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Finite-horizon resilient state estimation for complex networks with integral measurements from partial nodes

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Abstract This paper proposes a finite-horizon state estimation method for a kind of complex network that suffers from randomly occurring gain variations. The method involves utilizing integral measurements from a portion of nodes in such complex networks. Integral measurements are carried out to characterize time delays that occur in signal acquisition together with real-time signal processing. Measurements from only partial nodes reflect the fact that signals of several sensors are unacquirable. A Gaussian random variable is utilized to depict the random appearance of gain variations during the practical implementation of estimators. The aim of this paper is to construct finite-horizon resilient estimators for complex networks in view of integral measurements from a portion of nodes that fulfill the specified H_∞ performance demand involving a specified disturbance attenuation level. Necessary and sufficient conditions are put forward to ensure that such ideal estimators exist by employing stochastic analysis as well as using the completing squares method. The gain parameters of the finite-horizon estimators are expressed by adopting the Moore-Penrose pseudoinverse and acquired through solving the solutions to a group of coupled backward recursive Riccati difference equations with constraint conditions. A confirmatory instance is carried out that demonstrates the feasibility of the newly developed estimation algorithm.

Keywords complex networks, finite-horizon H_∞ partial-nodes-based state estimation, gain variations, backward recursive Riccati difference equations, integral measurements

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1 Introduction

The complex network model, which contains a number of nodes and their inner/outer connections (couplings), can characterize diverse network systems in the real world, such as social, Internet, transportation, biological, computer, and power transmission network systems. Considering that the node state in complex networks contains much valuable information which cannot be measured, the goal of state estimation (SE) is to acquire accurate estimates of state under a certain performance constraint and establish a basis for subsequent system analysis and decision. Over the last few decades, the SE issue with respect to complex networks has gained increased attention; see [1] and [2] for the globally asymptotic and robustly exponential SE in the mean square, [3] for the H_∞ SE, [4] for the SE with the performance of exponential ultimate boundedness in the mean square, [5] for the set-membership SE, [6] for the recursive SE, [7] for the asynchronous dissipative SE, etc. Nowadays, most complex networks are known to exhibit time-varying features due to rapid changes in working conditions, unpredictable network environments, the need for working assignments, and physical device factors. Therefore, it is natural to

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consider time-varying parameters when solving the SE issue for complex networks, i.e., the finite-horizon SE problem. For example, in [8], considering the time-varying complex networks (TVCNs) containing stochastic factors, the design of bounded H_∞ state estimator has been discussed, which focuses on the transient behavior over a given finite time horizon.

It is recognized that the majority of study results on SE of complex networks stem from measuring all nodes. Measurements from some network nodes may be unavailable due to a faulty sensor, network disconnection between node and sensor, loss of sensor's transmission capacity, or harsh operational conditions. It is critical to realize the SE based on measurement signals from only partial nodes of the network. Initial research efforts were devoted to the analysis of complex networks, considering measurements from a portion of nodes [9–11]. For example, for delayed complex networks, the SE of the whole network has been realized in [10] based on measurement signals of a portion of nodes. Current instant measurements may sometimes be relevant to time delays due to the existence of time intervals between signal acquisition and real-time processing, which is known as integral measurements. So far, integral measurements have been discussed when dealing with the analysis problem of actual systems (e.g., physical reaction [12], quality-of-service guarantees [13], chemical process [14], self-referencing SPR-sensor [15], and electron energy albedos [16]). In particular, Ref. [17] developed a discrete-time model for integral measurements, and the measurement signal is expressed with both the present state and some previous states. However, the finite-horizon SE issue has not yet been solved for TVCNs in terms of integral measurements of a part of nodes, which is the motivation behind our work.

A number of methods have been proposed to cope with finite-horizon analysis and synthesis issues of estimating time-varying systems. These methods include, but are not limited to, the minimum covariance method [6,18–20], an auxiliary Krein space state-space model [21], the recursive linear matrix inequality [2, 5, 8, 22, 23], and the coupled backward recursive Riccati difference equation (RRDE) method [24–27]. The advantages of using the RRDE method include low computation burdens, fast calculation of gains, acquisition of necessary and sufficient conditions (NSCs), less conservativeness, as well as easy realization. Such a method proved effective in investigating H_∞ containment control [24], finite-horizon H_∞ bipartite consensus control [25], and finite-horizon H_∞ control [26] and filtering [27], etc. Among these references, the H_∞ performance indices guarantee that the influences are restrained below a prescribed disturbance attenuation level (DAL). These influences include both the noises and the initial states on the controlled tracking errors, the consensus error, the controlled output, and the estimation error output filtering. Specifically, NSCs are obtained for designing desired observer/controller/filter such that the corresponding H_∞ performance indices are met. Nevertheless, considering time-varying complex networks with integral measurements of partial nodes, the establishment of NSCs has not yet been sufficiently investigated for the beingness of finite-horizon H_∞ state estimators, which also inspires us to carry out this work.

During the actual implementation of estimators, some gain variations/perturbations may appear due to analog-digital conversion, rounding error, finite accuracy, and noises. Such gain variations may cause system fragilities, such as increased estimation error, loss of estimation performance, unreliability, abnormal behavior, and wrong estimator operation. It is thus critical to devise resilient estimators that are insensitive to such gain variations. Research results on resilient/non-fragile estimation for complex networks are available in the literature; see [5, 28] and the references therein. In this paper, the main task is to realize the SE for complex networks with time-varying parameters, integral measurements of a portion of nodes as well as randomly occurring gain variations under a finite-horizon H_∞ performance constraint. The difficulties that need to be addressed include (1) how to construct a uniform framework for the problem under investigation; (2) how to obtain NSCs that guarantee that such finite-horizon resilient H_∞ state estimators exist; and (3) how to acquire the time-varying gain parameters of such state estimators. In this paper, the major novelties are the following aspects: (1) the finite-horizon partial-nodes-based SE issue is, firstly, solved for TVCNs with integral measurements and randomly occurring gain variations; (2) NSCs are established with the help of the completing squares method (CSM), which ensure that the estimation error dynamics exhibits the finite-horizon H_∞ performance; and (3) the gain parameters of the time-varying state estimators are conveniently calculated by finding the solutions to certain coupled backward RRDEs.

The framework of this paper is listed below. Section 2 presents specified models and the issue that needs to be addressed. Section 3 presents the analysis of estimation performance and the development of the SE method. In Section 4, a confirmatory instance is carried out to verify the estimation performance of the developed estimators. Section 5 presents the conclusion of this paper.

Notation. In this paper, $\Re^p$ is the Euclidean space with dimension p. $l_2([0, L], \Re^q)$ is referred to the space of nonanticipatory square-summable vector-valued functions with dimension q over $[0, L]$. As for a real symmetric matrix S, $S > 0$ (or $S \geqslant 0$) illustrates that S is a positive definite (or positive semi-definite) matrix, S^{-1} and $|S|$ mean the inverse and the determinant of S, and $*$ depicts the symmetric term therein. diag$\{\cdots\}$ reflects a block-diagonal matrix. Considering a matrix O, O^{T}, $O^{\dagger}$, $\|O\|$, and $\|O\|_F$ show the transpose, the Moore-Penrose pseudoinverse, the norm, and the Frobenius norm, respectively. I (0) is the identity (zero) matrix. $\mathscr{E}\{\vartheta\}$ characterizes the mathematical expectation of a random variable ϑ. $\otimes$ represents the Kronecker product.

2 Problem formulation and preliminaries

For a given finite horizon $[0, \breve{D}]$, consider a kind of TVCNs involving U coupled nodes as follows:

$$\begin{aligned} x_\imath(\hbar+1) &= A_\imath(\hbar)x_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\Gamma x_j(\hbar) + E_\imath(\hbar)w(\hbar), \\ z_\imath(\hbar) &= H_\imath(\hbar)x_\imath(\hbar),\ \imath = 1, 2, \ldots, U,\ \hbar = 0, 1, \ldots, \breve{D}, \end{aligned} \tag{1}$$

where $x_\imath(\hbar) \in \Re^{n_x}$ and $z_\imath(\hbar) \in \Re^{n_z}$ denote the state vector and the vector we would like to estimate, respectively; $w(\hbar) \in l_2([0, \breve{D}], \Re^{n_w})$ is the process noise; $A_\imath(\hbar)$, $E_\imath(\hbar)$ and $H_\imath(\hbar)$ are given time-varying matrices whose dimensions are proper; and the matrix $B \triangleq [a_{\imath j}]_{U\times U}$ represents the outer-coupling configuration matrix of network (1) conforming to $a_{\imath j} \geqslant 0$ ($\imath \neq j$) but not all zeros. Generally, B is symmetric with the satisfaction of

$$\sum_{j=1}^{U} a_{\imath j} = \sum_{j=1}^{U} a_{j\imath} = 0,\ \imath = 1, 2, \ldots, U. \tag{2}$$

The matrix $\Gamma \triangleq \mathrm{diag}\{r_1, r_2, \ldots, r_{n_x}\} \geqslant 0$ stands for the inner-coupling matrix with a connection to the jth state variable when $r_j \neq 0$.

Assume that the measurement outputs from the first u ($u \leqslant U$) network nodes are available. The measurement signal of the $\imath$th ($\imath = 1, 2, \ldots, u$) node is expressed as

$$y_\imath(\hbar) = C_\imath(\hbar)\sum_{m=0}^{\wp} x_\imath(\hbar - m) + G_\imath(\hbar)w(\hbar) \in \Re^{n_y}, \tag{3}$$

where $\wp$ represents the time interval to collect the data; and time-varying matrices $C_\imath(\hbar)$ and $G_\imath(\hbar)$ are given having proper dimensions.

Remark 1. This paper mainly focuses on the finite-horizon SE issue in terms of measurements of a portion of nodes, considering the fact that measurements from some nodes may be inaccessible during the actual operation of complex networks due to complexity of working environments, sensor failure, temporary ineffectiveness of network of certain region, and linking inconvenience from the sensor to the estimator of several nodes. In later analysis, effort is devoted to conducting the analysis of estimation performance and estimator design on the basis of measurements from a part of rather than all the nodes.

Let $\breve{x}_\imath(\hbar) \triangleq [x_\imath^{\mathrm{T}}(\hbar)\ x_\imath^{\mathrm{T}}(\hbar-1)\ \cdots\ x_\imath^{\mathrm{T}}(\hbar-\wp)]^{\mathrm{T}}$. The expressions of (1) and (3) are changed to the following form:

$$\begin{aligned} \breve{x}_\imath(\hbar+1) &= \breve{A}_\imath(\hbar)\breve{x}_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\breve{\Gamma}\breve{x}_j(\hbar) + \breve{E}_\imath(\hbar)w(\hbar),\ \imath = 1, 2, \ldots, U, \\ z_\imath(\hbar) &= \breve{H}_\imath(\hbar)\breve{x}_\imath(\hbar),\ \imath = 1, 2, \ldots, U, \\ y_\imath(\hbar) &= \breve{C}_\imath(\hbar)\breve{x}_\imath(\hbar) + G_\imath(\hbar)w(\hbar),\ \imath = 1, 2, \ldots, u, \end{aligned} \tag{4}$$

where

$$\breve{A}_\imath(\hbar) \triangleq \begin{bmatrix} A_\imath(\hbar) & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ 0 & \cdots & \cdots & I & 0 \end{bmatrix}, \quad \breve{E}_\imath(\hbar) \triangleq \begin{bmatrix} E_\imath(\hbar) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \breve{C}_\imath(\hbar) \triangleq [\underbrace{C_\imath(\hbar)\ C_\imath(\hbar)\ \cdots\ C_\imath(\hbar)}_{\wp+1}],$$

$$\breve{\Gamma} \triangleq \mathrm{diag}\{\Gamma, 0, 0, \ldots, 0\}, \quad \breve{H}_\imath(\hbar) \triangleq \begin{bmatrix} H_\imath(\hbar) & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Remark 2. In order to be more realistic, in the measurement signal (3) of the $\imath$th ($\imath = 1, 2, \ldots, u$) node, integral measurements are considered to reflect the existence of time delays during the process of signal acquisition in view of real-time signal processing. By employing the mathematical manipulation of augmenting the vectors [17], the form of time delays in the expression of integral measurements (3) is involved in a single vector $\breve{x}_\imath(\hbar)$, which is convenient for executing analysis and design of estimators afterwards.

Construct the following estimators for TVCN (1) on node $\imath$:

$$\begin{aligned} &\hat{x}_\imath(\hbar+1) = \breve{A}_\imath(\hbar)\hat{x}_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\breve{\Gamma}\hat{x}_j(\hbar) + (K_\imath(\hbar) + \varsigma(\hbar)\Delta K_\imath(\hbar))(y_\imath(\hbar) - \breve{C}_\imath(\hbar)\hat{x}_\imath(\hbar)),\ \imath = 1, 2, \ldots, u, \\ &\hat{x}_\imath(\hbar+1) = \breve{A}_\imath(\hbar)\hat{x}_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\breve{\Gamma}\hat{x}_j(\hbar),\ \imath = u+1, u+2, \ldots, U, \\ &\hat{z}_\imath(\hbar) = \breve{H}_\imath(\hbar)\hat{x}_\imath(\hbar),\ \imath = 1, 2, \ldots, U, \end{aligned} \tag{5}$$

where $\hat{x}_\imath(\hbar) \in \Re^{n_x(\wp+1)}$ and $\hat{z}_\imath(\hbar) \in \Re^{n_z}$ characterize the estimates of the state $\breve{x}_\imath(\hbar)$ and the signal $z_\imath(\hbar)$ of the $\imath$th node, and $K_\imath(\hbar)$ is the gain parameter to be decided of the estimator. $\Delta K_\imath(\hbar)$ denotes the gain variations with the following expression:

$$\Delta K_\imath(\hbar) = M_\imath(\hbar)F_\imath(\hbar)N_\imath(\hbar), \tag{6}$$

where $M_\imath(\hbar)$ and $N_\imath(\hbar)$ stand for given matrices, and $F_\imath(\hbar)$ means an unknown matrix with $F_\imath^{\mathrm{T}}(\hbar)F_\imath(\hbar) \leqslant I$. The variable $\varsigma(\hbar)$ is introduced in (5) which reflects the random occurrence of gain variations, which is supposed as a sequence of Gaussian white noise with expectation $\bar{\varsigma}$ as well as variance $\hat{\varsigma}$.

Let $e_\imath(\hbar) \triangleq \breve{x}_\imath(\hbar) - \hat{x}_\imath(\hbar)$ and $\check{z}_{e\imath}(\hbar) \triangleq z_\imath(\hbar) - \hat{z}_\imath(\hbar)$ denote the estimation errors, and we derive the following estimation error dynamics (EED):

$$\begin{aligned} e_\imath(\hbar+1) = {} & (\breve{A}_\imath(\hbar) - K_\imath(\hbar)\breve{C}_\imath(\hbar) - \bar{\varsigma}\Delta K_\imath(\hbar)\breve{C}_\imath(\hbar))e_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\breve{\Gamma}e_j(\hbar) + (\breve{E}_\imath(\hbar) - K_\imath(\hbar)G_\imath(\hbar) - \bar{\varsigma} \\ & \times \Delta K_\imath(\hbar)G_\imath(\hbar))w(\hbar) - \tilde{\varsigma}(\hbar)\Delta K_\imath(\hbar)\breve{C}_\imath(\hbar)e_\imath(\hbar) - \tilde{\varsigma}(\hbar)\Delta K_\imath(\hbar)G_\imath(\hbar)w(\hbar),\ \imath = 1, 2, \ldots, u, \\ e_\imath(\hbar+1) = {} & \breve{A}_\imath(\hbar)e_\imath(\hbar) + \sum_{j=1}^{U} a_{\imath j}\breve{\Gamma}e_j(\hbar) + \breve{E}_\imath(\hbar)w(\hbar),\ \imath = u+1, u+2, \ldots, U, \\ \check{z}_{e\imath}(\hbar) = {} & \breve{H}_\imath(\hbar)e_\imath(\hbar),\ \imath = 1, 2, \ldots, U. \end{aligned} \tag{7}$$

Several new variables are introduced as follows:

$$\begin{aligned} &\nabla(\hbar) \triangleq \left[\nabla_1^{\mathrm{T}}(\hbar)\ \nabla_2^{\mathrm{T}}(\hbar)\ \cdots\ \nabla_U^{\mathrm{T}}(\hbar)\right]^{\mathrm{T}},\ \nabla = e, \check{z}_e, \breve{E},\ \partial(\hbar) \triangleq \mathrm{diag}\{\partial_1(\hbar), \partial_2(\hbar), \ldots, \partial_U(\hbar)\},\ \partial = \breve{A}, \breve{H}, \\ &G(\hbar) \triangleq \left[G_1^{\mathrm{T}}(\hbar)\ G_2^{\mathrm{T}}(\hbar)\ \cdots\ G_u^{\mathrm{T}}(\hbar)\right]^{\mathrm{T}},\ \sqsupset(\hbar) \triangleq \mathrm{diag}\{\sqsupset_1(\hbar), \sqsupset_2(\hbar), \ldots, \sqsupset_u(\hbar)\},\ \sqsupset = K, \Delta K, \breve{C}. \end{aligned} \tag{8}$$

In terms of (7) and (8), we acquire the following compact form of the EED (7):

$$e(\hbar+1) = (\mathcal{A}(\hbar) - \bar{\varsigma}\Delta\breve{K}(\hbar)\mathcal{C}(\hbar))e(\hbar) - \tilde{\varsigma}(\hbar)\Delta\breve{K}(\hbar)\mathcal{C}(\hbar)e(\hbar) + (\breve{E}(\hbar) - \breve{K}(\hbar)$$

$$\times G(\hbar) - \bar{\varsigma}\Delta\breve{K}(\hbar)G(\hbar))w(\hbar) - \tilde{\varsigma}(\hbar)\Delta\breve{K}(\hbar)G(\hbar)w(\hbar), \tag{9}$$

$$\breve{z}_e(\hbar) = \breve{H}(\hbar)e(\hbar), \tag{10}$$

where

$$\mathcal{A}(\hbar) \triangleq \breve{A}(\hbar) + B \otimes \breve{\Gamma} - \breve{K}(\hbar)\mathcal{C}(\hbar),\ \breve{K}(\hbar) \triangleq \begin{bmatrix} K^{\mathrm{T}}(\hbar) & 0 \end{bmatrix}^{\mathrm{T}},\ \Delta\breve{K}(\hbar) \triangleq \begin{bmatrix} \Delta K^{\mathrm{T}}(\hbar) & 0 \end{bmatrix}^{\mathrm{T}},\ \mathcal{C}(\hbar) \triangleq \begin{bmatrix} \breve{C}(\hbar) & 0 \end{bmatrix}.$$

In this paper, the aim lies in coping with the finite-horizon resilient SE issue for the TVCN (1) based on integral measurements of the form (3). In particular, we are devoted to designing the gain parameters $K_\imath(\hbar)$ ($\imath = 1, 2, \ldots, u$, $\hbar = 0, 1, \ldots, \breve{D}$) of time-varying estimators (5), for all nonzero $w(\hbar)$, which guarantee that the estimation error $\breve{z}_e(\hbar)$ in (10) meets the performance constraint below:

$$W \triangleq \mathscr{E}\left\{\sum_{\hbar=0}^{\breve{D}} \|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2 \sum_{\hbar=0}^{\breve{D}} \|w(\hbar)\|^2\right\} - \bar{\chi}^2 e^{\mathrm{T}}(0)Qe(0) < 0, \tag{11}$$

where $Q > 0$ is a known weighting matrix, and $\bar{\chi} > 0$ stands for the specified DAL.

Considering (6), one has

$$\Delta\breve{K}(\hbar) = \breve{M}(\hbar)F(\hbar)N(\hbar), \tag{12}$$

where

$$\breve{M}(\hbar) \triangleq \begin{bmatrix} M^{\mathrm{T}}(\hbar) & 0 \end{bmatrix}^{\mathrm{T}},\quad O(\hbar) \triangleq \mathrm{diag}\{O_1(\hbar), O_2(\hbar), \ldots, O_u(\hbar)\},\ O = M, F, N.$$

To deal with the gain variations in (9), an effective method is to view them as one source of the disturbances [26, 27]. Thus, we are to resist the impact of all the disturbances on the estimation error in accordance with the specified H_∞ performance constraint (11). Then, we rewrite (9) in the following expression:

$$\begin{aligned} &e(\hbar+1) = \mathcal{A}(\hbar)e(\hbar) + \mathcal{G}(\hbar)\varpi(\hbar) - \tilde{\varsigma}(\hbar)\mathcal{M}(\hbar)\varpi(\hbar), \\ &\breve{z}_e(\hbar) = \breve{H}(\hbar)e(\hbar), \end{aligned} \tag{13}$$

where

$$\begin{aligned} &\varpi(\hbar) \triangleq \begin{bmatrix} w^{\mathrm{T}}(\hbar) & (\zeta(\hbar)F(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar))^{\mathrm{T}} & (\zeta(\hbar)F(\hbar)N(\hbar)G(\hbar)w(\hbar))^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &\mathcal{G}(\hbar) \triangleq \begin{bmatrix} \breve{E}(\hbar) - \breve{K}(\hbar)G(\hbar) & -\zeta^{-1}(\hbar)\bar{\varsigma}\breve{M}(\hbar) & -\zeta^{-1}(\hbar)\bar{\varsigma}\breve{M}(\hbar) \end{bmatrix},\ \ \mathcal{M}(\hbar) \triangleq \begin{bmatrix} \zeta^{-1}(\hbar)\breve{M}(\hbar) & \zeta^{-1}(\hbar)\breve{M}(\hbar) \end{bmatrix}. \end{aligned}$$

Here, $\zeta(\hbar) > 0$ means a function which denotes the scaling concerning the perturbation, and the involvement of it is to offer more flexibility into the estimation problem. Besides, for all nonzero $\varpi(\hbar)$, we utilize the auxiliary index as follows:

$$\begin{aligned} \bar{W} \triangleq \mathscr{E}\Bigg\{ &\sum_{\hbar=0}^{\breve{D}} \|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2 \sum_{\hbar=0}^{\breve{D}} (\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2) \Bigg\} \\ &- \bar{\chi}^2 e^{\mathrm{T}}(0)Qe(0) < 0. \end{aligned} \tag{14}$$

3 Main results

Before presenting the major results, several lemmas are shown which are necessary for performance analysis and design of estimators (5).

Lemma 1 ([29])**.** Let W, Y and X denote nonzero matrices with suitable dimensions. The solution S with respect to $\min_S \|WSX - Y\|_F$ is $W^{\dagger}YX^{\dagger}$.

Lemma 2. Concerning the noise $\varpi(\hbar)$ and the initial state $e(0)$, regard $e(\hbar)$ as the appropriate solution to the system (13) over $[0,\breve{D}]$. Then, one acquires

$$W_1(e(0),\varpi(\hbar)) \triangleq \mathscr{E}\left\{\sum_{\hbar=0}^{\breve{D}}\|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2\sum_{\hbar=0}^{\breve{D}}(\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2)\right\}$$
$$= \sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\bar{e}^{\mathrm{T}}(\hbar)\Omega(\hbar)\bar{e}(\hbar)\right\} + \mathscr{E}\{e^{\mathrm{T}}(0)\breve{P}(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1)\}. \quad (15)$$

Moreover, if $|\Omega_{22}(\hbar+1)| \neq 0$ for all $\hbar \in [0,\breve{D}]$, by choosing $\varpi(\hbar) \triangleq \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$ and defining $\breve{Z}(\hbar) \triangleq -\breve{K}(\hbar)\mathcal{C}(\hbar)e(\hbar)$, one has

$$W_2(\breve{Z}(\hbar),\varpi(\hbar)) \triangleq \sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\|\breve{z}_e(\hbar)\|^2 + \|\breve{Z}(\hbar)\|^2\right\}$$
$$= \sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\breve{e}^{\mathrm{T}}(\hbar)\Theta(\hbar)\breve{e}(\hbar)\} + \mathscr{E}\{e^{\mathrm{T}}(0)X(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)X(\breve{D}+1)e(\breve{D}+1)\}, \quad (16)$$

where $\{\breve{P}(\hbar)\}_{0\leqslant\hbar\leqslant\breve{D}+1} > 0$ and $\{X(\hbar)\}_{0\leqslant\hbar\leqslant\breve{D}+1} > 0$ denote two groups of matrices with $T(\hbar) \triangleq [T_{ij}(\hbar)]_{Un_x(\wp+1)\ \times Un_x(\wp+1)}$ $(T = P, X)$, and

$$\Omega(\hbar+1) \triangleq \begin{bmatrix}\Omega_{11}(\hbar+1) - \breve{P}(\hbar) & * \\ \Omega_{12}^{\mathrm{T}}(\hbar+1) & -\Omega_{22}(\hbar+1)\end{bmatrix},\ \Theta(\hbar+1) \triangleq \begin{bmatrix}\Theta_1(\hbar+1) & * \\ \Theta_{12}^{\mathrm{T}}(\hbar+1) & \Theta_{22}(\hbar+1)\end{bmatrix},$$
$$\Omega_{11}(\hbar+1) \triangleq \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) + \mathcal{A}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{A}(\hbar) + \bar{\chi}^2\zeta^2(\hbar)\mathcal{C}^{\mathrm{T}}(\hbar)N^{\mathrm{T}}(\hbar)N(\hbar)\mathcal{C}(\hbar),$$
$$\Omega_{12}(\hbar+1) \triangleq \mathcal{A}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{G}(\hbar),\ \Theta_1(\hbar+1) = \Theta_{11}(\hbar+1) + \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) - X(\hbar),$$
$$\Omega_{22}(\hbar+1) \triangleq -\mathcal{G}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{G}(\hbar) - \hat{\varsigma}\mathcal{M}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{M}(\hbar) + \bar{\chi}^2 I - \bar{\chi}^2\zeta^2(\hbar)S^{\mathrm{T}}G^{\mathrm{T}}(\hbar)N^{\mathrm{T}}(\hbar)$$
$$\times N(\hbar)G(\hbar)S,\ \breve{\mathcal{A}}(\hbar) \triangleq \breve{A}(\hbar) + B\otimes\breve{\Gamma},\ \Theta_{12}(\hbar+1) \triangleq (\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1))^{\mathrm{T}}X(\hbar+1),$$
$$\breve{e}(\hbar) \triangleq \begin{bmatrix}e^{\mathrm{T}}(\hbar) & \breve{Z}^{\mathrm{T}}(\hbar)\end{bmatrix}^{\mathrm{T}},\ S \triangleq \begin{bmatrix}I & 0 & 0\end{bmatrix},\ \bar{e}(\hbar) \triangleq \begin{bmatrix}e^{\mathrm{T}}(\hbar) & \varpi^{\mathrm{T}}(\hbar)\end{bmatrix}^{\mathrm{T}},\ \Lambda(\hbar+1) \triangleq \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1),$$
$$\Theta_{11}(\hbar+1) \triangleq (\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1))^{\mathrm{T}}X(\hbar+1)(\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1))$$
$$+ \hat{\varsigma}\Lambda^{\mathrm{T}}(\hbar+1)\mathcal{M}^{\mathrm{T}}(\hbar)X(\hbar+1)\mathcal{M}(\hbar)\Lambda(\hbar+1),\ \Theta_{22}(\hbar+1) \triangleq X(\hbar+1) + I. \quad (17)$$

Proof. Let $\Upsilon(\hbar) \triangleq e^{\mathrm{T}}(\hbar+1)\breve{P}(\hbar+1)e(\hbar+1) - e^{\mathrm{T}}(\hbar)\breve{P}(\hbar)e(\hbar)$. According to system (13), one derives

$$\mathscr{E}\{\Upsilon(\hbar)\} = e^{\mathrm{T}}(\hbar)\mathcal{A}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{A}(\hbar)e(\hbar) + 2e^{\mathrm{T}}(\hbar)\mathcal{A}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{G}(\hbar)\varpi(\hbar) + \varpi^{\mathrm{T}}(\hbar)\mathcal{G}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)$$
$$\times \mathcal{G}(\hbar)\varpi(\hbar) + \hat{\varsigma}\varpi^{\mathrm{T}}(\hbar)\mathcal{M}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\mathcal{M}(\hbar)\varpi(\hbar) - e^{\mathrm{T}}(\hbar)\breve{P}(\hbar)e(\hbar). \quad (18)$$

Combining $\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\breve{z}_e(\hbar)\|^2\}$ with the zero term $\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\Upsilon(\hbar)-\Upsilon(\hbar)\}+\bar{\chi}^2\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\varpi(\hbar)\|^2-\|\zeta(\hbar)N(\hbar)\times\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2 - (\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2)\}$ and noticing $w(\hbar) = S\varpi(\hbar)$, we deduce that

$$\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\breve{z}_e(\hbar)\|^2\} = \sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\bar{e}^{\mathrm{T}}(\hbar)\Omega(\hbar+1)\bar{e}(\hbar)\right\} + \bar{\chi}^2\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2$$
$$- \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2\} - \mathscr{E}\{e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1) - e^{\mathrm{T}}(0)\breve{P}(0)e(0)\}. \quad (19)$$

In similarity, considering $\breve{Z}(\hbar) = -\breve{K}(\hbar)\mathcal{C}(\hbar)e(\hbar)$, we have

$$\mathcal{A}(\hbar)e(\hbar) = \breve{\mathcal{A}}(\hbar)e(\hbar) + \breve{Z}(\hbar). \quad (20)$$

In addition, under $|\Omega_{22}(\hbar+1)| \neq 0$ for all $\hbar \in [0,\breve{D}]$, the selection of $\varpi(\hbar) \triangleq \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$ leads to

$$\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\breve{z}_e(\hbar)\|^2\} = \sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\breve{H}(\hbar)e(\hbar)\|^2 - \|\breve{Z}(\hbar)\|^2 + \|\breve{Z}(\hbar)\|^2\} - \mathscr{E}\{e^{\mathrm{T}}(\breve{D}+1)X(\breve{D}+1)e(\breve{D}+1) - e^{\mathrm{T}}(0)$$

$$
\begin{aligned}
&\times X(0)e(0)\} + \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{e^{\mathrm{T}}(\hbar)(\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1))^{\mathrm{T}} X(\hbar+1)(\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1)) \\
&\times e(\hbar) - e^{\mathrm{T}}(\hbar)X(\hbar)e(\hbar) + \hat{\varsigma} e^{\mathrm{T}}(\hbar)\Lambda^{\mathrm{T}}(\hbar+1)\mathcal{M}^{\mathrm{T}}(\hbar)X(\hbar+1)\mathcal{M}(\hbar)\Lambda(\hbar+1)e(\hbar) \\
&+ 2e^{\mathrm{T}}(\hbar)(\breve{\mathcal{A}}(\hbar) + \mathcal{G}(\hbar)\Lambda(\hbar+1))^{\mathrm{T}} X(\hbar+1)\breve{Z}(\hbar) + \breve{Z}^{\mathrm{T}}(\hbar)X(\hbar+1)\breve{Z}(\hbar)\} \\
=\,& \mathscr{E}\{e^{\mathrm{T}}(0)X(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)X(\breve{D}+1)e(\breve{D}+1)\} - \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{\|\breve{Z}(\hbar)\|^2\} \\
&+ \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{\breve{e}^{\mathrm{T}}(\hbar)\Theta(\hbar+1)\breve{e}(\hbar)\}. \qquad (21)
\end{aligned}
$$

It is clear to see that Eqs. (15) and (16) are guaranteed, respectively, via (19) and (21).

Through employing the CSM as well as proofs by contradiction, NSCs would be put forward for designing the time-varying estimators (5) under the prescribed performance requirement (11).

Lemma 3. Note that the DAL $\bar{\chi} > 0$ and the matrix $Q > 0$ are known. Considering the system (13) with all nonzero $\{\varpi(\hbar)\}_{0\leqslant\hbar\leqslant\breve{D}}$, the performance requirement (14) is fulfilled if and only if there exists a series of matrices $\{\breve{P}(\hbar) > 0\}_{0\leqslant\hbar\leqslant\breve{D}}$ (under the final condition $\breve{P}(\breve{D}+1) = 0$) which ensures that the backward RRDE below:

$$
\Omega_{11}(\hbar+1) + \Omega_{12}(\hbar+1)\Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1) = \breve{P}(\hbar) \qquad (22)
$$

holds with

$$
\Omega_{22}(\hbar+1) > 0 \quad \text{and} \quad \breve{P}(0) < \bar{\chi}^2 Q. \qquad (23)
$$

Proof. Sufficiency. In terms of matrices $\{\breve{P}(\hbar) > 0\}_{0\leqslant\hbar\leqslant\breve{D}+1}$ in (22), by considering (19), we have

$$
\begin{aligned}
&\sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{\|\breve{z}_e(\hbar)\|^2\} - \bar{\chi}^2 \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2\} \\
&= \mathscr{E}\{e^{\mathrm{T}}(0)\breve{P}(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1)\} + \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{e^{\mathrm{T}}(\hbar)(\Omega_{11}(\hbar+1) - \breve{P}(\hbar))e(\hbar) \\
&\quad + 2e^{\mathrm{T}}(\hbar)\Omega_{12}(\hbar+1)\varpi(\hbar) - \varpi^{\mathrm{T}}(\hbar)\Omega_{22}(\hbar+1)\varpi(\hbar)\} \\
&= \mathscr{E}\{e^{\mathrm{T}}(0)\breve{P}(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1)\} \\
&\quad + \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\big\{ - (\varpi(\hbar) - \varpi^*(\hbar))^{\mathrm{T}}\Omega_{22}(\hbar+1)(\varpi(\hbar) - \varpi^*(\hbar))\big\}, \qquad (24)
\end{aligned}
$$

where $\varpi^*(\hbar) \triangleq \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$.

Owing to $\Omega_{22}(\hbar+1) > 0$ and $\breve{P}(0) < \bar{\chi}^2 Q$, for all nonzero $\varpi(\hbar)$, we derive from $\breve{P}(\breve{D}+1) = 0$ that

$$
\begin{aligned}
\bar{W} &< \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\left\{\|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2(\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2)\right\} \\
&\quad - e^{\mathrm{T}}(0)\breve{P}(0)e(0) \\
&= - \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\{(\varpi(\hbar) - \varpi^*(\hbar))^{\mathrm{T}}\Omega_{22}(\hbar+1)(\varpi(\hbar) - \varpi^*(\hbar))\} < 0, \qquad (25)
\end{aligned}
$$

which is equivalent to (14).

Necessity. In this stage, we are to validate that, as long as Eq. (14) is met, there exists a feasible solution $\breve{P}(\hbar)$ $(0 \leqslant \hbar \leqslant \breve{D}+1)$ to (22) meeting (23) for all nonzero $(\{\varpi(\hbar)\}, e(0))$. Actually, taking into account the final condition $\breve{P}(\breve{D}+1) = 0$, the RRDE (22) is solved backward at all the time if $\Omega_{22}(\hbar+1) > 0$ and $\breve{P}(0) < \bar{\chi}^2 Q$ for $\hbar \in [0,\ \breve{D}]$, which indicates that Eq. (22) ceases the recursion for some $\hbar = \acute{\tau} \in [0,\ \breve{D}]$ if $\Omega_{22}(\acute{\tau}+1)$ or $\bar{\chi}^2 Q - \breve{P}(0)$ has one or more non-positive eigenvalues.

The subsequent part of the proof is carried through contradiction. For the assumption that there is at least one non-positive eigenvalue for $\Omega_{22}(\hbar+1)$ or $\bar{\chi}^2 Q - \breve{P}(0)$ at certain time step $\hbar = \acute{\tau} \in [0,\ \breve{D}]$, we would like to verify that $\bar{W} < 0$ cannot be fulfilled.

Case 1. We are to verify

$$\varrho_\ell(\Omega_{22}(\hbar+1)) \leqslant 0, \quad \forall \hbar \in [0,\ \breve{D}], \quad \ell = 1,2,\ldots, \quad n_w \Longrightarrow \bar{W} \geqslant 0, \tag{26}$$

where $\varrho_\ell(\Omega_{22}(\hbar+1))$ stands for the ℓth eigenvalue of $\Omega_{22}(\hbar+1)$.

In order to simplify the representation, we express the zero or negative eigenvalue of $\Omega_{22}(\hbar+1)$ at time step $\acute{\tau}$ as $\varrho(\acute{\tau})$, i.e., $\varrho(\acute{\tau}) \leqslant 0$. In the following part, we will employ $\varrho(\acute{\tau}) \leqslant 0$ to represent that there is some $(\{\varpi(\hbar)\}, e(0)) \neq 0$ which causes $\bar{W} \geqslant 0$. First of all, choose $e(0) \triangleq 0$ and

$$\varpi(\hbar) = \begin{cases} 0, & \text{if } h \in [0, \acute{\tau}), \\ \psi(\acute{\tau}), & \text{if } h = \acute{\tau}, \\ \varpi^*(\hbar), & \text{if } h \in (\acute{\tau}, \breve{D}+1), \end{cases}$$

where $\psi(\acute{\tau})$ is the eigenvector of $\Omega_{22}(\acute{\tau}+1)$ corresponding to $\varrho(\acute{\tau})$.

For $0 \leqslant \hbar < \acute{\tau}$, according to (13) with $e(0) = 0$ as well as $\varpi(\hbar) = 0$, we acquire $e(\hbar) = 0$ $(0 \leqslant \hbar \leqslant \acute{\tau})$, so that $\varpi^*(\hbar) = \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1)e(\hbar) = 0$ $(\hbar \in [0, \acute{\tau}])$.

According to (24) and $\varpi(\hbar)$, we have

$$\mathscr{E}\left\{\sum_{\hbar=0}^{\acute{\tau}-1} \|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2 \sum_{\hbar=0}^{\acute{\tau}-1}(\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2)\right\} = 0, \tag{27}$$

$$\begin{aligned} &\mathscr{E}\left\{\|\breve{z}_e(\acute{\tau})\|^2 - \bar{\chi}^2(\|\varpi(\acute{\tau})\|^2 - \|\zeta(\acute{\tau})N(\acute{\tau})\mathcal{C}(\acute{\tau})e(\acute{\tau})\|^2 - \|\zeta(\acute{\tau})N(\acute{\tau})G(\acute{\tau})v(\acute{\tau})\|^2) + \Upsilon(\acute{\tau}) - \Upsilon(\acute{\tau})\right\} \\ &\quad = \mathscr{E}\left\{-\varpi^{\mathrm{T}}(\acute{\tau})\Omega_{22}(\acute{\tau}+1)\varpi(\acute{\tau}) - e^{\mathrm{T}}(\acute{\tau}+1)\breve{P}(\acute{\tau}+1)e(\acute{\tau}+1)\right\} \end{aligned} \tag{28}$$

and

$$\begin{aligned} &\mathscr{E}\left\{\sum_{\hbar=\acute{\tau}+1}^{\breve{D}} \|\breve{z}_e(\hbar)\|^2 - \bar{\chi}^2 \sum_{\hbar=\acute{\tau}+1}^{\breve{D}} (\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2)\right\} \\ &\quad = \mathscr{E}\left\{e^{\mathrm{T}}(\acute{\tau}+1)\breve{P}(\acute{\tau}+1)e(\acute{\tau}+1) - e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1) - \sum_{\hbar=\acute{\tau}+1}^{\breve{D}}(\varpi(\hbar) - \varpi^*(\hbar))^{\mathrm{T}}\right. \\ &\qquad \left. \times\, \Omega_{22}(\hbar+1)(\varpi(\hbar) - \varpi^*(\hbar))\right\} = \mathscr{E}\{e^{\mathrm{T}}(\acute{\tau}+1)\breve{P}(\acute{\tau}+1)e(\acute{\tau}+1)\}. \end{aligned} \tag{29}$$

From (27)–(29), we derive

$$\bar{W} = -\varpi^{\mathrm{T}}(\acute{\tau})\Omega_{22}(\acute{\tau}+1)\varpi(\acute{\tau}) = -\psi^{\mathrm{T}}(\acute{\tau})\Omega_{22}(\acute{\tau}+1)\psi(\acute{\tau}) = -\varrho(\acute{\tau})\|\psi(\acute{\tau})\|^2 \geqslant 0,$$

which violates the condition $\bar{W} < 0$. Therefore, we conclude that $\Omega_{22}(\hbar+1) > 0$.

Case 2. It remains to testify

$$\Omega_{22}(\hbar+1) > 0 \text{ and } \breve{P}(0) \geqslant \bar{\chi}^2 Q, \quad \forall \hbar \in [0, \breve{D}] \Longrightarrow \bar{W} \geqslant 0. \tag{30}$$

Choose $\varpi(\hbar) \triangleq \varpi^*(\hbar)$. It is shown by (24) that

$$\begin{aligned} \bar{W} &= \mathscr{E}\{e^{\mathrm{T}}(0)\breve{P}(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1)\} + \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\big\{-(\varpi(\hbar) - \varpi^*(\hbar))^{\mathrm{T}}\Omega_{22}(\hbar+1) \\ &\quad \times (\varpi(\hbar) - \varpi^*(\hbar))\big\} - \bar{\chi}^2 e^{\mathrm{T}}(0)Qe(0) = \mathscr{E}\{e^{\mathrm{T}}(0)(\breve{P}(0) - \bar{\chi}^2 Q)e(0)\}. \end{aligned}$$

For $e(0) \neq 0$, it is clear that $\bar{W} \geqslant 0$, which contradicts the condition $\bar{W} < 0$.

Based on Lemmas 2 and 3, we are prepared to develop the algorithm for the design of estimators (5).

Theorem 1. Take the TVCN (1) over the finite horizon $[0, \breve{D}]$ into consideration. For a known DAL $\bar{\chi} > 0$ and a matrix $Q > 0$, the finite-horizon estimators (5) fulfil the performance constraint (11) for all nonzero $\{\varpi(\hbar)\}_{0 \leqslant \hbar \leqslant \breve{D}}$ if Eq. (22) and the following RRDE:

$$\Theta_{11}(\hbar+1) + \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) - \Theta_{12}(\hbar+1)\Theta_{22}^{-1}(\hbar+1)\Theta_{12}^{\mathrm{T}}(\hbar+1) = X(\hbar) \tag{31}$$

hold with a series of solutions $\{\aleph(\hbar)\}_{0 \leqslant \hbar \leqslant \breve{D}}$ $(\aleph = \breve{P}, X, K)$ conforming to

$$\breve{P}(\breve{D}+1) = X(\breve{D}+1) = 0, \tag{32}$$

$$\Theta_{22}(\hbar+1) > 0, \quad \Omega_{22}(\hbar+1) > 0, \quad \breve{P}(0) < \bar{\chi}^2 Q, \tag{33}$$

$$\breve{K}^*(\hbar) = \arg\min_{\breve{K}(\hbar)} \| - \breve{K}(\hbar)\mathcal{C}(\hbar) + \Theta_{22}^{-1}(\hbar+1)\Theta_{12}^{\mathrm{T}}(\hbar+1)\|_F, \tag{34}$$

where other parameters are shown in Lemma 2.

Proof. First, as long as there are $\{\breve{P}(\hbar)\}_{0 \leqslant \hbar \leqslant \breve{D}}$ meeting (22) and (33), it is convenient to obtain from Lemma 3 that the EED (13) fulfills the performance requirement (14). Under such a situation, the worst-case disturbance is able to be represented by $\varpi^*(\hbar) = \Omega_{22}^{-1}(\hbar+1)\Omega_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$. Then, through utilizing the worst-case disturbance, our target is to design the gain matrices $K_\imath(\hbar)$ $(\imath = 1, \ldots, u)$ of the estimators (5). By virtue of the CSM, we derive from Lemma 2 that

$$\begin{aligned}
W_2(\breve{Z}(\hbar), \varpi(\hbar)) =& \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\Big\{ e^{\mathrm{T}}(\hbar)(\Theta_{11}(\hbar+1) + \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) - X(\hbar) - \Theta_{12}(\hbar+1)\Theta_{22}^{-1}(\hbar+1) \\
& \times \Theta_{12}^{\mathrm{T}}(\hbar+1))e(\hbar) + (\breve{Z}(\hbar) - \breve{Z}^*(\hbar))^{\mathrm{T}}\Theta_{22}(\hbar+1)(\breve{Z}(\hbar) - \breve{Z}^*(\hbar))\Big\} \\
& + \mathscr{E}\{e^{\mathrm{T}}(0)X(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)X(\breve{D}+1)e(\breve{D}+1)\} \\
\leqslant & \sum_{\hbar=0}^{\breve{D}} \mathscr{E}\Big\{ e^{\mathrm{T}}(\hbar)(\Theta_{11}(\hbar+1) + \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) - X(\hbar) - \Theta_{12}(\hbar+1)\Theta_{22}^{-1}(\hbar+1) \\
& \times \Theta_{12}^{\mathrm{T}}(\hbar+1))e(\hbar) + \| - \breve{K}(\hbar)\mathcal{C}(\hbar) + \Theta_{22}^{-1}(\hbar+1)\Theta_{12}^{\mathrm{T}}(\hbar+1)\|_F^2 \\
& \times \|\Theta_{22}(\hbar+1)\|_F \|e(\hbar)\|^2 \Big\} + \mathscr{E}\{e^{\mathrm{T}}(0)X(0)e(0) - e^{\mathrm{T}}(\breve{D}+1)X(\breve{D}+1)e(\breve{D}+1)\},
\end{aligned} \tag{35}$$

where $\breve{Z}^*(\hbar) \triangleq -\Theta_{22}^{-1}(\hbar+1)\Theta_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$. Moreover, it is proved that the gain parameters $K_\imath(\hbar)$ $(\imath = 1, \ldots, u)$ fulfill (31) and (34), which finishes the proof.

Remark 3. In this stage, NSCs have been acquired in Theorem 1 for the beingness of estimators (5) by virtue of Lemmas 2 and 3, which ensure the fulfillment of the prescribed performance constraint (11). To be more specific, from one aspect, if there exists a solution to the backward RRDEs (22) and (31) satisfying (33) and (34), then the performance constraint (11) is satisfied. From another aspect, under the condition that the EED (9)-(10) fulfils the performance constraint (11), then the backward RRDEs (22) and (31) have a solution conforming to (33) and (34).

Usually, it is of some difficulties to deal with the optimization problem (34). For enhancing the convenience of realization, the gains $K_\imath(\hbar)$ $(\imath = 1, \ldots, u)$ can be computed through adopting the Moore-Penrose pseudoinverse, and it would be discussed in the theorem below.

Theorem 2. Set the DAL $\bar{\chi} > 0$, the matrix $Q > 0$, the constants $\theta(\hbar) > 0$, and $\epsilon(\hbar) > 0$. The EED (9)-(10) fulfills the H_∞ performance requirement (11) for all nonzero noise sequence $\{\varpi(\hbar)\}_{0 \leqslant \hbar \leqslant \breve{D}}$ as long as there is a set of solutions $\{\breve{P}(\hbar), X(\hbar), K_\imath(\hbar)\}_{0 \leqslant \hbar \leqslant \breve{D}, \imath=1,\ldots,u}$ to the RRDEs below:

$$\Omega_{11}(\hbar+1) + \bar{\Omega}_{12}(\hbar+1)\bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\Omega}_{12}^{\mathrm{T}}(\hbar+1) = \breve{P}(\hbar), \tag{36}$$

$$\bar{\Theta}_{11}(\hbar+1) + \breve{H}^{\mathrm{T}}(\hbar)\breve{H}(\hbar) - \bar{\Theta}_{12}(\hbar+1)\Theta_{22}^{-1}(\hbar+1)\bar{\Theta}_{12}^{\mathrm{T}}(\hbar+1) = X(\hbar) \tag{37}$$

with

$$\breve{P}(\breve{D}+1) = X(\breve{D}+1) = 0, \tag{38}$$

$$\bar{\Omega}_{22}(\hbar+1)>0,\quad \breve{P}(0)<\bar{\chi}^2Q,\quad \Theta_{22}(\hbar+1)>0, \tag{39}$$

$$\breve{K}^*(\hbar)=Z^{\dagger}(\hbar+1)\Phi(\hbar+1)\mathcal{C}^{\dagger}(\hbar), \tag{40}$$

$$Y(\hbar)\leqslant\epsilon(\hbar)I, \tag{41}$$

where

$$\begin{aligned}
&\bar{\mathcal{G}}(\hbar)\triangleq\Big[\breve{E}(\hbar)\ -\theta^{-1}(\hbar)I\ -\zeta^{-1}(\hbar)\bar{\varsigma}\breve{M}(\hbar)\ -\zeta^{-1}(\hbar)\bar{\varsigma}\breve{M}(\hbar)\Big],\ \bar{\mathcal{M}}(\hbar)\triangleq\Big[0\ 0\ \zeta^{-1}(\hbar)\breve{M}(\hbar)\ \zeta^{-1}(\hbar)\breve{M}(\hbar)\Big],\\
&\bar{\Omega}_{12}(\hbar+1)\triangleq\mathcal{A}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\bar{\mathcal{G}}(\hbar),\ \bar{\Lambda}(\hbar+1)\triangleq\bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\Omega}_{12}^{\mathrm{T}}(\hbar+1),\ \bar{S}\triangleq\Big[I\ 0\ 0\ 0\Big],\\
&\bar{\Omega}_{22}(\hbar+1)\triangleq-\bar{\mathcal{G}}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\bar{\mathcal{G}}(\hbar)-\hat{\varsigma}\bar{\mathcal{M}}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1)\bar{\mathcal{M}}(\hbar)+\bar{\chi}^2I-\bar{\chi}^2\zeta^2(\hbar)\bar{S}^{\mathrm{T}}G^{\mathrm{T}}(\hbar)N^{\mathrm{T}}(\hbar)\\
&\qquad\qquad\times N(\hbar)G(\hbar)\bar{S}-\epsilon(\hbar)\bar{S}^{\mathrm{T}}\bar{S},\\
&\bar{\Theta}_{12}(\hbar+1)\triangleq(\breve{\mathcal{A}}(\hbar)+\bar{\mathcal{G}}(\hbar)\bar{\Lambda}(\hbar+1))^{\mathrm{T}}X(\hbar+1),\\
&\bar{\Theta}_{11}(\hbar+1)\triangleq(\breve{\mathcal{A}}(\hbar)+\bar{\mathcal{G}}(\hbar)\bar{\Lambda}(\hbar+1))^{\mathrm{T}}X(\hbar+1)(\breve{\mathcal{A}}(\hbar)+\bar{\mathcal{G}}(\hbar)\bar{\Lambda}(\hbar+1))+\hat{\varsigma}\bar{\Lambda}^{\mathrm{T}}(\hbar+1)\bar{\mathcal{M}}^{\mathrm{T}}(\hbar)\\
&\qquad\qquad\times X(\hbar+1)\bar{\mathcal{M}}(\hbar)\bar{\Lambda}(\hbar+1),\\
&Y(\hbar)\triangleq\bar{\chi}^2\theta^2(\hbar)G^{\mathrm{T}}(\hbar)\breve{K}^{\mathrm{T}}(\hbar)\breve{K}(\hbar)G(\hbar),\\
&Z(\hbar+1)\triangleq-I-\Theta_{22}^{-1}(\hbar+1)X(\hbar+1)\bar{\mathcal{G}}(\hbar)\bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\mathcal{G}}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1),\\
&\Phi(\hbar+1)\triangleq-\Theta_{22}^{-1}(\hbar+1)X(\hbar+1)(I+\bar{\mathcal{G}}(\hbar)\bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\mathcal{G}}^{\mathrm{T}}(\hbar)\breve{P}(\hbar+1))\breve{\mathcal{A}}(\hbar).
\end{aligned}\tag{42}$$

Proof. Let $v(\hbar)\triangleq\theta(\hbar)\breve{K}(\hbar)G(\hbar)w(\hbar)$, where $\theta(\hbar)>0$ is introduced to produce additional degree of freedom in the process of designing the estimators. With the selection of $\breve{w}(\hbar)\triangleq[\,w^{\mathrm{T}}(\hbar)\ v^{\mathrm{T}}(\hbar)\ (\zeta(\hbar)F(\hbar)\times N(\hbar)\mathcal{C}(\hbar)e(\hbar))^{\mathrm{T}}\ (\zeta(\hbar)F(\hbar)N(\hbar)G(\hbar)w(\hbar))^{\mathrm{T}}\,]^{\mathrm{T}}$, Eq. (13) is rewritten in the expression as follows:

$$e(\hbar+1)=\mathcal{A}(\hbar)e(\hbar)+\bar{\mathcal{G}}(\hbar)\breve{w}(\hbar)-\tilde{\varsigma}(\hbar)\bar{\mathcal{M}}(\hbar)\breve{w}(\hbar), \tag{43}$$

$$\breve{z}_e(\hbar)=\breve{H}(\hbar)e(\hbar). \tag{44}$$

Furthermore, regarding Lemma 1, we recognize that Eq. (40) is a solution to the optimization issue below:

$$\min_{\breve{K}(\hbar)}\|Z(\hbar+1)\breve{K}(\hbar)\mathcal{C}(\hbar)-\Phi(\hbar+1)\|_F,$$

which is further described by

$$\min_{\breve{K}(\hbar)}\|-\breve{K}(\hbar)\mathcal{C}(\hbar)+\Theta_{22}^{-1}(\hbar+1)\bar{\Theta}_{12}^{\mathrm{T}}(\hbar+1)\|_F. \tag{45}$$

According to (19) and Theorem 1, by assuming that there is a series of solutions to the RRDEs (36) and (37) with (38)–(41), we have

$$\begin{aligned}
\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\|\breve{z}_e(\hbar)\|^2\right\}=&\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\Big\{e^{\mathrm{T}}(\hbar)(\Omega_{11}(\hbar+1)-\breve{P}(\hbar)+\bar{\Omega}_{12}(\hbar+1)\bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\Omega}_{12}^{\mathrm{T}}(\hbar+1))e(\hbar)\\
&-(\breve{w}(\hbar)-\breve{w}^*(\hbar))^{\mathrm{T}}\bar{\Omega}_{22}(\hbar+1)(\breve{w}(\hbar)-\breve{w}^*(\hbar))\Big\}-\mathscr{E}\big\{e^{\mathrm{T}}(\breve{D}+1)\breve{P}(\breve{D}+1)e(\breve{D}+1)\\
&-e^{\mathrm{T}}(0)\breve{P}(0)e(0)\big\}-\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\epsilon(\hbar)(\bar{S}\breve{w}(\hbar))^{\mathrm{T}}(\bar{S}\breve{w}(\hbar))\right\}\\
&+\bar{\chi}^2\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\|\breve{w}(\hbar)\|^2-\|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2-\|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2\right\}\\
<&\bar{\chi}^2e^{\mathrm{T}}(0)Qe(0)+\bar{\chi}^2\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\{\|\varpi(\hbar)\|^2-\|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2-\|\zeta(\hbar)N(\hbar)\\
&\times G(\hbar)w(\hbar)\|^2\}+\sum_{\hbar=0}^{\breve{D}}\mathscr{E}\left\{\varpi^{\mathrm{T}}(\hbar)S^{\mathrm{T}}(Y(\hbar)-\epsilon(\hbar)I)S\varpi(\hbar)\right\},
\end{aligned}\tag{46}$$

where $\breve{w}(\hbar) \triangleq \breve{w}^*(\hbar) = \bar{\Omega}_{22}^{-1}(\hbar+1)\bar{\Omega}_{12}^{\mathrm{T}}(\hbar+1)e(\hbar)$.

By employing (41), it is shown based on (46) that

$$\sum_{\hbar=0}^{\breve{D}} \mathscr{E}\left\{\|\breve{z}_e(\hbar)\|^2\right\} < \bar{\chi}^2 e^{\mathrm{T}}(0)Qe(0) + \bar{\chi}^2 \sum_{\hbar=0}^{\breve{D}} (\|\varpi(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)\mathcal{C}(\hbar)e(\hbar)\|^2 - \|\zeta(\hbar)N(\hbar)G(\hbar)w(\hbar)\|^2).$$

We make a conclusion that the estimators (5) ensure that system (9)-(10) fulfills the performance requirement (11).

Based on Theorem 2, the time-varying state estimator design (TVSED) algorithm is developed which is shown in Algorithm 1.

Algorithm 1 TVSED algorithm

1: Choose the DAL $\bar{\chi}$ and the matrix $Q > 0$, and set $\hbar = \breve{D}$.
2: Calculate $\Theta_{22}(\hbar+1)$ and $\bar{\Omega}_{22}(\hbar+1)$ with given $X(\hbar+1)$ and $\breve{P}(\hbar+1)$ according to (17) and (42), respectively. Besides, the gain parameters $K_\imath(\hbar)$ ($\imath = 1, \ldots, u$) of the estimators (5) are computed in accordance with (40).
3: If $\bar{\Omega}_{22}(\hbar+1) \neq 0$ and $\Theta_{22}(\hbar+1) \neq 0$, then compute (36) and (37) to acquire $\breve{P}(\hbar)$ and $X(\hbar)$, respectively, and continue to Step 4; otherwise this algorithm is infeasible and quit.
4: In case $\hbar \neq 0$, $\bar{\Omega}_{22}(\hbar+1) > 0$ and $\Theta_{22}(\hbar+1) > 0$, set $\hbar = \hbar - 1$ and go back to Step 2; else continue to Step 5.
5: If $\breve{P}(0) \geqslant \bar{\chi}^2 Q$ or $\bar{\Omega}_{22}(\hbar+1) \leqslant 0$ or $\Theta_{22}(\hbar+1) \leqslant 0$, this algorithm is infeasible and quit.

Remark 4. Via the CSM, the finite-horizon resilient H_∞ estimation issue has already been solved with respect to TVCN (1). The occurrence of gain variations may be random which is represented by the probability distribution of a Gaussian random variable. The results of Theorem 2 contain the total important factors of the complex networks such as time-varying parameters and the probabilistic information of the randomly occurring gain variations. NSCs are put forward to guarantee that the H_∞ performance demand (11) is fulfilled concerning the overall EED (9)-(10), and the estimator gains are expressed in (40) via utilizing the Moore-Penrose pseudoinverse. To be specific, Theorem 2 possesses little conservatism due to the fact that the obtained conditions are both sufficient and necessary.

Remark 5. Until now, we have finished discussing the finite-horizon H_∞ performance of the resilient state estimators for TVCNs involving integral measurements of partial nodes, and designing gains of such estimators. Under the existing research framework of SE for TVCNs, the innovations of this paper are stressed below: (1) the finite-horizon resilient SE issue is new, which is based on integral measurements from a portion of network nodes; and (2) the developed design algorithm of time-varying state estimators is new as it is composed of NSCs which guarantee that the finite-horizon resilient estimators exist. To testify the correctness of the design algorithm of the finite-horizon/time-varying state estimators, a specific simulation is done in Section 4.

4 Simulation results

Consider a 6-node TVCN (1) over the finite horizon $[0, 60]$, where the following parameters are used:

$$\wp = 2, \quad \bar{\chi} = 1, \quad \zeta(\hbar) = 1, \quad \bar{\varsigma} = 0.5, \quad \hat{\varsigma} = 0.02, \quad Q = 3I,$$

$$\Gamma = \mathrm{diag}\{0.53, 0.55\}, \quad E_\imath(\hbar) = \begin{bmatrix} 0.2\sin(3\hbar) & 0.5 \end{bmatrix}^{\mathrm{T}},$$

$$B = \begin{bmatrix} -0.4 & 0.15 & 0.08 & 0.12 & 0.03 & 0.02 \\ 0.15 & -0.4 & 0.02 & 0.07 & 0.08 & 0.08 \\ 0.08 & 0.02 & -0.22 & 0.04 & 0.04 & 0.04 \\ 0.12 & 0.07 & 0.04 & -0.3 & 0.04 & 0.03 \\ 0.03 & 0.08 & 0.04 & 0.04 & -0.28 & 0.09 \\ 0.02 & 0.08 & 0.04 & 0.03 & 0.09 & -0.26 \end{bmatrix}, \quad A_\imath(\hbar) = \begin{bmatrix} -0.02\sin(5\hbar) & 0.02 \\ 0.04 & 0.04 \end{bmatrix}, \quad H_\imath(\hbar) = \begin{bmatrix} 0.3 & 0.1 \\ 0.5 & 0.2 \end{bmatrix},$$

$$M_\imath(\hbar) = \begin{bmatrix} I \\ I \\ I \end{bmatrix}, \quad N_\imath(\hbar) = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, \quad \imath = 1, 2, \ldots, 6,$$

Table 1 Gain parameters of finite-horizon partial-nodes-based state estimators

	$\hbar=0$	$\hbar=1$	$\dots$	$\hbar=59$
$K_1(\hbar)$	$\begin{bmatrix} -0.1558 & -0.0167 \\ -0.1542 & 0.15 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \end{bmatrix}$	$\begin{bmatrix} -0.1997 & 0.0312 \\ -0.2201 & 0.2028 \\ 1.1899 & -0.2853 \\ 1.4874 & -1.1899 \\ 1.1899 & -0.2853 \\ 1.4874 & -1.1899 \end{bmatrix}$	$\dots$	$\begin{bmatrix} -0.5796 & -0.8951 \\ -0.2214 & -0.3416 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
$K_2(\hbar)$	$\begin{bmatrix} -0.1558 & -0.0167 \\ -0.1542 & 0.15 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \end{bmatrix}$	$\begin{bmatrix} -0.1997 & 0.0312 \\ -0.2201 & 0.2028 \\ 1.1899 & -0.2853 \\ 1.4874 & -1.1899 \\ 1.1899 & -0.2853 \\ 1.4874 & -1.1899 \end{bmatrix}$	$\dots$	$\begin{bmatrix} -0.5788 & -0.894 \\ -0.2210 & -0.3412 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
$K_3(\hbar)$	$\begin{bmatrix} -0.0763 & -0.0167 \\ -0.0510 & 0.0675 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \\ 0.8333 & 0 \\ 1.0417 & -0.8333 \end{bmatrix}$	$\begin{bmatrix} -0.0862 & 0.0040 \\ -0.0728 & 0.0849 \\ 1.1899 & -0.2853 \\ 1.4873 & -1.1899 \\ 1.19 & -0.2853 \\ 1.4876 & -1.19 \end{bmatrix}$	$\dots$	$\begin{bmatrix} -0.5928 & -0.9466 \\ -0.2263 & -0.3614 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$C_j(\hbar)=\begin{bmatrix}0.4 & 0.1\sin(5\hbar)\\ 0.5 & -0.4\end{bmatrix},\quad G_j(\hbar)=\begin{bmatrix}0.2\\ 0.5\end{bmatrix},\quad j=1,2,3.$$

By employing the TVSED algorithm in Algorithm 1 via the Matlab software, we obtain the estimator gains with $\theta(\hbar)=2.5$ and $\epsilon(\hbar)=0.25$, which are listed in Table 1. Choose the initial conditions $x_\imath(-2)=[0.4\ -0.1]^{\rm T}$, $x_\imath(-1)=[0.2\ -0.1]^{\rm T}$, $x_\imath(0)=[0.3\ -0.2]^{\rm T}$ and $\hat{x}_\imath(0)=[0\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2]^{\rm T}$ ($\imath=1,2,\dots,6$). Select the noise signal and the uncertain matrix in (6) as $w(\hbar)=0.4\cos(3\hbar){\rm e}^{-0.01\hbar}$ and $F_j(\hbar)={\rm diag}\{\cos(3\hbar),\sin(2\hbar)\}$ ($j=1,2,3$), respectively. Emulation curves are shown in Figures 1–8. Figure 1 demonstrates the state estimation error $e_\imath(\hbar)$ ($\imath=1,2,\dots,6$), where $e_{\imath\Im}(\hbar)$ ($\Im=1,2$) is the $\Im$th element of $e_\imath(\hbar)$. Figure 2 draws the estimation errors $\check{z}_{e\imath}(\hbar)$ of all the nodes. On the basis of the performance requirement (11), Figure 3 depicts the estimation performance curve, through which it is seen that

$$\phi(\hbar)=\frac{\sum_{\varsigma=0}^{\hbar}\mathscr{E}\left\{\|\check{z}_e(\varsigma)\|^2\right\}}{\sum_{\varsigma=0}^{\hbar}(\|\varpi(\varsigma)\|^2-\|\zeta(\varsigma)N(\varsigma)\mathcal{C}(\varsigma)e(\varsigma)\|^2-\|\zeta(\varsigma)N(\varsigma)G(\varsigma)w(\varsigma)\|^2)+e^{\rm T}(0)Qe(0)}<\bar{\chi}^2$$

for $\hbar=0,1,\dots,\check{D}$; i.e., the developed estimation method is able to satisfy the performance requirement (11). Figure 4 (or 6) and Figure 5 (or 7) show the estimation errors and performance with different values of $\bar{\varsigma}$ (or $\Delta K_\imath(\hbar)$), which illustrate that the bigger the value of $\bar{\varsigma}$ (the intensity of $\Delta K_\imath(\hbar)$), the bigger the resulted estimation error and the worse the estimation performance. Figure 8 represents the estimation performance of general and resilient estimators, and it is seen that the performance of the resilient estimators is better than that of the general estimators.

5 Conclusion

This paper has addressed a finite-horizon partial-nodes-based SE issue for TVCNs with integral measurements as well as randomly occurring gain variations. Time-varying parameters have been involved in the model to depict the changing characteristics of complex networks with time. Measurements from only partial nodes have been taken into account to reflect the presence of unmeasurable network nodes under specific working situations. Integral measurements have been considered to reflect the time delays in the measurement signals under practical circumstances. A Gaussian stochastic variable and the norm-bounded uncertainty have been used to represent randomly occurring gain variations. NSCs were found to devise the resilient H_∞ state estimators by employing the stochastic analysis skill as well as the CSM. The estimator parameters have been obtained by recursively solving certain coupled backward

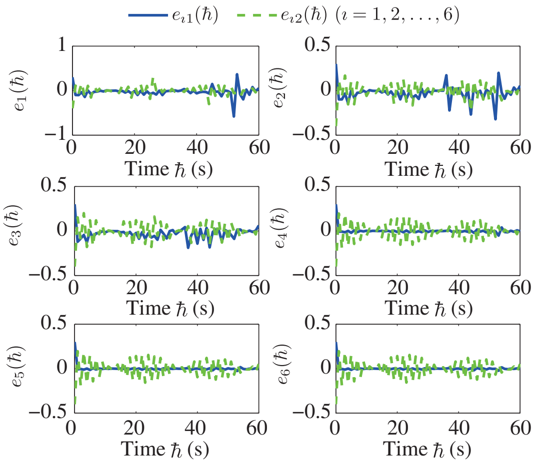

Figure 1 (Color online) The state estimation error $e_\imath(\hbar)$ $(\imath = 1, 2, \ldots, 6)$.

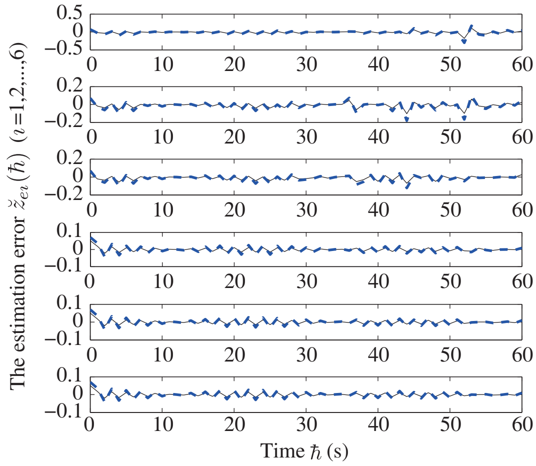

Figure 2 (Color online) The estimation error $\breve{z}_{e\imath}(\hbar)$ $(\imath = 1, 2, \ldots, 6)$.

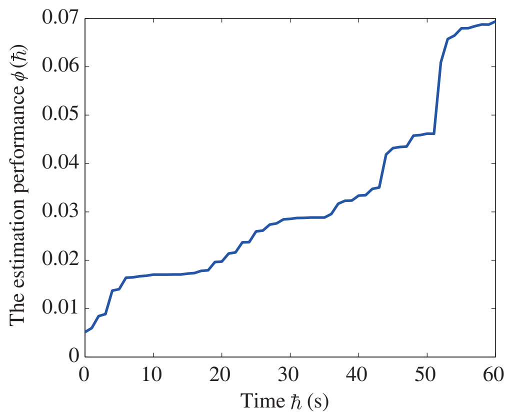

Figure 3 (Color online) The estimation performance $\phi(\hbar)$.

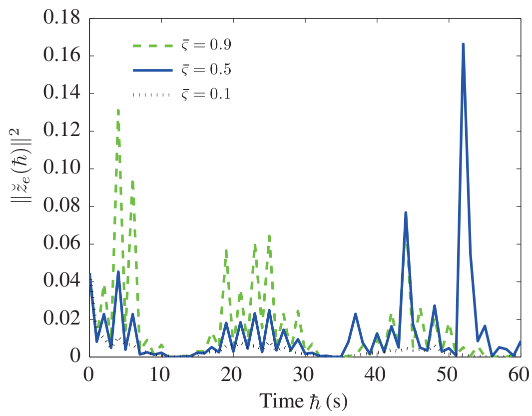

Figure 4 (Color online) The norm of estimation error with different $\bar{\varsigma}$.

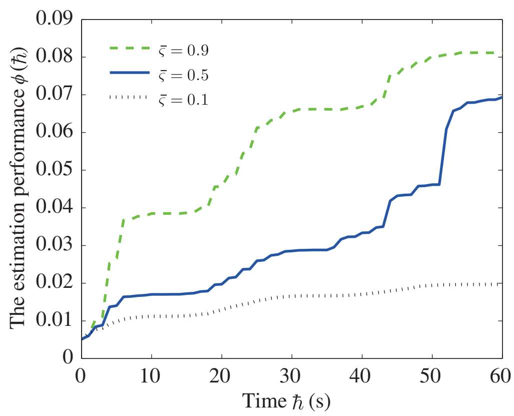

Figure 5 (Color online) The estimation performance with different $\bar{\varsigma}$.

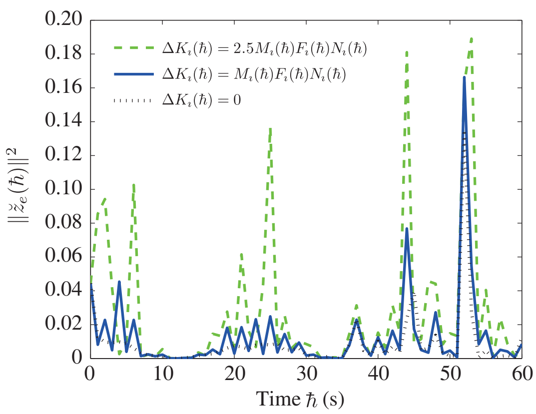

Figure 6 (Color online) The norm of estimation error with different $\Delta K_\imath(\hbar)$.

RRDEs and computing the Moore-Penrose pseudoinverse of the corresponding matrices. The results of the illustrative emulation example were used to demonstrate the usefulness of the developed method. Future research endeavors will address the estimation for systems with communication protocols [30,31], Markovian jumping parameters [32], time delays [33–35], nonlinearities [36], or stochastic coupling [37],

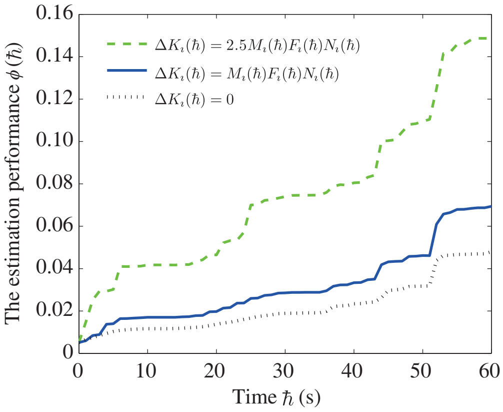

Figure 7 (Color online) The estimation performance with different $\Delta K_\imath(\hbar)$.

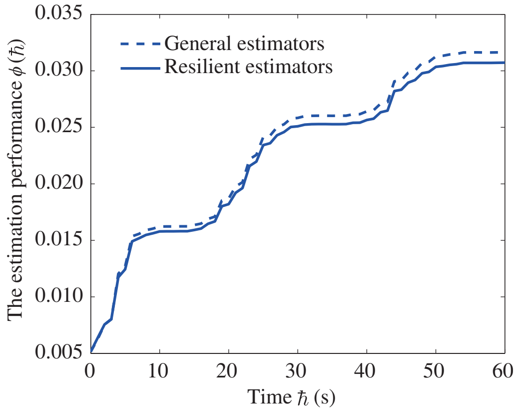

Figure 8 (Color online) The estimation performance of general and resilient estimators.

the synchronization [38] and the pinning control [39] issues.

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