}}'_1 & \hat{\boldsymbol{\theta}}_{2,k-1}^{\mathrm{H}}\hat{\boldsymbol{\theta}}'_2\\ \hat{\boldsymbol{\theta}}_{1,k-1}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\theta}}_{2,k-1}^{\mathrm{H}}\hat{\boldsymbol{\beta}}_2\end{bmatrix}^{-1}\begin{bmatrix}\hat{f}_1(k)\\ \hat{f}_2(k)\end{bmatrix}$;

5: Bias compensation update
$\hat{\boldsymbol{\theta}}_k=\hat{\boldsymbol{\theta}}_{k-1}+\frac{\mu}{\epsilon+p(k)}\left[\boldsymbol{\phi}_{y,k}(y_2(k)-\boldsymbol{\phi}_{y,k}^{\mathrm{H}}\hat{\boldsymbol{\theta}}_{k-1})+\begin{bmatrix}\hat{\sigma}_2^2\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \hat{\sigma}_1^2\boldsymbol{I}\end{bmatrix}\hat{\boldsymbol{\theta}}_{k-1}\right]$;

6: Repeat from **2**–**5** to update the estimation results until convergence.

5 Application to blind channel equalization

Then the estimates can be used to recover the signal transmitted by the channels.

By stacking the channel output vector $\boldsymbol{y}_k$, we have

$$\boldsymbol{y}_k=[y_1(k),\ldots,y_1(k-M+1),y_2(k),\ldots,y_2(k-M+1)]^{\mathrm{T}}. \tag{57}$$

Define

$$\boldsymbol{s}_k=[s(k),s(k-1),\ldots,s(k-L-M+1)]^{\mathrm{T}}, \tag{58}$$

$$\boldsymbol{v}_k=[v_1(k),\ldots,v_1(k-M+1),v_2(k),\ldots,v_2(k-M+1)]^{\mathrm{T}}, \tag{59}$$

where M is called the stack number or smoothed factor and $M\geqslant L$.

Then we have

$$\boldsymbol{y}_k=\boldsymbol{H}\boldsymbol{s}_k+\boldsymbol{v}_k, \tag{60}$$

where the channel convolution matrix $\boldsymbol{H}$ is a $2M\times(L+M)$ Sylvester matrix with full column rank:

$$\boldsymbol{H}=\begin{bmatrix} h'_{1,0} & \cdots & h'_{1,L} & 0 & 0 & \cdots & 0\\ 0 & \cdots & h'_{1,L-1} & h'_{1,L} & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & h'_{1,L}\\ h'_{2,0} & \cdots & h'_{2,L} & 0 & 0 & \cdots & 0\\ 0 & \cdots & h'_{2,L-1} & h'_{2,L} & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & h'_{2,L} \end{bmatrix} \tag{61}$$

with $h'_{i,k} = h_{i,k}/h_{1,0}$, $i = 1, 2$, and $k = 0, 1, \ldots, L$.

The best linear least squares estimate of $\boldsymbol{s}_k$ is given by

$$\hat{\boldsymbol{s}}_{0,k} = \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{y}_k. \tag{62}$$

By substituting $h'_{i,k}$ with the their unbiased estimate obtained by (29), the estimate value $\hat{\boldsymbol{H}}$ can be obtained. Then, Eq. (62) can be expressed as

$$\hat{\boldsymbol{s}}_k = \left(\hat{\boldsymbol{H}}^{\mathrm{H}}\hat{\boldsymbol{H}}\right)^{-1}\hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{y}_k. \tag{63}$$

Therefore, the transmitted signal can be recovered by (63).

6 Simulation results

In the simulation, we consider a scenario with a two-ray multipath channel presented in [16]. The single-input double-output system transfer function is described as

$$\begin{aligned} H_1(q^{-1}) &= -1.1836 + 0.4906q^{-1} - 0.3093q^{-2} + 0.4011q^{-3} + 0.1269q^{-4} - 1.8522q^{-5}, \\ H_2(q^{-1}) &= 1.2965 + 0.0525q^{-1} + 0.3410q^{-2} - 0.0260q^{-3} + 0.3991q^{-4} + 0.8817q^{-5}, \end{aligned} \tag{64}$$

where both $H_1(q^{-1})$ and $H_2(q^{-1})$ are nonminimum phase filters.

The noise-free input $s(k)$ is generated from a series of non-return-to-zero code and the additive noise $v_1(k)$ and $v_1(k)$ are Gaussian distributions. We set the smoothing factor $\alpha = 0.998$. The number of iteration is chosen as 50000 and a Monte Carlo simulation of 100 independent trials independently is conducted under the same simulation scenario.

The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 20\log_{10}\frac{\mathrm{std}\,(x_k)}{\mathrm{std}\,(v_k)}\ (\mathrm{dB}), \quad k = 1, 2. \tag{65}$$

According to different requirements for SNR, we determine the variances of $v_1(k)$ and $v_2(k)$.

Here, we introduce mean-square deviation (MSD) as the criteria for assessing the accuracy of the estimation results, which is defined as

$$\mathrm{MSD}_{\boldsymbol{\theta}} = 20\log_{10}\left(\frac{1}{N}\sum_{k=1}^{N}\|\widehat{\boldsymbol{\theta}}_{k,i} - \boldsymbol{\theta}\|\right)\ (\mathrm{dB}), \tag{66}$$

where N denotes the number of the independent trials, and $\boldsymbol{\theta}_{k,i}$ denotes the estimate of $\boldsymbol{\theta}$ at the ith iteration in kth trial.

Figure 3 gives the mean square error (MSE) curves of the LMS method, NLMS method, the VSS-NLMS method, the BC-NLMS method and the BC-VSS-NLMS method when the step-size $\mu = 0.1$ and SNR = 15 dB.

The curves demonstrate that the results of the traditional NLMS algorithms are biased when considering the background noises of each channel. Compared with the method without bias-compensate, there is a significant improvement in terms of estimation accuracy by implementing the proposed BC-NLMS algorithm and BC-VSS-NLMS algorithm.

In Figure 4 the symbol error rate (SER) performance of each algorithm is shown for the selected SNR levels.

It is clear from Figure 4 that the SER performance of the bias-compensate methods is better compared with both the NLMS method and the VSS-NLMS method under different SNR conditions. SER decreases with the increase of SNR. However, when SNR is reduced to a certain extent, there is a tendency that the SER curves of the four algorithms approach to each other. This is because with the reduction of noise variance, the benefit of bias compensation for noise is less and less obvious, and the accuracy of the four methods is closer.

Figures 5(a) and (b) show the constellation of the recovered signal by implementing the NLMS method and VSS-NLMS method when the step-size $\mu = 0.01$ at SNR = 12 dB, respectively. Figures 5(c) and

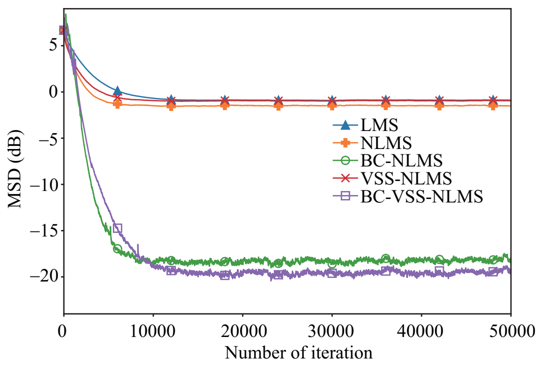

Figure 3 (Color online) MSD curves.

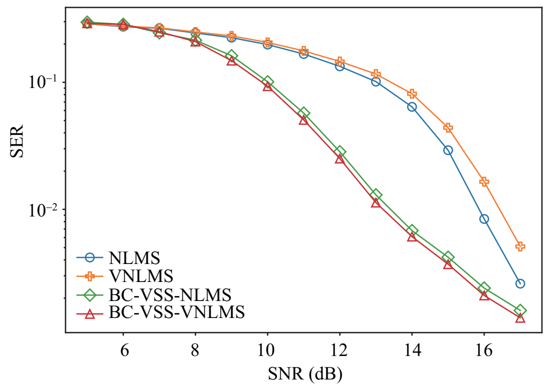

Figure 4 (Color online) SER curves.

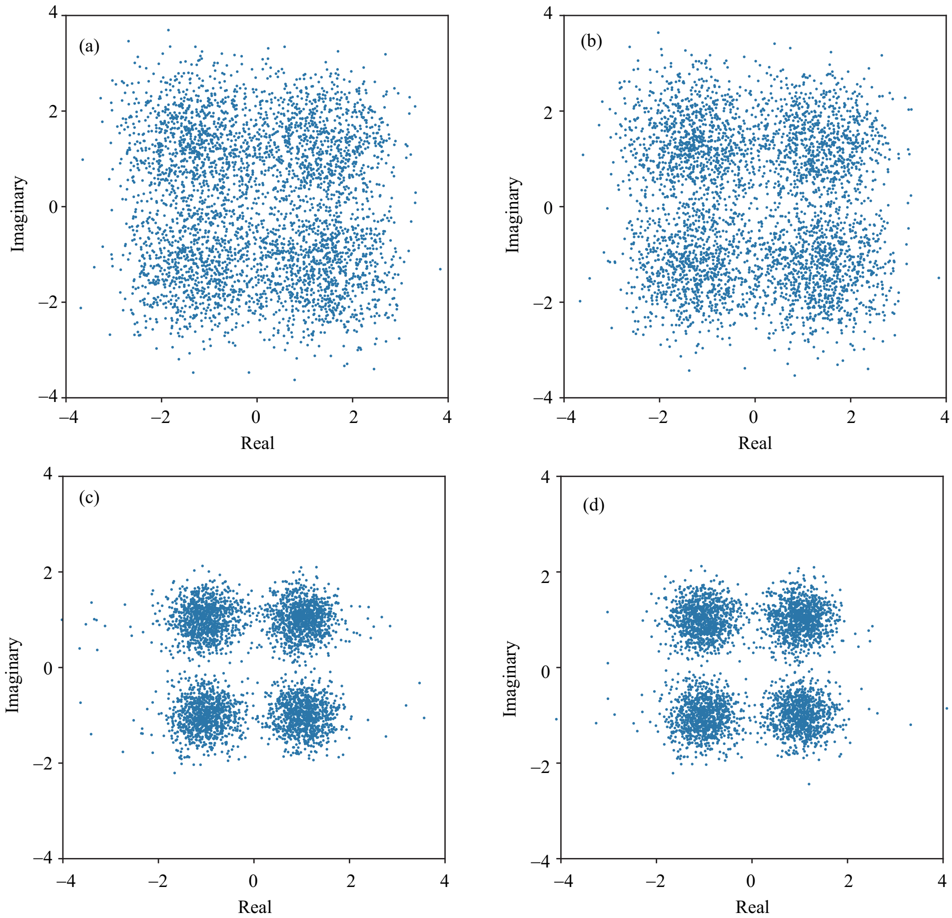

Figure 5 (Color online) Constellation of the equalized signal. (a) NLMS method; (b) VSS-NLMS method; (c) BC-NLMS method; (d) BC-VSS-NLMS.

(d) show the recovered signal constellation by implementing the BC-NLMS method and BC-VSS-NLMS method at SNR= 12 dB, respectively.

It indicates that the channel is not well equalized by methods without bias-compensate. Compared with the traditional NLMS methods, there is a significant improvement in terms of equalization performance by implementing the proposed BC-NLMS and BC-VSS-NLMS method as the retrieved signal constellation are tightly clustered, the points on the constellation are more convergent to the true values.

Simulation results above illustrate that both the BC-NLMS and BC-VSS-NLMS methods can effectively improve the performance of the NLMS and VSS-NLMS method. In other words, the BC-NLMS and BC-VSS-NLMS algorithms can both effectively solve the problem of bind equalization.

7 Conclusion

In this paper, we address the problem of blind channel equalization that the transmission channel is corrupted by noise with unknown statistics by proposing BC-NLMS and BC-VSS-NLMS methods. The blind equalization problem is converted to an EIV parameter estimation problem by receiving signals with two antennas. The proposed algorithms contain an approach to estimate the unknown additive noise variance of the transmission channel online, and then the effect of the noise-induced bias can be removed. Therefore, the estimate of channel characters can be obtained in real time to equalize the channel and recover the transmitted signal. We compared the numerical performance behavior of the proposed methods with other adaptive methods. Simulation results demonstrate the effectiveness of the proposed methods for blind channel equalization on the accuracy of equalization.

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