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Observer-based boundary control for an asymmetric output-constrained flexible robotic manipulator

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Dear editor,

With the rapid development of robotics, flexible robotic manipulators have played an important role in industrial production and manufacture because of their lightweight, higher operation speed and better energy efficiency [1]. However, flexible materials will result in vibration. Besides, in practical applications, external disturbances and constraints caused by environmental factors or technique requirements diffusely exist, which will affect the operation accuracy of the manipulators. Thus, we focus on the angle tracking and vibration problem of the flexible robotic manipulator with external disturbances and output constraints.

Disturbance observers have been widely recognized as a valid way to eliminate the effect of disturbances [2–6]. In [3], an adaptive neural control with disturbance observers was designed for uncertain nonlinear systems. Chen et al. [5] proposed anti-disturbance control based on disturbance observers for hypersonic flight vehicles. Most of the aforementioned methods require strong assumptions about the first time derivative of disturbances, which is not so pratical.

Many researches have been done to deal with the constraint problems, and diverse control approaches have been proposed. In [7], an adaptive neural control was proposed to deal with output constraints. He et al. [8] developed a vibration controller for a flexible manipulator with input deadzone. However, the foregoing studies only discussed the effects of the symmetric constraints for the control systems, but the asymmetric constraints are crucial to practical requirement.

Thus, in this study, boundary control schemes that can trace the prescribed position and suppress the vibration are developed based on a partial differential equation (PDE) model. An asymmetric barrier Lyapunov function is introduced to handle the asymmetric constraints. Novel disturbance observers are proposed to attenuate the effects of boundary disturbances. The well-posedness and the stability of the control system are discussed.

Problem formulation. Notations $(\cdot) = (\cdot)(t)$, $(\cdot) = (\cdot)(x,t)$, $\dot{(\cdot)} = \frac{\partial(\cdot)}{\partial t}$ and $(\cdot)' = \frac{\partial(\cdot)}{\partial x}$ are used throughout the study. Define the displacement of the manipulator as $z = x\theta + y$, where y is the elastic deformation and θ is the angular position. Let $e = \theta - \theta_d$ be the tracking error of the angle and $z_e = xe+y$ be the offset of the manipulator, where θ_d is the desired angle. Besides, d_1 and d_2 represent the boundary disturbances, and u and τ are the designed control laws. Let $a_1, a_2 > 0$ be the end-point displacement output constraints, $b_1, b_2 > 0$ be the angle output constraints, such that the output constraints $-a_1 < z_e(l,t) < a_2$ and $-b_1 < e < b_2$ are guaranteed by u and τ, where l represents the length of the flexible manipulator.

According to [9], consider the flexible manipulator system model as

$$\rho\ddot{z} + EIy'''' - Ty'' + c\dot{z} = 0, \tag{1}$$

$\forall x \in (0,l)$ and $t \in [0,\infty)$, where the tension, bending stiffness, density and damping coefficient are represented by T, EI, ρ and c respectively.

The boundary conditions are presented as

$$\begin{cases} I_h\ddot{\theta} - Ty(l,t) - EIy''(0,t) = \tau + d_1, \\ Ty'(l,t) - EIy'''(l,t) + m\ddot{z}(l,t) = u + d_2, \\ y(0,t) = y'(0,t) = y''(l,t) = 0, \end{cases} \tag{2}$$

$\forall t \in [0,\infty)$, where the inertia of the hub is represented by I_h and m is the mass of payload.

Assumption 1. Suppose that there is a positive constant $\mathcal{D}$ satisfying $|\ddot{d}_i| \leqslant \mathcal{D}$ $(i = 1,2)$.

Control design. Define a disturbance matrix $D = [d_1, d_2]^{\mathrm{T}}$. Then $\dot{D} = [\dot{d}_1, \dot{d}_2]^{\mathrm{T}}$ is the time derivative of D, and $\hat{D} = [\hat{d}_1, \hat{d}_2]^{\mathrm{T}}$ and $\hat{\dot{D}} = [\hat{\dot{d}}_1, \hat{\dot{d}}_2]^{\mathrm{T}}$ are the estimates of D and $\dot{D}$, respectively. We propose the following disturbance

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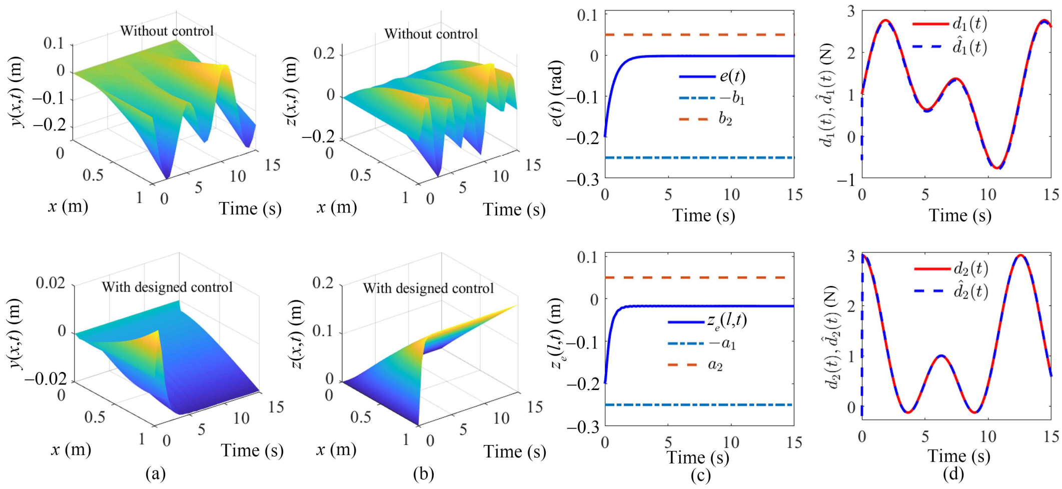

Figure 1 (Color online) (a) Elastic deformation y; (b) position response z; (c) output tracking errors; (d) estimates of disturbances.

observers

$$\begin{cases} \widehat{D} = -\Psi_1 + [\gamma_1 I_h \dot{\theta}, \gamma_2 m \dot{z}(l,t)]^{\mathrm{T}}, \\ \hat{\dot{D}} = -\Psi_2 + [\gamma_1 I_h \dot{\theta}, \gamma_2 m \dot{z}(l,t)]^{\mathrm{T}}, \\ \dot{\Psi}_1 = \dot{\Psi}_2 - \hat{\dot{D}}, \\ \dot{\Psi}_2 = [\gamma_1(P\Phi_1 + \hat{d}_1 + \tau), \gamma_2(P\Phi_2 + \hat{d}_2 + u)]^{\mathrm{T}}, \end{cases} \tag{3}$$

where $\gamma_1, \gamma_2 > 0$, $P = [EI, T]^{\mathrm{T}}$, $\Phi_1 = [y''(0,t), y(l,t)]^{\mathrm{T}}$ and $\Phi_2 = [y'''(l,t), -y'(l,t)]^{\mathrm{T}}$.

Define the estimate errors of D and $\dot{D}$ as

$$\begin{cases} \tilde{D} = [\tilde{d}_1, \tilde{d}_2]^{\mathrm{T}} = [d_1 - \hat{d}_1, d_2 - \hat{d}_2]^{\mathrm{T}}, \\ \tilde{\dot{D}} = [\tilde{\dot{d}}_1, \tilde{\dot{d}}_2]^{\mathrm{T}} = [\dot{d}_1 - \hat{\dot{d}}_1, \dot{d}_2 - \hat{\dot{d}}_2]^{\mathrm{T}}. \end{cases} \tag{4}$$

The boundary control laws are designed as

$$\begin{cases} \tau = -\hat{d}_1 - \dfrac{J(e)e}{b_2^2 - e^2} - k_1\dot{\theta} - \dfrac{(1-J(e))e}{b_1^2 - e^2} - k_2 e, \\ u = -\hat{d}_2 - \dfrac{J(z_e(l,t))z_e(l,t)}{a_2^2 - z_e^2(l,t)} - k_3\dot{z}(l,t) \\ \quad - \dfrac{(1-J(z_e(l,t)))z_e(l,t)}{a_1^2 - z_e^2(l,t)} - k_4 z_e(l,t), \end{cases} \tag{5}$$

where k_1, k_2, k_3, $k_4 > 0$. The function $J(\chi)$ is

$$J(\chi) = \begin{cases} 1, & \chi > 0, \\ 0, & \chi \leqslant 0. \end{cases} \tag{6}$$

Well-posedness. Define the state space as

$$\begin{cases} L^2 = \left\{ f : [0,l] \times [0,T] \to R \,\middle|\, \sup\limits_{t\in[0,T]} \left| \displaystyle\int_0^l f^2 \mathrm{d}x \right| < \infty \right\}, \\ H_l^2 = \{ f \in L^2 | f', f'' \in L^2, f(0,t) = f''(l,t) = 0 \}. \end{cases} \tag{7}$$

The inner product of H is considered as

$$\begin{aligned} &\langle Y_1, Y_2 \rangle_H \\ &= \beta EI \int_0^l g_1'' g_2'' \mathrm{d}x + \sigma\rho \int_0^l (v_1 + w_1)(v_2 + w_2)\mathrm{d}x \\ &\quad + \sigma(c-\rho)\int_0^l v_1 v_2 \mathrm{d}x + \rho(\beta - \sigma)\int_0^l w_1 w_2 \mathrm{d}x \\ &\quad + \beta T \int_0^l (g_1' - g_1'(0,t))(g_2' - g_2'(0,t))\mathrm{d}x \\ &\quad + \sigma I_h (v_1'(0,t) + w_1'(0,t))(v_2'(0,t) + w_2'(0,t)) \\ &\quad + \sigma m (v_1(l,t) + w_1(l,t))(v_2(l,t) + w_2(l,t)) \\ &\quad + I_h(k_1\sigma + k_2\beta)v_1'(0,t)v_2'(0,t) + \beta m T_{11}T_{12} \\ &\quad + m(k_3\sigma + k_4\beta)v_1(l,t)v_2(l,t) + \beta I_h T_{21}T_{22} \\ &\quad - \sigma m w_1(l,t)w_2(l,t) - \sigma I_h w_1'(0,t)w_2'(0,t), \end{aligned} \tag{8}$$

where β, $\sigma > 0$, and $Y_i = [g_i, v_i, w_i, T_{1i}, T_{2i}]^{\mathrm{T}} \in H$, $i = 1, 2$.

We can defind a linear operator $\mathcal{A}$ which satisfys that $\mathcal{A}$ is dissipative in H and its inverse is compact in H, so that the closed-loop system can be described as

$$\frac{\partial Y}{\partial t} = \mathcal{A}Y + F, \quad Y(x,0) = Y_0(x), \tag{9}$$

where $Y = [z, z_e, \dot{z}, \dot{z}(l,t), \dot{\theta}]^{\mathrm{T}}$, $F = [0, 0, 0, \frac{\tilde{d}_1}{m}, \frac{\tilde{d}_2}{I_h}]^{\mathrm{T}}$, and $Y_0(x)$ is the system initial state. The details of $\mathcal{A}$ are shown in Appendix A.

Combining the Lumer-Phillips theorem, $\mathcal{A}$ is a generator of contraction C_0-semigroup. From Assumption 1, it can be concluded that F is locally Lipschitz continuous. Hence, the well-posedness of the proposed control system is verified. If $Y_0(x) \in H$, there will be a unique solution for the closed-loop system, which can be expressed as

$$Y = TY_0(x) + \int_0^t T(x, t-s)F(x,s)\mathrm{d}s, \tag{10}$$

where T is the semigroup associated with $\mathcal{A}$.

Uniform ultimately bundedness. Define the asymmetric barrier Lyapunov function as

$$\mathbb{V} = \mathbb{V}_s + \mathbb{V}_a + \mathbb{V}_b + \mathbb{V}_m + \mathbb{V}_u + \mathbb{V}_d, \tag{11}$$

where

$$\begin{cases} \mathbb{V}_s = \sigma(mz_e(l,t)\dot{z}(l,t) + I_h\dot{\theta}e) + \sigma\rho\int_0^l z_e\dot{z}\mathrm{d}x, \\ \mathbb{V}_a = \frac{\beta(1-J(z_e(l,t)))}{2}\ln\frac{a_1^2}{a_1^2 - z_e^2(l,t)} \\ \qquad + \frac{\beta J(z_e(l,t))}{2}\ln\frac{a_2^2}{a_2^2 - z_e^2(l,t)}, \\ \mathbb{V}_b = \frac{\beta(1-J(e))}{2}\ln\frac{b_1^2}{b_1^2 - e^2} + \frac{\beta J(e)}{2}\ln\frac{b_2^2}{b_2^2 - e^2}, \\ \mathbb{V}_m = \frac{\beta\rho}{2}\int_0^l \dot{z}^2\mathrm{d}x + \frac{\beta EI}{2}\int_0^l (y'')^2\mathrm{d}x \\ \qquad + \frac{\sigma c}{2}\int_0^l z_e^2\mathrm{d}x + \frac{\beta T}{2}\int_0^l (y')^2\mathrm{d}x, \\ \mathbb{V}_u = \frac{\sigma k_1 + \beta k_2}{2}e^2 + \frac{\beta I_h}{2}\dot{\theta}^2 + \frac{\beta m}{2}\dot{z}^2(l,t) \\ \qquad + \frac{\sigma k_3 + \beta k_4}{2}z_e^2(l,t), \\ \mathbb{V}_d = \frac{1}{2}\tilde{D}^{\mathrm{T}}\tilde{D} + \frac{1}{2}[\tilde{D} - \tilde{\hat{D}}]^{\mathrm{T}}[\tilde{D} - \tilde{\hat{D}}]. \end{cases} \tag{12}$$

Theorem 1. By applying the control schemes (5) and the Lyapunov function (11), it can be concluded that the system is uniformly ultimately bounded and the tracking errors $z_e(l,t)$ and e never violate the prescribed asymmetric constraints. That is, $-a_1 < z_e(l,t) < a_2$ and $-b_1 < e < b_2$. The proof is shown in Appendix B.

Simulations. Figure 1 presents the simulation results. It is shown that the vibration is suppressed and the end-point is controlled to the prescribed position with the proposed control. Besides, external disturbances are estimated precisely by observers and both angle and displacement tracking errors obey the constraints $-a_1 < z_e(l,t) < a_2$ and $-b_1 < e < b_2$. The simulation settings are shown in Appendix C.

Conclusion. Boundary control laws with disturbance observers have been constructed to restrain the vibration and regulate the position of the flexible robotic manipulator subject to external disturbances and asymmetric output constraints. The stabilities and well-posedness of the system have been demonstrated. Numerical simulation results have illustrated the feasibility of control schemes.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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