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Robust interval stability/stabilization and H_∞ feedback control for uncertain stochastic Markovian jump systems based on the linear operator

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Abstract This paper focuses on the problem of robust H_∞ state feedback control with interval pole constraints for uncertain stochastic Markovian jump systems (MJSs). First, we present the sufficient conditions of robust interval stability using a linear operator and its spectrum. A robust interval stabilization controller is designed for MJSs; it ensures the stability of MJSs and adjusts the rate of convergence. In addition, the robust H_∞ controller with interval pole constraints is designed with the admissible parametric uncertainties and a prescribed H_∞ disturbance attenuation level; this guarantees that the closed-loop system is robust and asymptotically stable with an ideal rate of convergence. A numerical example is provided to demonstrate the effectiveness of the proposed method.

Keywords Markovian jump systems, linear operator, interval stability, interval stabilization, robust H_∞ control

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1 Introduction

Markovian jump systems (MJSs) are special cases of switched hybrid systems with switching signals governed by a Markovian chain; these systems also be considered as special types of stochastic systems with system matrices randomly changing at discrete-time controlled by a Markov process. Over the past decades, the investigation of MJSs has become an active area because of their wide applications in the real world. Many practical systems have been modeled as MJSs, e.g., power systems, networks control systems, economic systems, communication systems and flight systems.

The stability analysis of MJSs has attracted a great deal of attention from many scholars. The stability of MJSs has been studied in [1–5]; the stability of the nonlinear stochastic differential equation with Markovian switching has been considered in [6–8]; robust stability and stabilization of MJSs have been investigated in [9–14]; and the stability and stabilization of discrete-time MJSs have been discussed in [15–19]. Robust H_∞ control of MJSs is a highly attractive field of research. Since it was first formulated by Zames in [20], many researchers have studied related issues from different perspectives, such as the theoretical analysis of H_∞ control of MJSs [21, 22], and the applications of H_∞ control of MJSs [23–25].

As is well known, there are spectrum criteria or eigenvalue criteria of asymptotic stability for the time-invariant deterministic systems [26]. The advantages of spectral characterization is its direct linkage with the system dynamic behavior of the system. As a concentrated reflection of the spectral characterization, pole assignment plays an important role in systems theory, and is a generalization of this theory, which is essential for accurate control of the system [27–29]. As a technical tool, operator theory plays a vital role in the generalization of the spectral criteria [30–33].

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Although MJSs have received increasing interest, there have been few efforts toward developing the interval stability conditions of MJSs, and very few researchers have investigated the design of the interval controller for MJSs as far as we know. Motivated by the above discussion, we focus on revealing the stability properties of uncertain stochastic Markovian jump systems (USMJSs) and improving their performance in controller design. This paper presents the interval stability conditions, the interval state feedback controller design and robust H_∞ state feedback control problem of USMJSs. The main contributions and novelty of this paper are threefold.

First, robust interval stability criteria of USMJSs are presented using a linear symmetric operator and its eigenvalues. Using the new standard, the stability of the USMJSs as well as the speed of convergence can be accurately judged. As a special case, the sufficient and necessary condition of interval stability for MJSs without robustness is also addressed.

Second, with the help of robust interval stability criteria, the new robust interval stabilization problem of USMJSs is solved by controlling their poles in an appropriate zonal interval perpendicular to the real axis; this guarantees system stability and accurately adjusts the speed of convergence.

Third, a robust H_∞ feedback controller with interval pole constraints is designed with all the admissible parametric uncertainties and a prescribed H_∞ disturbance attenuation level; this guarantees that the closed-loop stochastic system is robust and asymptotically stable with an ideal rate of convergence.

The rest of this paper is organized as follows. Section 2 introduces the definition of the linear symmetric operator, the system formulation and several lemmas. Section 3 lists the conditions for the interval stability and stabilization of USMJSs. Section 4 is devoted to deriving the results of the robust H_∞ controller with interval pole constraints of USMJSs. Section 5 illustrates the feasibility of the obtained results through an example. The conclusion is given in Section 6.

Notation. $\mathbb{R}(\mathcal{C})$: the set of all real (complex) numbers. $\mathbb{R}^{m\times n}$: the set of all $m\times n$ real matrices. $\mathcal{S}_n$: the set of all $n\times n$ symmetric matrices. I_n: the identity matrix $n\times n$. A^{T}: the matrix transpose of A. $\sigma(A)$: the spectrum set of the matrix A. $\mathcal{E}(\cdot)$: mathematical expectation operator. $\|\cdot\|$ denotes the norm of a matrix or vector. $\mathcal{C}_{-a}^{-b}$: the vertical strip region from $-a$ to $-b$ in the complex plane. $\mathcal{C}^-$: left half complex plane.

2 Definitions and preliminaries

Consider the following USMJS:

$$\begin{cases} \mathrm{d}x(t)=(A(t,r_t)x(t)+C(t,r_t)u(t,r_t))\mathrm{d}t+B(t,r_t)x(t)\mathrm{d}w(t),\\ x(0)=x_0, \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is state vector, $u(t,r_t)\in\mathbb{R}^m$ is control inputs and r_t is a Markov chain taking values in $S=\{1,2,\ldots,N\}$ with the transition rates matrix $\prod=[\pi_{ij}]$; $w(t)$ is independent random variable, standard 1-D Wiener processes defined on the filtered probability space $(\Omega,\mathcal{F},\mathcal{P};\mathcal{F}_t)$. The probability space $(\Omega,\mathcal{F},\mathcal{P})$ is complete and the filtration satisfies the usual conditions. When $r_t=i$,

$$A(t,r_t):=A_i(t)=A_i+\triangle A_i(t),\ B(t,r_t):=B_i(t)=B_i+\triangle B_i(t),$$

$$C(t,r_t):=C_i(t)=C_i+\triangle C_i(t),\ u(t,r_t)=u_i(t),$$

where $\triangle A_i(t),\triangle B_i(t)$ and $\triangle C_i(t)$ are unknown real matrices reflecting the parameter uncertainty in the system, as well as

$$[\triangle A_i(t)\ \ \triangle B_i(t)\ \ \triangle C_i(t)]=G_iF_i(t)[E_{1i}\ E_{2i}\ E_{3i}].$$

$A_i,B_i,C_i,G_i,E_{1i},E_{2i},E_{3i}$ are given real matrices with appropriate dimensions, $F_i(t)$ is an unknown time-varying matrix and

$$F_i^{\mathrm{T}}(t)F(t)_i\leqslant I. \tag{2}$$

Lemma 1 ([6]). Let $x(t)\in\mathbb{R}^n$ be an Itô process expressed by a stochastic differential equation (1), $V(x,t)\in\mathcal{C}^{2,1}(\mathbb{R}^n\times\mathbb{R}^+;\mathbb{R}^{p\times q}),V(x,t)=(v_{ij})_{p\times q}$. Then $V(x,t)$ is a matrix-valued Itô process whose component (v_{ij}) is given by

$$\mathrm{d}v_{ij}(x(t),t)=\frac{\partial v_{ij}(x(t),t)}{\partial t}\mathrm{d}t+\left(\frac{\partial v_{ij}(x(t),t)}{\partial x}\right)^{\mathrm{T}}\mathrm{d}x+\frac{1}{2}\mathrm{d}x^{\mathrm{T}}(t)\frac{\partial^2 v_{ij}(x(t),t)}{\partial x^2}\mathrm{d}x(t).$$

Definition 1. The linear operator $\mathcal{L}_{A_i(t),\ B_i(t)}$ associated with system (1) is defined as

$$\mathcal{L}_{A_i(t),B_i(t)} : X_i \in \mathcal{S}_n \longmapsto X_iA_i(t) + A_i^{\mathrm{T}}(t)X_i + B_i(t)^{\mathrm{T}}X_iB_i(t) + \sum_{i=1}^{N}\pi_{ij}X_j \in \mathcal{S}_n.$$

If there are a constant $\lambda \in \mathcal{C}$ and a nonzero $X_i \in \mathcal{S}_n$, such that $\mathcal{L}_{A_i(t),B_i(t)}X_i = \lambda X_i$, λ is named as an eigenvalue and X_i is named as an eigenvector of $\mathcal{L}_{A_i(t),B_i(t)}$ corresponding to λ. $\sigma(\mathcal{L}_{A_i(t),B_i(t)})$ denotes the spectral set of $\mathcal{L}_{A_i(t),B_i(t)}$, i.e., $\sigma(\mathcal{L}_{A_i(t),B_i(t)}) = \{\lambda_i : \mathcal{L}_{A_i(t),B_i(t)}X_i = \lambda_i X_i, X_i \neq 0 \in \mathcal{S}_n\}$.

Remark 1. The linear operator method is used to study linear stochastic systems and good results are obtained in [30, 31]. We extend this method to the SMJS, which reflects the properties of matrices $A_i(t)$ and $B_i(t)$ in $\mathcal{L}_{A_i(t),B_i(t)}$. Meanwhile, the matrices $A_i(t)$ and $B_i(t)$ have a great influence on the SMJS. The distribution of the generalized eigenvalues or the poles for $\mathcal{L}_{A_i(t),B_i(t)}$ can determine the exact nature of the SMJS. Therefore, we establish more accurate stability conditions for the SMJS by the linear operator method.

Definition 2 ([31])**.** System (1) has $(-a, -b)$-stability with $0 \leqslant b < a$, i.e., interval stability, if

$$\sigma(\mathcal{L}_{A_i(t),B_i(t)}) \subset \mathcal{C}_{-a}^{-b} := \{\lambda : -a < \mathrm{Re}(\lambda) < -b\}.$$

Remark 2. The interval stability is a generalization of the stability. As defined in Definition 2, the interval stability becomes the usual stability when $b = 0$ and $a = -\infty$. In other words, ordinary stability only requires that the poles of the system are located in the negative half plane, while interval stability requires that the poles are distributed in a specific strip. The accurate judgment of pole distribution is conducive to the understanding of system performance.

Lemma 2 ([33])**.** Let $Y \in S_n$, D, E, F be the appropriate dimensions matrices, and $F^{\mathrm{T}}F \leqslant I$. Then

$$Y + DFE + (DFE)^{\mathrm{T}} < 0,$$

if and only if there exists a contant $\varepsilon > 0$, such that

$$Y + \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}EE^{\mathrm{T}} < 0.$$

3 Robust interval stability and stabilization

We first consider the robust interval stability of the USMJS (1); i.e., USMJS (1) has robust interval stability in mean square sense when $u(t) = 0$.

Theorem 1. The unforced system (1) ($u(t) = 0$) is asymptotically robustly $(-a, -b)$-stable in mean square sense if for all $i \in S$, there exist constants $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$ and matrices $P_i > 0$, such that the following linear matrix inequalities (LMIs) hold:

$$\begin{bmatrix} \Gamma_{1i} & B_i^{\mathrm{T}}P_i & P_iG_i & 0 \\ * & -P_i & 0 & P_iG_i \\ * & * & -\varepsilon_{1i}I & 0 \\ & * & * & -\varepsilon_{2i}I \end{bmatrix} < 0, \tag{3}$$

$$\begin{bmatrix} \Gamma_{1i} & B_i^{\mathrm{T}}P_i & P_iG_i & 0 \\ * & -P_i & 0 & P_iG_i \\ * & * & -\varepsilon_{1i}I & 0 \\ & * & * & -\varepsilon_{2i}I \end{bmatrix} < 0, \tag{4}$$

where

$$\Gamma_{1i} = -(A_i + aI)^{\mathrm{T}}P_i - P_i(A_i + aI) + \sum_{j=1}^{N}\pi_{ij}P_j + \varepsilon_{1i}E_{1i}^{\mathrm{T}}E_{1i} + \varepsilon_{2i}E_{2i}^{\mathrm{T}}E_{2i},$$

$$\Gamma_{2i} = (A_i + bI)^{\mathrm{T}}P_i + P_i(A_i + bI) + \sum_{j=1}^{N}\pi_{ij}P_j + \varepsilon_{1i}E_{1i}^{\mathrm{T}}E_{1i} + \varepsilon_{2i}E_{2i}^{\mathrm{T}}E_{2i}.$$

Proof. The system (1) is asymptotically robustly $(-a,-b)$-stable in mean square sense if and only if both following systems are asymptotically robustly mean-square stable. That is, $\sigma(\mathcal{L}_{A_i(t),B_i(t)})\subset\mathcal{C}_{-a}^{-b}$ if and only if $\sigma(-\mathcal{L}_{A_i(t),B_i(t)}-aI)\subset\mathcal{C}^-$ and $\sigma(\mathcal{L}_{A_i(t),B_i(t)}+bI)\subset\mathcal{C}^-$.

$$\begin{cases}\mathrm{d}x(t)=-(A(t,r_t)+aI)x(t)\mathrm{d}t-B(t,r_t)x(t)\mathrm{d}w(t),\\ x(0)=x_0,\end{cases}\tag{5}$$

and

$$\begin{cases}\mathrm{d}x(t)=(A(t,r_t)+bI)x(t)\mathrm{d}t+B(t,r_t)x(t)\mathrm{d}w(t),\\ x(0)=x_0.\end{cases}\tag{6}$$

For the system (5), select the Lyapunov-Krasvskii functional $V(t,x(t))=x^{\mathrm{T}}(t)Px(t)$. According to Lemma 1,

$$\mathcal{L}V(t,x(t))=2x^{\mathrm{T}}(t)\left[-P_i(A_i(t)+aI)^{\mathrm{T}}-(A_i(t)+aI)P_i+B_i(t)P_iB_i^{\mathrm{T}}(t)+\sum_{j=1}^{N}\pi_{ij}P_j\right]x(t).$$

The system (5) is asymptotically robustly stable in mean square sense if and only if

$$\mathcal{L}V(t,x(t))<0.$$

According to the Schur complement, the above inequality can be rewritten as

$$\begin{bmatrix}-(A_i(t)+aI)^{\mathrm{T}}P_i-P_i(A_i(t)+aI)+\sum_{j=1}^{N}\pi_{ij}P_j & B_i^{\mathrm{T}}(t)\\ * & -P_i^{-1}\end{bmatrix}<0.$$

Multipling matrix $\left[\begin{smallmatrix}I&0\\0&P_i\end{smallmatrix}\right]$ on left side and right side of the above inequality, we have

$$\begin{bmatrix}-(A_i(t)+aI)^{\mathrm{T}}P_i-P_i(A_i(t)+aI)+\sum_{j=1}^{N}\pi_{ij}P_j & B_i^{\mathrm{T}}(t)P_i\\ * & -P_i\end{bmatrix}<0.$$

Let

$$Y=\begin{bmatrix}-(A_i+aI)^{\mathrm{T}}P_i-P_i(A_i+aI)+\sum_{j=1}^{N}\pi_{ij}P_j & B_i^{\mathrm{T}}P_i\\ * & -P_i\end{bmatrix}.$$

The above inequality is equivalent to the following inequality:

$$Y-\begin{bmatrix}P_iG_i\\0\end{bmatrix}F_i(t)[E_{1i}\ 0]-\begin{bmatrix}E_{1i}^{\mathrm{T}}\\0\end{bmatrix}F_i^{\mathrm{T}}(t)[G_i^{\mathrm{T}}P_i\ \ 0]+\begin{bmatrix}0\\P_iG_i\end{bmatrix}F_i(t)[E_{2i}\ 0]+\begin{bmatrix}E_{2i}^{\mathrm{T}}\\0\end{bmatrix}F_i^{\mathrm{T}}(t)[0\ \ G_i^{\mathrm{T}}P_i]<0.$$

By Lemma 2, the above formula is true for all matrices $F(t)$ that satisfy $F_i(t)F_i^{\mathrm{T}}(t)\leqslant I$ if and only if there are constants ε_{1i} and ε_{2i}, such that

$$\begin{aligned}&Y+\varepsilon_{1i}^{-1}\begin{bmatrix}P_iG_i\\0\end{bmatrix}[G_i^{\mathrm{T}}P_i\ 0]+\varepsilon_{1i}\begin{bmatrix}E_{1i}^{\mathrm{T}}\\0\end{bmatrix}F_i(t)F_i^{\mathrm{T}}(t)[E_{1i}\ 0]+\varepsilon_{2i}^{-1}\begin{bmatrix}0\\P_iG_i\end{bmatrix}[0\ G_i^{\mathrm{T}}P_i]\\ &+\varepsilon_{2i}\begin{bmatrix}E_{2i}^{\mathrm{T}}\\0\end{bmatrix}F_i(t)F_i^{\mathrm{T}}(t)[E_{2i}\ 0]\ \leqslant\ -Y+\varepsilon_{1i}^{-1}\begin{bmatrix}P_iG_i\\0\end{bmatrix}[G_i^{\mathrm{T}}P_i\ 0]+\varepsilon_{1i}\begin{bmatrix}E_{1i}^{\mathrm{T}}\\0\end{bmatrix}[E_{1i}\ 0]\\ &+\varepsilon_{2i}^{-1}\begin{bmatrix}0\\P_iG_i\end{bmatrix}[0\ G_i^{\mathrm{T}}P_i]+\varepsilon_{2i}\begin{bmatrix}E_{2i}^{\mathrm{T}}\\0\end{bmatrix}[E_{2i}\ 0]<0.\end{aligned}\tag{7}$$

By the Schur complement, we can get (3). Meanwhile, the system (6) is asymptotically robustly stable in mean square sense if and only if

$$(A_i(t)+bI)^{\mathrm{T}}P_i+P_i(A_i(t)+bI)+B_i^{\mathrm{T}}(t)P_iB_i(t)+\sum_{j=1}^{N}\pi_{ij}P_j<0.$$

By the similar process, we can get (8).

Remark 3. Theorem 1 gives a sufficient condition for the generalized eigenvalues of the USMJSs to fall in a specific interval, which can judge not only the stability of the system but also the speed of convergence of the USMJSs. In other words, when the eigenvalues or poles of the linear operator $\mathcal{L}_{A_i(t),B_i(t)}$ are distributed in a given interval $(-a,-b)$, the USMJSs are asymptotically robustly mean-square stable and the rates of convergence are within a certain range.

As a special case of Theorem 1, the usual stability conditions can be presented as follows.

Corollary 1. The unforced system (1) is asymptotically robustly stable in mean square sense if for all $i \in S$, there exist $\varepsilon_{1i} > 0, \varepsilon_{2i} > 0$ and $P_i > 0$, such that

$$\begin{bmatrix} \Gamma_{1i} & B_i^{\mathrm{T}} P_i & P_i G_i & 0 \\ * & -P_i & 0 & P_i G_i \\ * & * & -\varepsilon_{1i} I & 0 \\ & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0,$$

where

$$\Gamma_i = A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij} P_j + \varepsilon_{1i} \alpha E_{1i}^{\mathrm{T}} E_{1i} + \varepsilon_{2i} \beta E_{2i}^{\mathrm{T}} E_{2i}.$$

Without considering the robustness, a sufficient and necessary condition for the interval stability in the mean square sense of the USMJS (1) can be obtained according to Theorem 1.

Theorem 2. The unforced system (1) $(u(t) = 0)$ is asymptotically $(-\infty, -b)$-stable in mean square sense if and only if for all $i \in S$, there exists $P_i > 0$, such that the following LMI holds:

$$\begin{bmatrix} \hat{\Gamma}_{2i} & B_i^{\mathrm{T}} P_i \\ * & -P_i \end{bmatrix} < 0, \tag{8}$$

where

$$\hat{\Gamma}_{2i} = (A_i + bI)^{\mathrm{T}} P_i + P_i (A_i + bI) + \sum_{j=1}^{N} \pi_{ij} P_j.$$

Based on stochastic stability analysis of the USMJS (1), the controller for the USMJS (1) is designed according to Theorem 1 to realize the stabilization.

Theorem 3. The closed-loop system (1) achieves the asymptotic robust $(-a,-b)$-stabilization in mean square sense if for all $i \in S$, there exist constants $\delta_{1i} > 0, \delta_{2i} > 0$ and matrices $X_i > 0$, such that the following LMIs hold:

$$\begin{bmatrix} \hat{\Xi}_{1i} & \hat{A}_i \\ * & \hat{B}_i \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} \hat{\Xi}_{2i} & \hat{A}_i \\ * & \tilde{B}_i \end{bmatrix} < 0, \tag{10}$$

where

$$\Xi_{1i} = -X_i A_i^{\mathrm{T}} - Y_i^{\mathrm{T}} C_i^{\mathrm{T}} - A_i X_i - C_i Y_i - a X_i + \pi_{ii} X_i + \delta_{1i} G_i G_i^{\mathrm{T}},$$

$$\Xi_{2i} = X_i A_i^{\mathrm{T}} + Y_i^{\mathrm{T}} C_i^{\mathrm{T}} + A_i X_i + C_i Y_i + b X_i + \pi_{ii} X_i + \delta_{2i} G_i G_i^{\mathrm{T}},$$

$$\hat{\Xi}_{1i} = \begin{bmatrix} \Xi_{1i} & \sqrt{\pi_{i1}} X_1 & \cdots & \sqrt{\pi_{iN}} X_N \\ * & -X_i & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_i \end{bmatrix}, \quad \hat{\Xi}_{2i} = \begin{bmatrix} \Xi_{2i} & \sqrt{\pi_{i1}} X_1 & \cdots & \sqrt{\pi_{iN}} X_N \\ * & -X_i & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_i \end{bmatrix},$$

$$\hat{A}_i = \begin{bmatrix} B_i^{\mathrm{T}} X_i & X_i E_{1i}^{\mathrm{T}} + Y_i E_{3i}^{\mathrm{T}} & X_i E_{2i}^{\mathrm{T}} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{B}_i = \begin{bmatrix} -X_i + \delta_{1i} G_i G_i^{\mathrm{T}} & 0 & 0 \\ 0 & -\delta_{1i} I & 0 \\ 0 & 0 & -\delta_{2i} I \end{bmatrix},$$

$$\tilde{B}_i = \begin{bmatrix} -X_i + \delta_{2i} G_i G_i^{\mathrm{T}} & 0 & 0 \\ 0 & -\delta_{1i} I & 0 \\ 0 & 0 & -\delta_{2i} I \end{bmatrix}.$$

The USMJS (1) can achieve asymptotic robust interval stabilization by using controllers $u_i(t) = K_i x(t)$, and the corresponding control gain matrices can be realized by

$$K_i = Y_i X_i^{-1}.$$

Proof. Replacing A_i and E_{1i} in (3) and (8) with $A_i + C_i K_i$ and $E_{1i} + E_{3i} K_i$ respectively, we have

$$\begin{bmatrix} \Omega_{1i} & B_i^{\mathrm{T}} P_i & P_i G_i & 0 \\ * & -P_i & 0 & P_i G_i \\ * & * & -\varepsilon_{1i} I & 0 \\ & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \Omega_{2i} & B_i^{\mathrm{T}} P_i & P_i G_i & 0 \\ * & -P_i & 0 & P_i G_i \\ * & * & -\varepsilon_{1i} I & 0 \\ & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{aligned} \Omega_{1i} = & -(A_i + C_i K_i + aI)^{\mathrm{T}} P_i - P_i (A_i + C_i K_i + aI) + \sum_{j=1}^{N} \pi_{ij} P_j \\ & + \varepsilon_{1i} \alpha (E_{1i} + E_{3i} K_i)^{\mathrm{T}} (E_{1i} + E_{3i} K_i) + \varepsilon_{2i} \alpha E_{2i}^{\mathrm{T}} E_{2i}, \end{aligned}$$

$$\begin{aligned} \Omega_{2i} = & (A_i + C_i K_i + bI)^{\mathrm{T}} P_i + P_i (A_i + C_i K_i + bI) + \sum_{j=1}^{N} \pi_{ij} P_j \\ & + \varepsilon_{1i} \beta (E_{1i} + E_{3i} K_i)^{\mathrm{T}} (E_{1i} + E_{3i} K_i) + \varepsilon_{2i} \beta E_{2i}^{\mathrm{T}} E_{2i}. \end{aligned}$$

By means of congruent transformation in matrix $\mathrm{diag}\{P_i^{-1}, P_i^{-1}, I, I\}$, and the replacement

$$X_i = P_i^{-1}, \quad Y_i = K_i X_i,$$

the equivalence inequalities of (11) and (12) can be presented as

$$\begin{bmatrix} \hat{\Omega}_{1i} & X_i B_i^{\mathrm{T}} & G_i X_i & 0 \\ * & -X_i & 0 & G_i X_i \\ * & * & -\varepsilon_{1i} I & 0 \\ & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \hat{\Omega}_{2i} & X_i B_i^{\mathrm{T}} & G_i X_i & 0 \\ * & -X_i & 0 & G_i X_i \\ * & * & -\varepsilon_{1i} I & 0 \\ & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{14}$$

where

$$\hat{\Omega}_{1i} = - X_i (A_i + aI)^{\mathrm{T}} + Y_i^{\mathrm{T}} C_i^{\mathrm{T}} - (A_i + aI) X_i + C_i Y_i + \sum_{j=1}^{N} \pi_{ij} P_j$$

$$+\ \varepsilon_{1i}(E_{1i}X_i+E_{3i}Y_i)^{\mathrm{T}}(E_{1i}X_i+E_{3i}Y_i)+\varepsilon_{2i}E_{2i}^{\mathrm{T}}E_{2i},$$

$$\begin{aligned}\hat{\Omega}_{2i} &= X_i(A_i+bI)^{\mathrm{T}}+Y_i^{\mathrm{T}}C_i^{\mathrm{T}}+(A_i+bI)X_i+C_iY_i+\sum_{j=1}^{N}\pi_{ij}P_j\\ &\quad+\varepsilon_{1i}(E_{1i}X_i+E_{3i}Y_i)^{\mathrm{T}}(E_{1i}X_i+E_{3i}Y_i)+\varepsilon_{2i}E_{2i}^{\mathrm{T}}E_{2i}.\end{aligned}$$

By the Schur complement, the above two inequalities are equivalent to (16) and (17). So the controllers can be given as $K_i=Y_iX_i^{-1}$.

Remark 4. Theorem 3 provides an interval stabilization method, which can satisfy precise control according to the stabilization needs of different USMJSs. Meanwhile, this method is between the usual stabilization and pole assignment, and its conditions are weaker than those of pole assignment. Interval stabilization method controls the poles within a certain strip using a convenient LMIs, which can control the speed of the state convergence.

Corollary 2. The closed-loop system (1) achieves the asymptotic robust stabilization in mean square sense if for all $i\in S$, there exist matrices $P_i>0$, such that the following LMI holds:

$$\begin{bmatrix}\hat{\Xi}_i & \hat{A}_i\\ * & \tilde{B}_i\end{bmatrix}<0, \tag{15}$$

where

$$\Xi_i=X_iA_i^{\mathrm{T}}+Y_i^{\mathrm{T}}C_i^{\mathrm{T}}+A_iX_i+C_iY_i+\pi_{ii}X_i+\delta_{2i}G_iG_i^{\mathrm{T}},$$

$$\hat{\Xi}_i=\begin{bmatrix}\Xi_i & \sqrt{\pi_{i1}}X_1 & \cdots & \sqrt{\pi_{iN}}X_N\\ * & -X_i & \cdots & 0\\ * & * & \cdots & 0\\ * & * & \cdots & -X_i\end{bmatrix},$$

and $\hat{A}_i,\hat{B}_i,\tilde{B}_i$ are defined as same as in Theorem 3. So the USMJS (1) can achieve robust stabilization by using controllers $u_i(t)=K_ix(t)$, and the corresponding control gain matrices are $K_i=Y_iX_i^{-1}$.

Without considering the robustness, a sufficient and necessary condition for the interval stabilization in the mean square of USMJSs can be obtained according to Theorem 2.

Theorem 4. The state feedback system (1) is asymptotically $(-a,-b)$-stable in mean square sense if and only if there exists $X_i>0$, such that the following LMIs hold:

$$\begin{bmatrix}\tilde{\bar{\Xi}}_{1i} & B_i^{\mathrm{T}}X_i\\ * & \bar{\tilde{B}}_i\end{bmatrix}<0, \tag{16}$$

$$\begin{bmatrix}\tilde{\bar{\Xi}}_{2i} & B_i^{\mathrm{T}}X_i\\ * & \bar{\bar{B}}_i\end{bmatrix}<0, \tag{17}$$

where

$$\bar{\Xi}_{1i}=-X_iA_i^{\mathrm{T}}-Y_i^{\mathrm{T}}C_i^{\mathrm{T}}-A_iX_i-C_iY_i-aX_i+\pi_{ii}X_i,$$

$$\bar{\Xi}_{2i}=X_iA_i^{\mathrm{T}}+Y_i^{\mathrm{T}}C_i^{\mathrm{T}}+A_iX_i+C_iY_i+bX_i+\pi_{ii}X_i,$$

$$\tilde{\bar{\Xi}}_{1i}=\begin{bmatrix}\bar{\Xi}_{1i} & \sqrt{\pi_{i1}}X_1 & \cdots & \sqrt{\pi_{iN}}X_N\\ * & -X_i & \cdots & 0\\ * & * & \cdots & 0\\ * & * & \cdots & -X_i\end{bmatrix},\quad \tilde{\bar{\Xi}}_{2i}=\begin{bmatrix}\bar{\Xi}_{2i} & \sqrt{\pi_{i1}}X_1 & \cdots & \sqrt{\pi_{iN}}X_N\\ * & -X_i & \cdots & 0\\ * & * & \cdots & 0\\ * & * & \cdots & -X_i\end{bmatrix},$$

$$\bar{\tilde{B}}_i=\begin{bmatrix}-X_i\\ 0\\ 0\end{bmatrix},\quad \bar{\bar{B}}_i=\begin{bmatrix}-X_i\\ 0\\ 0\end{bmatrix}.$$

So the SMJS (1) can achieve robust interval stabilization by using controllers $u_i(t)=K_ix(t)$, and the corresponding control gain matrices $K_i=Y_iX_i^{-1}$.

Remark 5. The basic idea of generalized pole assignment and interval stabilization is consistent. The difference is that the pole assignment is accurate to each pole, while the interval stabilization is the pole in a certain region. That is, pole assignment is more accurate in theory. Interval stabilization is more practical in practice.

4 Robust H_∞ state feedback control with interval pole constraints

Consider the following USMJS:

$$\begin{cases} \mathrm{d}x(t) = (A(t,r_t)x(t) + C(t,r_t)u(t,r_t) + C_v v(t))\mathrm{d}t + (B(t,r_t)x(t) + B_v(t,r_t)v(t))\mathrm{d}w(t), \\ z(t) = D_1 x(t) + D_2 u(t), \\ x(0) = x_0, \end{cases} \tag{18}$$

where $z(t) \in \mathbb{R}^q$ is the controlled output of the system, $v(t) \in \mathbb{R}^l$ is the interference output on $\mathcal{L}_2[0,\ \infty]$, C_v, B_v, D_1, D_2 are the real matrices of appropriate dimensions, the other system parameters are the same as the USMJS (1).

Define the performance metric

$$J = \mathcal{E}\left\{\int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 v^{\mathrm{T}}(t)v(t)]\mathrm{d}t\right\},$$

where γ is a given degree of interference suppression.

In the following, we design the state feedback controllers $u_i(t) = K_i x(t)$, such that the closed-loop system has asymptotic robust interval mean-square stability as $v(t) = 0$, and the closed-loop system (18) has a given γ of H_∞ interference suppression, i.e., $J < 0$.

Theorem 5. For the given scalar γ and the state feedback USMJS (18), if for all $i \in S$, there exist $\delta_{1i} > 0, \delta_{2i} > 0$ and $X_i > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \Delta_{1i} & \Upsilon \\ * & \Phi_1 \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} \Delta_{2i} & \Upsilon \\ * & \Phi_2 \end{bmatrix} < 0, \tag{20}$$

where

$$\bar{\Delta}_{1i} = -A_i X_i - C_i Y_i - X_i A_i^{\mathrm{T}} - aX_i - Y_i^{\mathrm{T}} C_i^{\mathrm{T}} + \pi_{ii} X_i + \delta_{1i} G_i G_i^{\mathrm{T}},$$
$$\bar{\Delta}_{2i} = A_i X_i + C_i Y_i + X_i A_i^{\mathrm{T}} + bX_i + Y_i^{\mathrm{T}} C_i^{\mathrm{T}} + \pi_{ii} X_i + \delta_{2i} G_i G_i^{\mathrm{T}},$$

$$\Delta_{1i} = \begin{bmatrix} \bar{\Delta}_1 & \sqrt{\pi_{i1}} X_i & \cdots & \sqrt{\pi_{iN}} X_i \\ * & -X_1 & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_N \end{bmatrix}, \quad \Delta_{2i} = \begin{bmatrix} \bar{\Delta}_2 & \sqrt{\pi_{i1}} X_i & \cdots & \sqrt{\pi_{iN}} X_i \\ * & -X_1 & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_N \end{bmatrix},$$

$$\Upsilon = \begin{bmatrix} C_v X_i & X_i B_i^{\mathrm{T}} & X_i E_{1i}^{\mathrm{T}} + Y_i E_{3i}^{\mathrm{T}} & X_i E_{2i}^{\mathrm{T}} & X_i D_1^{\mathrm{T}} + Y_i^{\mathrm{T}} D_2^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} -\gamma^2 I & X_i B_v^{\mathrm{T}} & 0 & 0 & 0 \\ * & -X_i + \delta_{1i} G_i G_i^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & -\delta_{1i}^{-1} I & 0 & 0 \\ * & * & * & -\delta_{2i}^{-1} I & 0 \\ * & * & * & * & -I \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} -\gamma^2 I & X_i B_v^{\mathrm{T}} & 0 & 0 & 0 \\ * & -X_i + \delta_{2i} G_i G_i^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & -\delta_{1i} I & 0 & 0 \\ * & * & * & -\delta_{2i} I & 0 \\ * & * & * & * & -I \end{bmatrix},$$

then $u_i(t) = K_i x(t)$ is a robust H_∞ controller of the USMJS (18). Meanwhile, the controller gains are constructed by

$$K_i = Y_i X_i^{-1}.$$

Proof. Select the Lyapunov-Krasvskii functional $V(t, x(t)) = x^{\mathrm{T}}(t)Px(t)$. Using Itô formula, we have

$$\begin{aligned}\mathcal{L}V(t,x(t)) &= 2x^{\mathrm{T}}(t)P_i[-(A_i(t)+aI+C_iK_i)x(t)+C_vv(t)]+(B_ix(t)+B_vv(t))^{\mathrm{T}}P_i(B_ix(t)+B_vv(t))\\ &= [x^{\mathrm{T}}(t)\ v^{\mathrm{T}}(t)]\begin{bmatrix}\Delta_i & P_iC_v+B_i^{\mathrm{T}}P_iB_v\\ * & B_v^{\mathrm{T}}P_iB_v\end{bmatrix}\begin{bmatrix}x(t)\\ v(t)\end{bmatrix}<0,\end{aligned}$$

where $\Delta_i = -P_i(A_i(t)+aI+C_iK_i)-(A_i(t)+\frac{a}{2}I+C_iK_i)^{\mathrm{T}}P_i^{\mathrm{T}}+B_i^{\mathrm{T}}P_iB_i+\sum_{j=1}^{N}\pi_{ij}P_j$.

Similar to the proof of Theorem 1, $\mathcal{L}V(t,x(t))<0$, if and only if

$$\begin{bmatrix}\Delta_i & P_iC_v+B_i^{\mathrm{T}}P_iB_v\\ * & B_v^{\mathrm{T}}P_iB_v\end{bmatrix}<0.$$

According to the system zero initial condition and asymptotic stability condition, it follows that:

$$\begin{aligned}J &= \mathcal{E}\left\{\int_0^{\mathrm{T}}[z^{\mathrm{T}}(t)z(t)-\gamma^2v^{\mathrm{T}}(t)v(t)]\mathrm{d}t\right\}\\ &< \mathcal{E}\left\{\int_0^{\mathrm{T}}[z^{\mathrm{T}}(t)z(t)-\gamma^2v^{\mathrm{T}}(t)v(t)]\mathrm{d}t\right\}+\mathcal{E}\left\{\int_0^{\mathrm{T}}\mathrm{d}V(t)\right\}\\ &= \mathcal{E}\left\{\int_0^{\mathrm{T}}[z^{\mathrm{T}}(t)z(t)-\gamma^2v^{\mathrm{T}}(t)v(t)+\mathcal{L}V(t)]\mathrm{d}t\right\}\\ &\leqslant \mathcal{E}\left\{\int_0^{\mathrm{T}}[x^{\mathrm{T}}(t)\ v^{\mathrm{T}}(t)]\Theta_i\begin{bmatrix}x(t)\\ v(t)\end{bmatrix}\mathrm{d}t\right\},\end{aligned}\tag{21}$$

where $\Theta_i = \left[\begin{smallmatrix}\hat{\Delta}_i & P_iC_v+B_i^{\mathrm{T}}P_iB_v\\ * & B_v^{\mathrm{T}}P_iB_v-\gamma^2I\end{smallmatrix}\right]$, $\hat{\Delta}_i = \Delta_i+(D_1+D_2K_i)^{\mathrm{T}}(D_1+D_2K_i)$.

Based on (21), if $\Theta_i<0$, then $J<0$. The matrix $\Theta_i<0$ can be rewritten as the following inequality by the Schur complement:

$$\begin{bmatrix}\hat{\hat{\Delta}}_i & P_iC_v & B_i^{\mathrm{T}}P_i & P_iG_i & 0 & (D_1+D_2K_i)^{\mathrm{T}}\\ * & -\gamma^2I & B_v^{\mathrm{T}}P_i & 0 & P_iG_i & 0\\ * & * & -P_i & 0 & 0 & 0\\ * & * & * & -\epsilon_{1i}I & 0 & 0\\ * & * & * & 0 & -\epsilon_{2i}I & 0\\ * & * & * & 0 & 0 & -I\end{bmatrix}<0,\tag{22}$$

where $\hat{\hat{\Delta}}_i = -P_i(A_i+aI+C_iK_i)-(A_i+aI+C_iK_i)^{\mathrm{T}}P_i^{\mathrm{T}}+\sum_{j=1}^{N}\pi_{ij}P_j+\epsilon_{1i}G_iG_i^{\mathrm{T}}$.

By means of congruent transformation in matrix diag$\{P_i^{-1},P_i^{-1},P_i^{-1},I,I,I\}$, and the replacement $X_i=P_i^{-1}, Y_i=K_iX_i$, the equivalence inequality of (22) can be rewritten as (19). By the same way, formula (20) holds.

Corollary 3. For the given scalar γ and the state feedback USMJS (18), if for all $i\in S$, there exist $\delta_{2i}>0$ and $P_i>0$ satisfying the following LMI:

$$\begin{bmatrix}\tilde{\Delta} & \Upsilon\\ * & \Phi_2\end{bmatrix}<0,\tag{23}$$

where

$$\check{\Delta} = A_iX_i+C_iY_i+X_iA_i^{\mathrm{T}}+Y_i^{\mathrm{T}}C_i^{\mathrm{T}}+\pi_{ii}X_i+\delta_{2i}G_iG_i^{\mathrm{T}},$$

$$\tilde{\Delta} = \begin{bmatrix} \check{\Delta} & \sqrt{\pi_{i1}}X_i & \cdots & \sqrt{\pi_{iN}}X_i \\ * & -X_1 & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_N \end{bmatrix},$$

and Υ, Φ_2 are defined in Theorem 5, then $u_i(t) = K_i x(t)$ is a robust H_∞ controller of the USMJS (18). Meanwhile, the controller gain matrices are constructed by $K_i = Y_i X_i^{-1}$.

Without considering the robustness, a state feedback controller of the MJS (1) with simple LMIs can be designed according to Theorem 5.

Corollary 4. For the given scalar γ and the state feedback USMJS (18), if there exist $\delta_{1i} > 0, \delta_{2i} > 0$ and $P_i > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \tilde{\Delta}_{1i} & \tilde{\Upsilon} \\ * & \tilde{\Phi}_1 \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{\Delta}_{2i} & \tilde{\Upsilon} \\ * & \tilde{\Phi}_2 \end{bmatrix} < 0,$$

where

$$\tilde{\bar{\Delta}}_{1i} = -A_iX_i - C_iY_i - X_iA_i^{\mathrm{T}} - aX_i - Y_i^{\mathrm{T}}C_i^{\mathrm{T}} + \pi_{ii}X_i,$$

$$\tilde{\bar{\Delta}}_{2i} = A_iX_i + C_iY_i + X_iA_i^{\mathrm{T}} + bX_i + Y_i^{\mathrm{T}}C_i^{\mathrm{T}} + \pi_{ii}X_i,$$

$$\tilde{\Delta}_{1i} = \begin{bmatrix} \bar{\Delta}_1 & \sqrt{\pi_{i1}}X_i & \cdots & \sqrt{\pi_{iN}}X_i \\ * & -X_1 & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_N \end{bmatrix}, \quad \tilde{\Delta}_{2i} = \begin{bmatrix} \bar{\Delta}_2 & \sqrt{\pi_{i1}}X_i & \cdots & \sqrt{\pi_{iN}}X_i \\ * & -X_1 & \cdots & 0 \\ * & * & \cdots & 0 \\ * & * & \cdots & -X_N \end{bmatrix},$$

$$\tilde{\Upsilon} = \begin{bmatrix} C_vX_i & X_iB_i^{\mathrm{T}} & X_iD_1^{\mathrm{T}} + Y_i^{\mathrm{T}}D_2^{\mathrm{T}} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\Phi}_1 = \begin{bmatrix} -\gamma^2 I & X_iB_v^{\mathrm{T}} & 0 \\ * & -X_i & 0 \\ * & * & -I \end{bmatrix}, \quad \tilde{\Phi}_2 = \begin{bmatrix} -\gamma^2 I & X_iB_v^{\mathrm{T}} & 0 \\ * & -X_i & 0 \\ * & * & -I \end{bmatrix},$$

then $u_i(t) = K_i x(t)$ is a robust H_∞ controller of the USMJS (18). Meanwhile, the controller gains are constructed by $K_i = Y_i X_i^{-1}$.

5 Numerical example

Consider the USMJS (1) with two operation modes and the following matrices:

$$A_1 = \begin{bmatrix} 1 & 0.5 \\ 1 & -1.2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 0.9 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.2 & 0.5 \\ 1 & -0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.5 & 0.1 \\ 0.2 & -0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0.9 \\ 0 & 0.1 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -0.1 & 0.5 \\ 0.1 & 0.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.5 & 0.1 \\ 0.9 & -0.2 \end{bmatrix},$$

$$E_{11} = E_{12} = F_1 = F_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_{21} = E_{22} = E_{31} = E_{32} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad a = 10, \quad b = 6.$$

The transition rates matrix of the operation modes is given by

$$\prod = \begin{bmatrix} -0.6 & 0.6 \\ 0.2 & -0.2 \end{bmatrix}.$$

According to Corollary 1, the system with given matrices cannot achieve the asymptotic mean-square stability. The trajectories of system states are presented in Figure 1.

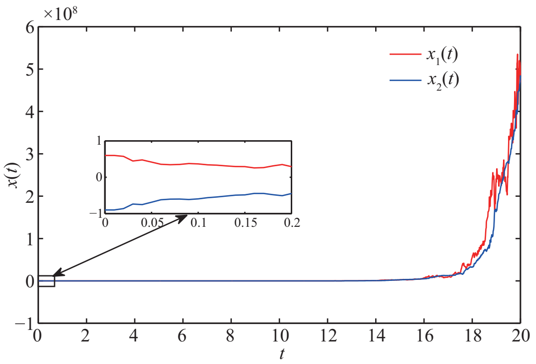

Figure 1 (Color online) State responses of the open-loop system.

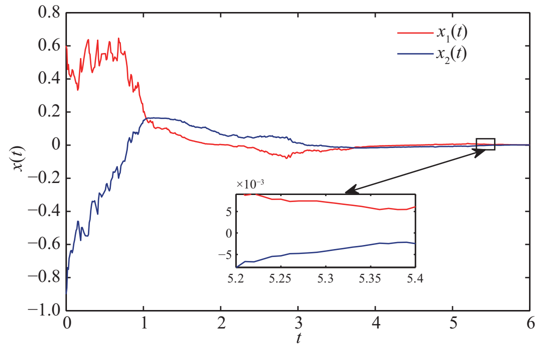

Figure 2 (Color online) State responses of the closed-loop system.

As a usual stabilization method, we can use Corollary 2 to solve the system controllers:

$$K_1 = \begin{bmatrix} -2.9069 & -0.5937 \\ -0.8970 & 0.0556 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -4.7877 & 2.4588 \\ 2.8745 & -4.4718 \end{bmatrix}.$$

The trajectories of the closed-loop system states are presented in Figure 2. When $t = 5.4$ s, $|x_1(t)| > 5 \times 10^{-3}$.

Based on Theorem 3, we can control the convergence rate of the system by keeping its eigenvalues within an appropriate interval location. By solving the conditions provided in Theorem 3, the controller gains that guarantee the poles of the system in appropriate interval location $[-10, -6]$ are calculated as

$$K_1 = \begin{bmatrix} -6.5553 & -1.3488 \\ -0.1215 & -2.5483 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -4.6739 & 9.1737 \\ -1.6059 & -15.7950 \end{bmatrix},$$

which can ensure the corresponding convergence rate. The trajectories of the system states under other controllers are shown in Figure 3. When $t = 1.3$ s, $|x_1(t)| < 5 \times 10^{-3}$ and $|x_2(t)| < 5 \times 10^{-3}$. Figure 4 shows the possible operation modes. Comparing Figures 2 and 3, the time of usual stabilization is more than four times that of interval stabilization when the states converge to less than 5×10^{-3}.

Let

$$B_v = \begin{bmatrix} -0.5 & 0 \\ 1.2 & -0.2 \end{bmatrix}, \quad C_v = \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -1.2 & 0 \\ 1 & -0.2 \end{bmatrix},$$

and $\gamma = 0.5$. Solving Theorem 5 can result in

$$K_1 = \begin{bmatrix} -6.3116 & -1.4143 \\ -0.0469 & -2.2393 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -5.6155 & 6.2027 \\ -0.1041 & -11.3624 \end{bmatrix}.$$

Remark 6. Corresponding to the existing stabilization methods, a unique controller can be designed to ensure the stability of the closed-loop system, which cannot adjust the convergence rate of the closed-loop system (see Figure 2). According to Theorem 3, we can adjust the characteristics of the system by setting the values of a and b, which can ensure not only the convergence but also a reasonable convergence rate of the system (see Figure 3).

6 Conclusion

In this paper, we first established a new robust interval stability criterion of USMJSs using a linear operator and its spectrum. A robust interval stabilization controller was also designed for the USMJS, which can not only ensure the stability of the USMJS, but also adjust the rate of convergence. Moreover, a robust H_∞ controller with interval pole constraints was designed with all the admissible parametric uncertainties and a prescribed H_∞ disturbance attenuation level, which guarantees that the closed-loop

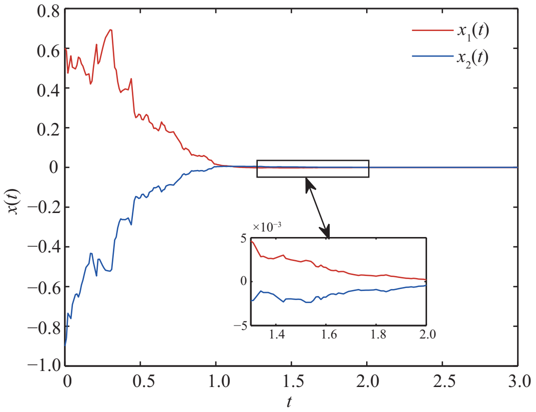

Figure 3 (Color online) Control results of the system.

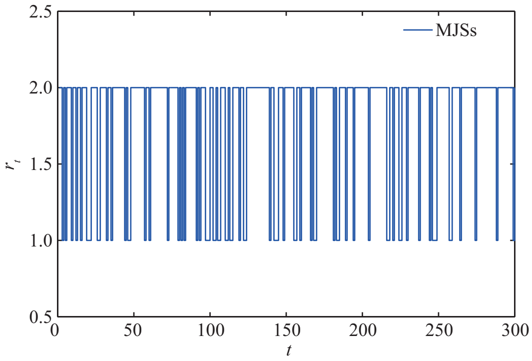

Figure 4 (Color online) Mode responses of the closed-loop system.

system is robust and asymptotically stable with an ideal rate of convergence. We believe that this method is more suitable for achieving an accurate control of USMJSs than the current control methods; moreover, it can be extended to other systems, such as switching systems, semi-MJSs, and singular systems.

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