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Finite-time boundedness analysis and composite anti-disturbance control for uncertain semi-Markovian jump systems with time delay

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Dear editor,

Markovian jump systems (MJSs) are a special type of hybrid systems that have been employed to yield a variety of results [1, 2]. Unlike MJSs, semi-Markovian jump systems (S-MJSs) relax the restriction requiring transition rate (TR) to always be constant. Due to their broad application prospects, S-MJSs have attracted the attention of many scholars. For example, the authors of [3, 4] addressed the problem of sliding mode control for S-MJSs. However, most research results have primarily been described as Lyapunov stability, which chiefly addresses the problem of convergence at infinite intervals. In many practical systems, the performance at a finite-time interval is also critical. The authors of [5, 6] addressed the finite-time problem based on multi-agent systems, while the authors of [7] analyzed finite-time stability for MJSs. In this study, the multiple-disturbance problem and finite-time performance are considered in a unified system model. To match the practical system, we used the idea of composite control to address the multiple-disturbance problem. Disturbance observer-based control (DOBC) was used to suppress exogenous disturbance by feed forward compensation, and the H_∞ method was applied to attenuate norm bounded disturbance. The main contributions of this study are summarized as follows: (i) a sufficient condition of finite-time boundedness is given for time-delay S-MJSs; (ii) uncertain parameters and generally uncertain TRs for S-MJSs are addressed to guarantee the system's performance; (iii) a disturbance observer for S-MJSs is designed to estimate true disturbance; (iv) a composite anti-disturbance controller is designed to address the multiple-disturbance problem for S-MJSs.

Preliminaries. Time-delay S-MJSs with multiple disturbances and uncertain parameters are described as

$$\begin{aligned}\dot{x}(t) &= [A(\gamma_t)+\Delta A(t,\gamma_t)]x(t)+[A_d(\gamma_t)\\&\quad+\Delta A_d(t,\gamma_t)]x(t-\tau)+B(\gamma_t)(u(t)+d(t))\\&\quad+H(\gamma_t)d_1(t),\\x(s)&=\varphi_1(s),\quad \forall s\in[-\tau,0],\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $u(t)\in\mathbb{R}^m$ is the control input, $d(t)\in\mathbb{R}^r$ is the unknown disturbance, and $d_1(t)$ refers to the norm bounded disturbance. $\tau>0$ denotes the time delay. $\varphi_1(s)$ is the vector valued function. $\{\gamma_t, t\geqslant 0\}$ denotes the right continuous semi-Markovian process taking values in the set $S=\{1,2,\ldots,N\}$. $Z=\{\lambda_{ij}(h)\}$ denotes the TR matrix and $h>0$ denotes the sojourn time. $A(\gamma_t)$, $A_d(\gamma_t)$, and $B(\gamma_t)$ are constant matrices, and $\Delta A(t,\gamma_t)$ and $\Delta A_d(t,\gamma_t)$ denote time varying parametric uncertainties.

Remark 1. In this study, generally uncertain TRs are primarily considered. We can describe $\lambda_{ij}(h)=\lambda_{ij}+\Delta\lambda_{ij}(h)$, in which $\lambda_{ij}=\frac{1}{2}(\underline{\lambda}_{ij}+\overline{\lambda}_{ij})$, $|\Delta\lambda_{ij}(h)|\leqslant k_{ij}$, and $k_{ij}=\frac{1}{2}(\overline{\lambda}_{ij}-\underline{\lambda}_{ij})$.

Assumption 1 ([8])**.** The disturbance input employed in this study was generated by an exogenous power system

$$\begin{aligned}\dot{w}(t)&=W(\gamma_t)w(t)+M(\gamma_t)d_2(t),\\d(t)&=V(\gamma_t)w(t),\end{aligned}\tag{2}$$

where $w(t)\in\mathbb{R}^{m\times 1}$ denotes the system state, and $d_2(t)$ is the disturbance caused by parametric and structural uncertainties in an external system. $W(\gamma_t)\in\mathbb{R}^{m\times m}$ and $V(\gamma_t)\in\mathbb{R}^{1\times m}$ refer to the appropriate dimensional matrices.

Assumption 2. $(A(\gamma_t),B(\gamma_t))$ is controllable and $(W(\gamma_t),\ B(\gamma_t)V(\gamma_t))$ is observable.

The disturbance observer is designed as

$$\begin{aligned}\hat{d}(t)&=V(\gamma_t)\hat{w}(t),\quad \hat{w}(t)=v(t)-L(\gamma_t)x(t),\\\dot{v}(t)&=(W(\gamma_t)+L(\gamma_t)B(\gamma_t)V(\gamma_t))(v(t)\\&\quad-L(\gamma_t)x(t))+L(\gamma_t)(A(\gamma_t)x(t)\\&\quad+A_d(\gamma_t)x(t-\tau)+B(\gamma_t)u(t)).\end{aligned}\tag{3}$$

The DOBC controller is designed as $u(t)=-\hat{d}(t)+K(\gamma_t)x(t)$, where $\hat{d}(t)$ is the estimation of $d(t)$. $v(t)$ represents the indirect variable. $K(\gamma_t)$ and $L(\gamma_t)$ represent the control gain and observer gain. For simplicity, we created

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$\gamma_t = i$ using the following analysis. The closed-loop system can be given as

$$\begin{aligned} &\dot{\xi}(t) = \tilde{A}_i \xi(t) + \tilde{A}_{di}\xi(t-\tau) + \tilde{H}_i \tilde{d}(t), \\ &\xi(s) = \varphi(s),\ \forall s \in [-\tau, 0], \end{aligned} \tag{4}$$

where $\xi(t) = [x^{\mathrm{T}}(t)\ e_w^{\mathrm{T}}(t)]^{\mathrm{T}}$, $e_w(t) = w(t) - \hat{w}(t)$,

$$\begin{aligned} &\tilde{A}_i = \begin{bmatrix} A_i + \Delta A_i + B_i K_i & B_i V_i \\ 0 & W_i + L_i B_i V_i \end{bmatrix}, \\ &\tilde{A}_{di} = \begin{bmatrix} A_{di} + \Delta A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \\ &\tilde{d}(t) = [d_1^{\mathrm{T}}(t)\ d_2^{\mathrm{T}}(t)]^{\mathrm{T}}, \\ &\tilde{H}_i = \begin{bmatrix} H_i & 0 \\ L_i H_i & M_i \end{bmatrix}, \\ &\int_0^{\mathrm{T}} \tilde{d}^{\mathrm{T}}(t)\tilde{d}(t)\mathrm{d}t \leqslant d\ (d \geqslant 0). \end{aligned}$$

The reference output can be given as

$$z(t) = C_{1i}x(t) + C_{2i}e_w(t) = \tilde{C}_i \xi(t). \tag{5}$$

Definition 1. For $\forall \tilde{d} \in L_2[0, T)$, if the composite system in (4) and (5) satisfies the inequality $E\{\int_0^{\mathrm{T}} z(t)^{\mathrm{T}} z(t)\mathrm{d}t\} \leqslant \gamma^2\{\int_0^{\mathrm{T}} \tilde{d}(t)^{\mathrm{T}}\tilde{d}(t)\mathrm{d}t\}$, where $\gamma > 0$ is the given scalar, then the system is said to achieve H_∞ finite-time boundedness with respect to (c_1, c_2, T, R_i, d) in norm bounded γ.

Definition 2. For given positive constants T, c_1, c_2, d, and positive definite matrix R_i, if the closed-loop system in (4) and (5) satisfies $Ex^{\mathrm{T}}(t_1)R_i x(t_1) \leqslant c_1 \Rightarrow Ex^{\mathrm{T}}(t_2)R_i x(t_2) \leqslant c_2$, where $t_1 \in [-\tau\ 0]$, $t_2 \in [0\ T]$, $c_2 > c_1$, then the system is said to achieve finite-time boundedness with respect to (c_1, c_2, T, R_i, d).

Lemma 1 ([9]). For any real number ε, any matrix F, the inequality $\varepsilon(F + F^{\mathrm{T}}) \leqslant \varepsilon^2 D + FD^{-1}F^{\mathrm{T}}$ holds, where $D > 0$.

In the following let us investigate the finite-time boundedness.

Theorem 1. If there exist symmetric positive definite matrices P_i, Q_g, Q_{gi}, symmetric matrices J_i, J_{gi}, appropriate matrices N_{pi}, and positive constants α, T, c_1, c_2, d, γ, $\tilde{c}_1$, for $\forall i \in S$, $g = 1, 2$, and $p = 1, 2, 3$, such that the following inequalities hold:

$$\begin{bmatrix} \Pi_{1i}^{11} & \Pi_{1i}^{12} & \Pi_{1i}^{13} & P_i\tilde{H}_i & \tilde{C}_i^{\mathrm{T}} & \tau N_{1i}^{\mathrm{T}} & \Pi_{1i}^{17} \\ * & \Pi_{1i}^{22} & \Pi_{1i}^{23} & 0 & 0 & \tau N_{2i}^{\mathrm{T}} & 0 \\ * & * & \Pi_{1i}^{33} & 0 & 0 & \tau N_{3i}^{\mathrm{T}} & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\tau Q_{2i} & 0 \\ * & * & * & * & * & * & \Pi_{1i}^{77} \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} \sum_{j\in S_k^i}\lambda_{ij}(Q_{gj} - J_{gi}) + \sum_{j\in S_k^i}\frac{1}{4}k_{ij}^2 - Q_g & \Pi_{3i}^{12} \\ * & \Pi_{3i}^{22} \end{bmatrix} < 0, \tag{7}$$

$$P_j - J_i \leqslant 0,\ Q_{gj} - J_{gi} \leqslant 0,\ j \in S_{uk}^i,\ j \neq i, \tag{8}$$

$$P_j - J_i \geqslant 0,\ Q_{gj} - J_{gi} \geqslant 0,\ j \in S_{uk}^i,\ j = i, \tag{9}$$

$$\begin{aligned} &c_1\left\{\sigma_{\max}[\tilde{P}_i] + \tau\sigma_{\max}[\tilde{Q}_{1i}] + \frac{1}{2}\tau^2\sigma_{\max}[\tilde{\tilde{Q}}_{1i}]\right\} \\ &+ \tilde{c}_1\left\{\frac{1}{2}\tau^2\sigma_{\max}[\tilde{Q}_{2i}] + \frac{1}{6}\tau^3\sigma_{\max}[\tilde{\tilde{Q}}_{2i}]\right\} \\ &+ \frac{\gamma^2 d}{\alpha}(1 - \mathrm{e}^{\alpha T}) < c_2\sigma_{\min}(\tilde{P}_i)\mathrm{e}^{-\alpha T}, \end{aligned} \tag{10}$$

where $\Pi_{1i}^{11} = P_i\tilde{A}_i + \tilde{A}_i^{\mathrm{T}}P_i + Q_{1i} + \tau Q_1 + N_{1i} + N_{1i}^{\mathrm{T}} + \sum_{j\in S_k^i}\lambda_{ij}(P_j - J_i) + \sum_{j\in S_k^i}\frac{1}{4}k_{ij}^2 I - \alpha P_i$, $\Pi_{1i}^{12} = P_i\tilde{A}_{di} - N_{1i} + N_{2i}^{\mathrm{T}}$, $\Pi_{1i}^{13} = \tilde{A}_i^{\mathrm{T}}P_i + N_{3i}^{\mathrm{T}}$, $\Pi_{1i}^{17} = [P_{k_1^i} - J_i, \ldots, P_{k_m^i} - J_i]$, $\Pi_{1i}^{22} = -Q_{1i} - N_{2i} - N_{2i}^{\mathrm{T}}$, $\Pi_{1i}^{23} = \tilde{A}_{di}^{\mathrm{T}}P_i - N_{3i}^{\mathrm{T}}$, $\Pi_{1i}^{33} = -2P_i + \tau Q_{2i} + \frac{\tau^2}{2}Q_2$, $\Pi_{1i}^{77} = [-I_{k_1^i}, \ldots, -I_{k_m^i}]$, $\Pi_{3i}^{12} = [Q_{gk_1^i} - J_{gi}, \ldots, Q_{gk_m^i} - J_{gi}]$, $\Pi_{3i}^{22} = [-I_{k_1^i}, \ldots, -I_{k_m^i}]$, σ denotes the eigenvalue, $\tilde{P}_i = R_i^{-1/2}P_iR_i^{-1/2}$, $\tilde{Q}_{gi} = R_i^{-1/2}Q_gR_i^{-1/2}$, $\tilde{\tilde{Q}}_{gi} = R_i^{-1/2}Q_{gi}R_i^{-1/2}$, $0 < c_1 < c_2$, and $R_i > 0$, then the composite system in (4) and (5) can achieve finite-time boundedness with respect to (c_1, c_2, T, R_i, d).

The proof of Theorem 1 can be found in Appendix A.

The following theorem describes the disturbance observer design and the composite controller solution.

Theorem 2. If there exist symmetric positive definite matrices X_{1i}, P_{2i}, U_{1gi}, U_{2gi}, U_{1gij}, and U_{2gij}, symmetric matrices $\tilde{V}_{1i}$, $\tilde{V}_{2i}$, $\tilde{V}_{1gi}$, and $\tilde{V}_{2gi}$, appropriate matrices Y_i, H_{1i}, $\tilde{N}_{1pi}$, and $\tilde{N}_{2pi}$, the known constant matrices with the appropriate dimensions E_{11i}, E_{12i}, F_{11i}, and F_{12i}, positive constants α, T, c_1, c_2, d, γ, $\tilde{c}_1$, σ_1, σ_2, σ_3, σ_4, and σ_5, for $\forall i \in S$, $g = 1, 2$, $p = 1, 2, 3$, $0 < c_1 < c_2$ and $R_i > 0$, such that the following inequalities hold:

$$\begin{bmatrix} \Pi_{d1i}^{11} & \Pi_{d1i}^{12} & \Pi_{d1i}^{13} & \Pi_{d1i}^{14} & \Pi_{d1i}^{15} & \Pi_{d1i}^{16} & \Pi_{d1i}^{17} & \Pi_{d1i}^{18} & \Pi_{d1i}^{19} \\ * & \Pi_{d1i}^{22} & \Pi_{d1i}^{23} & 0 & 0 & \Pi_{d1i}^{26} & 0 & \Pi_{d1i}^{28} & 0 \\ * & * & \Pi_{d1i}^{33} & 0 & 0 & \Pi_{d1i}^{36} & 0 & 0 & 0 \\ * & * & * & \Pi_{d1i}^{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{d1i}^{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{d1i}^{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{d1i}^{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Pi_{d1i}^{88} & 0 \\ * & * & * & * & * & * & * & * & \Pi_{d1i}^{99} \end{bmatrix} < 0,\quad i \in S_k^i, \tag{11}$$

$$\begin{bmatrix} \tilde{\Pi}_{d1i}^{11} & \Pi_{d1i}^{12} & \Pi_{d1i}^{13} & \Pi_{d1i}^{14} & \Pi_{d1i}^{15} & \Pi_{d1i}^{16} & \Pi_{d1i}^{17} & \Pi_{d1i}^{18} & \tilde{\Pi}_{d1i}^{19} \\ * & \Pi_{d1i}^{22} & \Pi_{d1i}^{23} & 0 & 0 & \Pi_{d1i}^{26} & 0 & \Pi_{d1i}^{28} & 0 \\ * & * & \Pi_{d1i}^{33} & 0 & 0 & \Pi_{d1i}^{36} & 0 & 0 & 0 \\ * & * & * & \Pi_{d1i}^{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{d1i}^{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{d1i}^{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{d1i}^{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Pi_{d1i}^{88} & 0 \\ * & * & * & * & * & * & * & * & \tilde{\Pi}_{d1i}^{99} \end{bmatrix} < 0,\quad i \notin S_k^i, \tag{12}$$

$$\begin{bmatrix} \Pi_{d3i}^{11} & \Pi_{d3i}^{12} \\ * & \Pi_{d1i}^{77} \end{bmatrix} < 0, \tag{13}$$

$$\begin{aligned} &\begin{bmatrix} -\tilde{V}_{1i} & 0 & X_{1i} \\ * & P_{2j} - \tilde{V}_{2i} & 0 \\ * & * & -X_{1j} \end{bmatrix} \leqslant 0, \\ &\mathrm{diag}\{U_{1gij} - \tilde{V}_{1gi}, U_{2gij} - \tilde{V}_{2gi}\} \leqslant 0,\ j \in S_{uk}^i,\ j \neq i, \\ &\mathrm{diag}\{X_{1i} - \tilde{V}_{1i}, P_{2i} - \tilde{V}_{2i}\} \leqslant 0, \end{aligned} \tag{14}$$

$$\mathrm{diag}\{U_{1gii}-\tilde{V}_{1gi}, U_{2gii}-\tilde{V}_{2gi}\} \geqslant 0,\ j \in S^i_{uk},\ j=i, \quad (15)$$

$$R_i^{-1} < P_i < \sigma_1 R_i^{-1},\quad 0 < Q_{1i} < \sigma_2 R_i^{-1},$$
$$0 < Q_1 < \sigma_3 R_i^{-1},\quad 0 < Q_{2i} < \sigma_4 R_i^{-1},$$
$$0 < Q_2 < \sigma_5 R_i^{-1}, \quad (16)$$

$$c_1\left(\sigma_1+\tau\sigma_2+\frac{1}{2}\tau^2\sigma_3\right)+\tilde{c}_1\left(\frac{1}{2}\tau^2\sigma_4+\frac{1}{6}\tau^3\sigma_5\right)$$
$$+\frac{\gamma^2 d}{\alpha}(1-\mathrm{e}^{-\alpha T}) < c_2\mathrm{e}^{-\alpha T}, \quad (17)$$

where the descriptions of the variables can be found in Appendix B, then the composite system in (4) and (5) can achieve finite-time boundedness with respect to (c_1, c_2, T, R_i, d), and observer and controller gains are given as $L_i = P_{2i}^{-1}H_{2i}$, $K_i = Y_i X_{1i}^{-1}$.

The proof of Theorem 2 can be found in Appendix B.

Remark 2. According to Theorem 2, the optimization value of ρ can be obtained by solving an optimal problem as follows:

$$\min\ \ \rho$$
$$X_{1i}, P_{2i}, Y_i, H_{2i}, Q_g, Q_{gi}, \beta_i, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \tilde{V}_{1i},$$
$$\tilde{V}_{2i}, \tilde{V}_{1gi}, \tilde{V}_{2gi}, \tilde{N}_{1pi}, \tilde{N}_{2pi}, U_{1gi}, U_{2gi}, U_{1gij}, U_{2gij}$$
$$\text{s.t.}\quad \text{inequalities (11)–(17) hold},\ \rho=\gamma^2. \quad (18)$$

The numerical example can be found in Appendix C.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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