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Adaptive event-triggered state estimation for large-scale systems subject to deception attacks

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Abstract This paper addresses state estimation issues of large-scale systems with measurements subject to deception attacks, where the communication topology among sub-estimators is the same as the physical coupling structure of the subsystems. In consideration of the limited channel bandwidth, a novel adaptive event-triggered scheme is proposed for governing the data transmission among sub-estimators. With the help of Lyapunov analysis approaches, sufficient conditions are derived to ensure the input-to-state stability of the dynamics of estimation errors. Meanwhile, the bound of the estimation errors is obtained in the mean-square sense. The desired estimator parameters are presented in an analytical form dependent on the solution of a set of matrix inequalities. The developed scheme is related to the local information of the subsystems and thus satisfies the requirement of scalability. Finally, a simulation example of power systems is given to reveal the usefulness and effectiveness of the developed design scheme.

Keywords large-scale discrete-time systems, adaptive event-triggering communication, deception attacks, input-to-state stability, power systems

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1 Introduction

Large-scale systems are widely presented in engineering practice, involving power distribution systems, road traffic networks, and sensor/actuator networks [1]. State estimation is a fundamental task in executing control and monitoring to capture the dynamic behavior of these systems [2–4]. Generally speaking, such behavior can be predicted from that of the individual subsystems and their interconnections. However, reducing the calculation complexity and handling the coupling among subsystems remain challenging. Typical techniques used to analyze large-scale systems include vector dissipativity approaches [5], approaches based on the small-gain theorem [6], and other decoupling approaches. In recent years, many valuable studies have been published (see [7–10] for more details). For instance, a distributed filtering scheme was proposed by compensating for the lost information, which comes from the inherent multi-rate nature [7]. The issue of non-fragile H_∞ filtering was discussed in [8] for large-scale power systems measured by a sensor network, where the system dynamics was modeled by Takagi-Sugeno fuzzy models. Recently, an interesting scheme of local-condition-based consensus filtering over sensor networks was developed in [11] via vector dissipativity theory.

In most existing studies on the state estimation of large-scale systems, a sub-estimator for each subsystem needs to communicate with all its neighbors at each instant [12–14], which will consume large amounts of communication resources. As such, the means of improving transmission efficiency while guaranteeing the expected estimation performance is drawing considerable research interest [15]. As one of the most effective approaches, commutation protocols can govern the communication sequences,

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thereby realizing the constraints of both the limited bandwidth and the finite energy source [16,17]. Representative protocols include round-robin protocols [18], stochastic communication protocols described by Bernoulli sequences or Markov chains, try-once-discard protocols with/without weight matrices [19], and event-triggered protocols with various event generators [20, 21]. An essential feature of event-triggered protocols is that information is prioritized for transmission only when a certain event is activated. In comparison with other protocols, such a protocol can be easily designed and executed in application layers. Many interesting results combined with effective analysis approaches can be easily found in the literature, such as [22, 23] and the references therein. For instance, an event-based consensus protocol was proposed in [22] for discrete-time multi-agent systems with state-dependent stochastic noises, and a scalable scheme was provided based on two eigenvalues of a Laplace matrix. Furthermore, a decentralized event-triggered communication scheme was adopted in [23] for large-scale systems where the synchronization assumption of communication is not necessary.

A constant threshold is commonly adopted in existing event-triggered schemes. Such a protocol cannot dynamically adjust the communication frequency to improve system performance. As such, adaptive schemes have been receiving increasing research concern (see [24–26] for more details). For instance, an event-governed communication scheme involving an adaptive threshold was developed in [25] for a class of nonlinear systems described as a T-S model; under this model, an interesting approach of asynchronous premise reconstruction is adopted to relax the assumption of the synchronization requirement about the premises of the plant rules and the control ones. For the scheme adopted in [24], the adaptive threshold depends on the dynamic error of the system. Recently, an H_∞ fuzzy filtering issue with an adaptive event-triggered mechanism was systematically investigated in [26] for interval time-varying delayed systems characterized by an IT2 fuzzy model with asynchronously and imperfectly matched membership functions. The introduced adaptive rule is usually monotonously decreased and therefore the event-triggering threshold will become smaller, and the information transmission will become more frequent. As such, the frequency of information transmission cannot be reduced when the system dynamics tends to be stable. Nonetheless, an adaptive scheme should be designed for removing the monotonicity in existing schemes. Furthermore, considering the inherent physical coupling, it is also challenging to develop a scalable algorithm for large-scale networked systems. Therefore, one of the main motivations of the present paper is to propose a new adaptive rule to serve state estimation issues of large-scale networked systems.

Cyber vulnerabilities gradually emerge when various networks are introduced into practical systems for communication [27, 28]. Typical cyber-attacks usually include deception, denial-of-service (DoS), and replay attacks [29–31], all of which have been receiving extensive research attention from the control and computer communities. Among these attack forms, deception is evidently more general in terms of mathematical model and technical implementation (by affecting the integrity of data). Thus, security estimation has been drawing particular research interest as a result of the inevitable degradation of the expected estimation performance [29]. For instance, a state estimation algorithm with a distributed form was proposed in [32] for smart grids; a hybrid cyber-attack model was developed by integrating the characteristics of DoS and deception attacks. Security threat assessment was discussed in [33] for networked systems encountering deception attacks; the considered scheme consists of a proportional-integral controller in the regulatory layer and a model-based detector in the supervisory layer. However, the state estimation of large-scale networked systems subject to deception attacks remains an issue owing mainly to the intrinsic complexity of the systems themselves. Motivated by the above analysis, in this paper, we endeavor to investigate the adaptive event-triggered state estimation issue for discrete-time large-scale systems under deception attacks.

According to the summary above, the focus of this paper is on developing a novel scalable scheme for solving the security state estimation problem with an adaptive event-triggered mechanism. The main contributions of this paper can be highlighted by the following aspects. (a) An event-triggered scheme with a novel adaptive mechanism is proposed for governing the information transmission among sub-estimators coupled according to the inherent physical structure of large-scale systems subject to deception attacks. (b) Some sufficient conditions combined with the bound of estimation errors are obtained to check the input-to-state stability of estimation error dynamics in the mean-square sense. (c) A novel scalable approach of the desired gain design is developed by virtue of the matrix-inequality-based decoupling technique. (d) A simulation example on power systems is employed to verify the usefulness and effectiveness of the developed scheme.

Notation. In addition to standard notations, the following symbols are adopted in this paper. The

transpose and inverse of the matrix A are denoted as A^{T} and A^{-1}, respectively. I and $\mathbf{0}$ denote an identity matrix and a zero matrix with appropriate dimensions, respectively. The function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is deemed to be of class $\mathscr{K}$ if it is a continuous strictly increasing function with $\varphi(\mathbf{0}) = \mathbf{0}$. Furthermore, the function $\varphi : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is considered to be of class $\mathscr{K}$ if the mapping $\varphi(a, b)$ is of class $\mathscr{K}$ for each fixed a and is decreasing to zero as $b \to \infty$ for each fixed b.

2 Problem formulation and preliminaries

2.1 System model

Consider a discrete-time large-scale system consisting of N coupled subsystems. The coupled structure is characterized by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of finite subsystems, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges standing for the inherently physical connection. Let $\mathbb{S}_i = \{j : (j, i) \in \mathcal{E}\}$ denote the set of neighbors of subsystem $i \in \mathcal{V}$. Furthermore, let m_i denote the number of neighboring subsystems for the subsystem i.

The discrete-time dynamics of each subsystem $i \in \mathcal{V}$ can be described by the following equation:

$$x_i(k+1) = A_i x_i(k) + \sum_{j \in \mathbb{S}_i} A_{ij} x_j(k) + B_i w_i(k), \tag{1}$$

where $x_i(k) \in \mathbb{R}^{n_x}$ is the state vector of the i-th subsystem and $w_i(k) \in \mathbb{R}^{n_w}$ denotes the process noise obeying the Gaussian distribution $N(0, Q_{i,k})$. A_i, A_{ij} and B_i are known matrices of suitable dimensions.

The measurements of the i-th subsystem can be given as follows:

$$y_i(k) = C_i x_i(k) + D_i v_i(k), \tag{2}$$

where $y_i(k) \in \mathbb{R}^{n_y}$ is the measurement output at the instant k, and $v_i(k) \in \mathbb{R}^{n_v}$ means the measurement noise obeying the Gaussian distribution $N(0, R_{i,k})$. C_i and D_i are known system matrices of suitable dimensions. $x_i(0)$, $\omega_i(k)$ and $v_i(k)$ are assumed to be independent and identically distributed sequences.

In the ideal circumstance, the estimator of the i-th subsystem is constructed as

$$\hat{x}_i(k+1) = A_i \hat{x}_i(k) + \sum_{j \in \mathbb{S}_i} A_{ij} \hat{x}_j(k) + K_i(y_i(k) - C_i \hat{x}_i(k)). \tag{3}$$

Obviously, there are N coupled sub-estimators, which are coupled via the same structure with (1) in order to realize the unbiasedness of the state estimation.

Note that the sensor measurements could be insecure owing to missing adequate security protection, which leads to hijacked sensors, cache pollution and so forth. In other words, the sensors could be subject to malicious attacks such that the desired estimation performance is degraded.

2.2 Deception attacks

As mentioned above, attackers can launch an attack into the measurement device to give rise to incorrect measurements. Specifically, the attacker can hijack sensors or inject malicious data in the cache of measurement devices to change the real information. This kind of attacks are usually named as deception attacks, the implementation of which could possess the random nature owing to network conditions or the deployment of defense softwares. In this paper, a Bernoulli process is utilized to describe such a nature and therefore the actual measurement outputs $\tilde{y}_i(k)$ are given by

$$\tilde{y}_i(k) = y_i(k) + \beta_i(k) r_i(k), \tag{4}$$

where the random variable $\beta_i(k)$ complies with the Bernoulli distribution and takes the value 0 or 1 with the probabilities: $\mathrm{Prob}\{\beta_i(k) = 0\} = 1 - \bar{\beta}_i$ and $\mathrm{Prob}\{\beta_i(k) = 1\} = \bar{\beta}_i$. Here, $\bar{\beta}_i \in [0, 1)$ is a known constant that represents the success rate of deception attacks. Furthermore, the signals injected into sensors can be generated by the form $r_i(k) = -y_i(k) + \xi_i(k)$ where $\xi_i(k)$ is a bounded signal satisfying $\|\xi_i(k)\|^2 \leqslant \xi$ with a given scalar $\xi > 0$.

According to the above description, the actual measurement outputs are rewritten as follows:

$$\tilde{y}_i(k) = (1 - \beta_i(k)) y_i(k) + \beta_i(k) \xi_i(k). \tag{5}$$

2.3 Adaptive event-triggering mechanism

For the state estimation issues of large-scale systems, the estimated states of sub-estimators have to be transmitted to their coupled neighbors. Thus, the communication burden is greatly increased if data transmission is executed at each instant. In this paper, an adaptive event-triggered protocol is developed to relieve such a burden. Under this protocol, the event-triggering function $\mathcal{Q}_i(\cdot,\cdot) : \mathbb{R}^{n_f} \times \mathbb{R} \longrightarrow \mathbb{R}$ is defined as follows:

$$\mathcal{Q}_i(f_i(k), \delta_i(k)) = f_i^{\mathrm{T}}(k)\Psi_i f_i(k) - \delta_i(k) \tag{6}$$

with $f_i(k) = \hat{x}_i(k_d^i) - \hat{x}_i(k)$ where k_d^i is the last transmission instant of sub-estimator i, Ψ_i is a predetermined positive definite weight matrix, and $\delta_i(k)$ is a time-varying positive scalar to be designed. In light of such a function, the data exchange is triggered when the following event-triggering condition is satisfied:

$$f_i^{\mathrm{T}}(k)\Psi_i f_i(k) > \delta_i(k). \tag{7}$$

Hence, the sequence of event-triggered instants $0 \leqslant k_0^i < k_1^i < \cdots < k_d^i < \cdots$ can be determined recursively by

$$k_{d+1}^i = \min\left\{k \mid k > k_d^i, \mathcal{Q}_i(f_i(k), \delta_i(k)) > 0\right\}. \tag{8}$$

In order to realize the adaptive regulation, an adaptive rule on the scalar $\delta_i(k)$ is designed as follows:

$$\delta_i(k+1) = \min\left\{\max\left\{\delta_m,\ \delta_i(k)\left(1 - \frac{2a}{\pi}\arctan\left[\|f_i(k)\| - b_i\right]\right)\right\}, \delta_M\right\}, \tag{9}$$

where "arctan(·)" is the invert tangent function, δ_M and δ_m (satisfying $\delta_m \leqslant \delta_M$) are two predetermined scalars, and $0 < a < 1$ and $b_i > 0$ are two given constants to adjust the output of arctan(·). Furthermore, let $\delta_i(0) = \delta_m$.

According to the condition (9), we can easily access to the following lemma.

Lemma 1. For the adaptive event-triggering condition (8) with the given initial condition $\delta_i(0) = \delta_m > 0$, the scalar $\delta_i(k)$ satisfies $\delta_m \leqslant \delta_i(k) \leqslant \delta_M$ for all $k \in \mathbb{N}$, and the condition $f_i^{\mathrm{T}}(k)\Psi_i f_i(k) \leqslant \delta_M$ always holds too.

Remark 1. Notice that the function arctan(·) in (9) has the lower and upper bounds, that is, $\arctan(\cdot) \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Combined with the adjustable parameters a and b, the above property is used in this paper to adaptively adjust the event-triggering threshold $\delta_i(k)$. Specifically, when $\|f_i(k)\| > b_i$, one can derive that $0 < 1 - \frac{2a}{\pi}\arctan[\|f_i(k)\| - b_i] < 1$ which results in $\delta_i(k+1) < \delta_i(k)$. In this case, the event-based communication (8) uses a smaller $\delta_i(k+1)$ to set a faster communication frequency to reduce the influence of errors $f_i(k)$. In reverse, when $\|f_i(k)\| \leqslant b_i$, one has $\delta_i(k+1) \geqslant \delta_i(k)$ which means that a bigger $\delta_i(k+1)$ is adopted to realize a lower communication frequency for saving more communication bandwidth. Furthermore, the proposed rule can be degenerated into some familiar communication schemes via setting different δ_m and δ_M. Specifically, the adaptive event-triggering mechanism is degenerated into the one with a fixed threshold in [34] by selecting $\delta_M = \delta_m > 0$, and into the periodic communication mechanism by selecting $\delta_M = \delta_m = 0$. Obviously, the communication rate of our scheme can achieve a tradeoff between ones under the periodic communication mechanisms and the event-triggering mechanism with a fixed threshold.

2.4 Design objective of the considered state estimation

According to the above addressed scenario, the actual estimator with deception attacks (4) and the event-triggered protocol (6) with (7) is as follows:

$$\begin{aligned}\hat{x}_i(k+1) &= A_i\hat{x}_i(k) + \sum_{j\in\mathbb{S}_i} A_{ij}\hat{x}_j(k_d^j) + K_i(\tilde{y}_i(k) - C_i\hat{x}_i(k)) \\ &= A_i\hat{x}_i(k) + \sum_{j\in\mathbb{S}_i} A_{ij}\hat{x}_j(k) + \sum_{j\in\mathbb{S}_i} A_{ij}(\hat{x}_j(k_d^j) - \hat{x}_j(k)) + K_i(\tilde{y}_i(k) - C_i\hat{x}_i(k)) \\ &= A_i\hat{x}_i(k) + \sum_{j\in\mathbb{S}_i} A_{ij}\hat{x}_j(k) + \sum_{j\in\mathbb{S}_i} A_{ij}f_j(k) + K_i(\tilde{y}_i(k) - C_i\hat{x}_i(k)).\end{aligned} \tag{10}$$

In comparison with the centralized estimator, the main challenges lie in the coupling term describing the internal physical structure and the sensor measurements subject to cyber-attacks.

Let the local estimation error $e_i(k) = x_i(k) - \hat{x}_i(k)$. The dynamics of estimation errors of the i-th sub-estimator can be obtained by (1) and (10) as follows:

$$\begin{aligned} e_i(k+1) = {} & (A_i - K_iC_i)e_i(k) + \sum_{j\in\mathbb{S}_i} A_{ij}e_j(k) + \sum_{j\in\mathbb{S}_i} A_{ij}f_j(k) + \beta_i(k)K_iC_ix_i(k) \\ & + B_i\omega_i(k) - (1-\beta_i(k))K_iD_iv_i(k) - \beta_i(k)K_i\xi_i(k). \end{aligned} \tag{11}$$

Let $\eta_i(k) = [\, x_i^{\mathrm{T}}(k)\ e_i^{\mathrm{T}}(k)\,]^{\mathrm{T}}$ for the convenience of analysis. It follows from (1) and (11) that

$$\begin{aligned} \eta_i(k+1) = {} & \bar{A}_i\eta_i(k) + (\beta_i(k) - \bar{\beta}_i)\bar{B}_i\eta_i(k) + \sum_{j\in\mathbb{S}_i} \bar{A}_{ij}\eta_j(k) + \sum_{j\in\mathbb{S}_i} H_1A_{ij}f_j(k) \\ & - (1-\beta_i(k))H_1K_iD_iv_i(k) - \beta_i(k)H_1K_i\xi_i(k) + H_2B_i\omega_i(k), \end{aligned} \tag{12}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & \mathbf{0} \\ \bar{\beta}_iK_iC_i & A_i - K_iC_i \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ K_iC_i & \mathbf{0} \end{bmatrix},$$
$$H_1 = \begin{bmatrix} \mathbf{0} \\ I \end{bmatrix}, \qquad H_2 = \begin{bmatrix} I \\ I \end{bmatrix}, \quad \bar{A}_{ij} = \begin{bmatrix} A_{ij} & \mathbf{0} \\ \mathbf{0} & A_{ij} \end{bmatrix}.$$

Definition 1. The system (12) is said to be input-to-state stable in the mean-square sense if there exist a function $\varphi_1 \in \mathscr{K}$ and a positive scalar φ_2 dependent on $\sum_{i=1}^{N} \|\xi_i(k)\|_\infty$ such that

$$\mathrm{E}\left\{\sum_{i=1}^{N} \|\eta_i(k)\|\right\} \leqslant \varphi_1\left(k, \mathrm{E}\left\{\sum_{i=1}^{N} \|\eta_i(0)\|\right\}\right) + \varphi_2, \tag{13}$$

where $\|\xi_i(k)\|_\infty = \sup_k \{\|\xi_i(k)\|\}$.

The objective of this paper is to design a set of estimators (10) for the discrete-time large-scale systems (1) with deception attacks and event-triggered communication protocols. To be specific, we focus on determining the estimator gains K_i such that the dynamics of estimation errors (12) is input-to-state stable in the mean-square sense.

3 Main results

In this section, a sufficient condition for input-to-state stability in the mean-square sense of the system (11) is first established via Lyapunov analysis approaches and then utilized to realize the design of desired gains.

Theorem 1. Consider the large-scale discrete-time system (1) with measurement outputs (4) subject to deception attacks. For given positive scalars δ_m, δ_M, $\bar{\beta}_i$ $(i \in \mathcal{V})$, a, b_i, $\varepsilon_s > 0$ $(s = 1, 2, \ldots, 7)$, $0 < \mu < 1$ as well as matrices K_i, the estimation error dynamics (12) with the adaptive event-triggered transmission scheme (6)–(9) is input-to-state stable in the mean-square sense if there exist positive-definite matrices P_i and positive scalars λ_i $(i \in \mathcal{V})$ such that the following inequalities:

$$\Phi_{1i} + \Phi_{2i} + (\mu - 1)P_i < 0, \tag{14}$$

$$A_{ij}^{\mathrm{T}}H_1^{\mathrm{T}}P_iH_1A_{ij} \ < \Psi_j, \tag{15}$$

$$\Phi_{4i} < \lambda_iI \tag{16}$$

hold for all $i, j \in \mathcal{V}$ where

$$\begin{aligned} & \sigma_{1i} = 1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3\bar{\beta}_i, \quad \sigma_{2i} = \bar{\beta}_i(1 - \bar{\beta}_i)(1 + \varepsilon_4), \\ & \sigma_{3i} = 2(1 + \varepsilon_1^{-1} + \varepsilon_5 + \varepsilon_6\bar{\beta}_i), \\ & \sigma_{5i} = \bar{\beta}_i\big(1 + \varepsilon_3^{-1} + \varepsilon_4^{-1}(1 - \bar{\beta}_i) + \varepsilon_6^{-1} + \varepsilon_7^{-1}\big), \\ & \Phi_{1i} = \sigma_{1i}\bar{A}_i^{\mathrm{T}}P_i\bar{A}_i + \sigma_{2i}\bar{B}_i^{\mathrm{T}}P_i\bar{B}_i, \end{aligned}$$

$$\Phi_{2i} = \sigma_{3i} m_j \sum_{j\in\mathbb{S}_i} \bar{A}_{ji}^{\mathrm{T}} P_i \bar{A}_{ji}, \quad \Phi_{4i} = \sigma_{5i} K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i,$$
$$\Phi_{3i} = (1-\bar{\beta}_i) D_i^{\mathrm{T}} K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i D_i Q_{i,k} + B_i^{\mathrm{T}} H_2^{\mathrm{T}} P_i H_2 B_i R_{i,k}.$$

Furthermore, the upper bound of estimation error is

$$\frac{\theta}{\mu \min_{i\in\mathbb{N}} \lambda_{\min}(P_i)} \tag{17}$$

with $\sigma_{4i} = m_i(1+\varepsilon_2^{-1}+\varepsilon_5^{-1}+\varepsilon_7\bar{\beta}_i)$ and $\theta = \sum_{i=1}^N \mathrm{tr}\big(\Phi_{3i}\big) + \sum_{i=1}^N \lambda_i \xi + \sum_{i=1}^N \big\{\sigma_{4i} \sum_{j\in\mathbb{S}_i} \delta_M\big\}$.

Proof. First, construct the following Lyapunov function:

$$V(k) = \sum_{i=1}^N \eta_i^{\mathrm{T}}(k) P_i \eta_i(k). \tag{18}$$

Calculating its difference along the trajectory (12) and then taking the mathematical expectation result in

$$\begin{aligned}
\mathrm{E}\big\{\Delta V(k)\big\} &= \mathrm{E}\big\{V(k+1) - V(k)\big\} \\
&= \sum_{i=1}^N \Bigg\{ \eta_i^{\mathrm{T}}(k) \bar{A}_i^{\mathrm{T}} P_i \Bigg(\bar{A}_i \eta_i(k) + 2\sum_{j\in\mathbb{S}_i} \bar{A}_{ij}\eta_j(k) + 2\sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k) \\
&\quad - 2\bar{\beta}_i H_1 K_i \xi_i(k) \Bigg) + \bar{\beta}_i(1-\bar{\beta}_i)\eta_i^{\mathrm{T}}(k)\bar{B}_i^{\mathrm{T}} P_i \big(\bar{B}_i\eta_i(k) - 2H_1 K_i \xi_i(k)\big) \\
&\quad + \Bigg(\sum_{j\in\mathbb{S}_i} \eta_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}\Bigg) P_i \Bigg(\sum_{j\in\mathbb{S}_i} \bar{A}_{ij}\eta_j(k) + 2\sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k) - 2\bar{\beta}_i H_1 K_i \xi_i(k)\Bigg) \\
&\quad + \Bigg(\sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k)\Bigg)^{\mathrm{T}} P_i \Bigg(\sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k)\Bigg) \\
&\quad - 2\bar{\beta}_i \Bigg(\sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k)\Bigg)^{\mathrm{T}} P_i H_1 K_i \xi_i(k) + \bar{\beta}_i \xi_i^{\mathrm{T}}(k) K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i \xi_i(k) \\
&\quad + \mathrm{tr}\big\{(1-\bar{\beta}_i) D_i^{\mathrm{T}} K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i D_i Q_{i,k} + B_i^{\mathrm{T}} H_2^{\mathrm{T}} P_i H_2 B_i R_{i,k}\big\} - \eta_i^{\mathrm{T}}(k) P_i \eta_i(k) \Bigg\}.
\end{aligned} \tag{19}$$

In what follows, using the essential matrix inequality, one has

$$\begin{aligned}
&2\eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i \sum_{j\in\mathbb{S}_i} \bar{A}_{ij}\eta_j(k) \\
&\leqslant \varepsilon_1 \eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i \bar{A}_i \eta_i(k) + \varepsilon_1^{-1} \Bigg(\sum_{j\in\mathbb{S}_i} \eta_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}\Bigg) P_i \sum_{j\in\mathbb{S}_i} \bar{A}_{ij}\eta_j(k),
\end{aligned} \tag{20}$$

$$\begin{aligned}
&2\eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i \sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k) \\
&\leqslant \varepsilon_2 \eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i \bar{A}_i \eta_i(k) + \varepsilon_2^{-1} \Bigg(\sum_{j\in\mathbb{S}_i} f_j^{\mathrm{T}}(k) A_{ij}^{\mathrm{T}} H_1^{\mathrm{T}}\Bigg) P_i \sum_{j\in\mathbb{S}_i} H_1 A_{ij} f_j(k),
\end{aligned} \tag{21}$$

$$\begin{aligned}
&2\bar{\beta}_i \eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i H_1 K_i \xi_i(k) \\
&\leqslant \varepsilon_3 \bar{\beta}_i \eta_i^{\mathrm{T}}(k)\bar{A}_i^{\mathrm{T}} P_i \bar{A}_i \eta_i(k) + \varepsilon_3^{-1}\bar{\beta}_i \xi_i^{\mathrm{T}}(k)\ K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i \xi_i(k),
\end{aligned} \tag{22}$$

$$\begin{aligned}
&2\bar{\beta}_i(1-\bar{\beta}_i)\eta_i^{\mathrm{T}}(k)\bar{B}_i^{\mathrm{T}} P_i H_1 K_i \xi_i(k) \\
&\leqslant \varepsilon_4 \bar{\beta}_i(1-\bar{\beta}_i)\eta_i^{\mathrm{T}}(k)\bar{B}_i^{\mathrm{T}} P_i \bar{B}_i \eta_i(k) + \varepsilon_4^{-1}\bar{\beta}_i(1-\bar{\beta}_i)\xi_i^{\mathrm{T}}(k) K_i^{\mathrm{T}} H_1^{\mathrm{T}} P_i H_1 K_i \xi_i(k),
\end{aligned} \tag{23}$$

$$2\Bigg(\sum_{j\in\mathbb{S}_i}\eta_j^{\mathrm{T}}\bar{A}_{ij}^{\mathrm{T}}\Bigg)P_i\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k)$$
$$\leqslant\varepsilon_5\Bigg(\sum_{j\in\mathbb{S}_i}\eta_j^{\mathrm{T}}\bar{A}_{ij}^{\mathrm{T}}\Bigg)P_i\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)+\varepsilon_5^{-1}\Bigg(\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)A_{ij}^{\mathrm{T}}H_1^{\mathrm{T}}\Bigg)P_i\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k),\tag{24}$$
$$2\bar{\beta}_i\sum_{j\in\mathbb{S}_i}\eta_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_iH_1K_i\xi_i(k)$$
$$\leqslant\bar{\beta}_i\varepsilon_6\left(\sum_{j\in\mathbb{S}_i}\eta_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}\right)P_i\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)+\bar{\beta}_i\varepsilon_6^{-1}\xi_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}H_1^{\mathrm{T}}P_iH_1K_i\xi_i(k),\tag{25}$$
$$2\bar{\beta}_i\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}H_1^{\mathrm{T}}P_iH_1K_i\xi_i(k)$$
$$\leqslant\bar{\beta}_i\varepsilon_7\Bigg(\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}H_1^{\mathrm{T}}\Bigg)P_i\sum_{j\in\mathbb{S}_i}H_1\bar{A}_{ij}f_j(k)+\bar{\beta}_i\varepsilon_7^{-1}\xi_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}H_1^{\mathrm{T}}P_iH_1K_i\xi_i(k).\tag{26}$$

Now, taking (20)–(26) into (19) results in

$$\begin{aligned}\mathrm{E}\big\{\Delta V(k)\big\}\leqslant&\sum_{i=1}^{N}\Big\{\eta_i^{\mathrm{T}}(k)\Phi_{1i}\eta_i(k)-\eta_i^{\mathrm{T}}(k)P_i\eta_i(k)\Big\}\\&+\sum_{i=1}^{N}\left\{\sigma_{3i}\Bigg(\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)\Bigg)^{\mathrm{T}}P_i\Bigg(\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)\Bigg)\right\}\\&+\sum_{i=1}^{N}\left\{\sigma_{4i}\Bigg(\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k)\Bigg)^{\mathrm{T}}P_i\Bigg(\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k)\Bigg)\right\}\\&+\sum_{i=1}^{N}\big\{\xi_i^{\mathrm{T}}(k)\Phi_{4i}\xi_i(k)\big\}+\sum_{i=1}^{N}\mathrm{tr}\big(\Phi_{3i}\big).\end{aligned}\tag{27}$$

On the other hand, on the basis of (7) and (15) combined with Lemma 1, we know that

$$\begin{aligned}&\sum_{i=1}^{N}\left\{\Bigg(\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k)\Bigg)^{\mathrm{T}}P_i\Bigg(\sum_{j\in\mathbb{S}_i}H_1A_{ij}f_j(k)\Bigg)\right\}\\&\leqslant\sum_{i=1}^{N}\left\{\frac{1}{2}\sum_{j\in\mathbb{S}_i}\sum_{l\in\mathbb{S}_i}\big[\big(H_1A_{ij}f_j(k)\big)^{\mathrm{T}}P_i\big(H_1A_{ij}f_j(k)\big)^{\mathrm{T}}+\big(H_1A_{il}f_l(k)\big)^{\mathrm{T}}P_i\big(H_1A_{il}f_l(k)\big)\big]\right\}\\&=\sum_{i=1}^{N}\left\{m_i\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)\Psi_jf_j(k)\right\}\\&\leqslant\sum_{i=1}^{N}\sum_{j\in\mathbb{S}_i}m_i\delta_M.\end{aligned}\tag{28}$$

Similarly, we have

$$\begin{aligned}&\sum_{i=1}^{N}\left\{\Bigg(\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)\Bigg)^{\mathrm{T}}P_i\Bigg(\sum_{j\in\mathbb{S}_i}\bar{A}_{ij}\eta_j(k)\Bigg)\right\}\\&\leqslant\sum_{i=1}^{N}\left\{m_i\sum_{j\in\mathbb{S}_i}\eta_j^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P_i\bar{A}_{ij}\eta_j(k)\right\}\\&=\sum_{j=1}^{N}\left\{m_j\sum_{i\in\mathbb{S}_j}\eta_i^{\mathrm{T}}(k)\bar{A}_{ji}^{\mathrm{T}}P_j\bar{A}_{ji}\eta_i(k)\right\}.\end{aligned}\tag{29}$$

In what follows, it follows from (27) and (28) that

$$\begin{aligned}
&\mathrm{E}\{\Delta V(k)\}\\
&\leqslant \sum_{i=1}^{N}\Big\{\eta_i^{\mathrm{T}}(k)\big(\Phi_{1i}+\Phi_{2i}-P_i\big)\eta_i\Big\}\\
&\quad+\sum_{i=1}^{N}\left\{m_i\sigma_{4i}\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)A_{ij}^{\mathrm{T}}H_1^{\mathrm{T}}P_iH_1A_{ij}f_j(k)\right\}\\
&\quad+\sum_{i=1}^{N}\big\{\xi_i^{\mathrm{T}}(k)\Phi_{4i}\xi_i(k)\big\}+\sum_{i=1}^{N}\mathrm{tr}\big(\Phi_{3i}\big)\\
&\leqslant \sum_{i=1}^{N}\big\{\eta_i^{\mathrm{T}}(k)\big(\Phi_{1i}+\Phi_{2i}-P_i\big)\eta_i\big\}+\sum_{i=1}^{N}\mathrm{tr}\big(\Phi_{3i}\big)\\
&\quad+\sum_{i=1}^{N}\left\{m_i\sigma_{4i}\sum_{j\in\mathbb{S}_i}f_j^{\mathrm{T}}(k)\Psi_jf_j(k)\right\}+\sum_{i=1}^{N}\lambda_i\|\xi_i(k)\|^2\\
&\leqslant -\mu\sum_{i=1}^{N}\eta_i^{\mathrm{T}}(k)P_i\eta_i+\sum_{i=1}^{N}\mathrm{tr}(\Phi_{3i})+\sum_{i=1}^{N}\lambda_i\xi+\sum_{i=1}^{N}\left\{m_i\sigma_{4i}\sum_{j\in\mathbb{S}_i}\delta_M\right\}\\
&\leqslant -\mu\sum_{i=1}^{N}\big\{\eta_i^{\mathrm{T}}(k)P_i\eta_i\big\}+\theta,
\end{aligned}\tag{30}$$

which means $\mathrm{E}\{V(k+1)\}<\big(1-\mu\big)\mathrm{E}\{V(k)\}+\theta$.

Finally, let us disclose the bound of estimation errors. First, it is not difficult to see that

$$\begin{aligned}
\mathrm{E}\{V(k+1)\}&<\big(1-\mu\big)\mathrm{E}\{V(k)\}+\theta\\
&<\big(1-\mu\big)\big[\big(1-\mu\big)\mathrm{E}\{V(k-1)\}+\theta\big]+\theta\\
&<\big(1-\mu\big)^k\mathrm{E}\{V(0)\}+\theta\sum_{i=0}^{k-1}\big(1-\mu\big)^i\\
&=\big(1-\mu\big)^kV(0)+\theta\frac{1-(1-\mu)^k}{\mu}.
\end{aligned}\tag{31}$$

In what follows, selecting

$$\varphi_1=(1-\mu)^k\frac{\max_{i\in\mathbb{N}}\lambda_{\max}(P_i)}{\min_{i\in\mathbb{N}}\lambda_{\min}(P_i)}\mathrm{E}\left\{\left\|\sum_{i=1}^{N}\eta_i(0)\right\|^2\right\},\quad \varphi_2=\frac{\theta}{\mu\min_{i\in\mathbb{N}}\lambda_{\min}(P_i)},\tag{32}$$

one has $\mathrm{E}\big\{\sum_{i=1}^{N}\|\eta_i(k)\|\big\}\leqslant\varphi_1\big(k,\mathrm{E}\{\sum_{i=1}^{N}\|\eta_i(0)\|\}\big)+\varphi_2$. It follows from Definition 1 that the estimation error dynamics (12) is input-to-state in the mean-square sense, which completes the proof.

The performance analysis of estimators has been dealt with. Now, we are in a position to consider the design problem estimator gains for the large-scale discrete-time system (1) with measurement outputs (4) subject to deception attacks.

With the help of the well-known Schur complement lemma, the inequalities (14)–(16) are true if

$$\begin{bmatrix}(\mu-1)P_i+\Phi_{2i} & \sqrt{\sigma_{1i}}\bar{A}_i^{\mathrm{T}}P_i & \sqrt{\sigma_{2i}}\bar{B}_i^{\mathrm{T}}P_i\\ * & -P_i & \mathbf{0}\\ * & * & -P_i\end{bmatrix}<0,\tag{33}$$

$$\begin{bmatrix}-\Psi_j & (P_iH_1A_{ij})^{\mathrm{T}}\\ * & -P_i\end{bmatrix}<0,\tag{34}$$

$$\begin{bmatrix} -\lambda_i I & \sqrt{\sigma_{5i}}(P_i H_1 K_i)^{\mathrm{T}} \\ * & -P_i \end{bmatrix} < 0. \tag{35}$$

Selecting $P_i = \mathrm{diag}\{P_{1i}, P_{2i}\}$ and executing the variable substitution $W_i = P_{2i}K_i$, one can easily access to the following theorem.

Theorem 2. Consider the large-scale discrete-time system (1) with measurement outputs (4) subject to deception attacks. For the given scalars δ_m, δ_M, $\bar{\beta}_i$ ($i \in \mathcal{V}$), a, b_i, $\varepsilon_s > 0$ ($s = 1, 2, \ldots, 7$) and $0 < \mu < 1$ if there exist positive-definite matrices P_{1i} and P_{2i}, matrices W_i, and positive scalars λ_i for $i \in \mathcal{V}$ such that the following inequalities:

$$\begin{bmatrix} (\mu-1)P_i + \Phi_{2i} & \Gamma_{i,1} & \Gamma_{i,2} \\ * & -P_i & \mathbf{0} \\ * & \mathbf{0} & -P_i \end{bmatrix} < 0, \tag{36}$$

$$\begin{bmatrix} -\lambda_i I & \mathbf{0} & \sqrt{\sigma_{5i}}W_i^{\mathrm{T}} \\ * & P_{1i} & \mathbf{0} \\ * & \mathbf{0} & P_{2i} \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} -\Psi_j & (P_i H_1 A_{ij})^{\mathrm{T}} \\ * & -P_i \end{bmatrix} < 0 \tag{38}$$

with

$$\Gamma_{i,2} = \begin{bmatrix} \mathbf{0} & \sqrt{\sigma_{2i}}C_i^{\mathrm{T}}W_i^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \Gamma_{i,1} = \begin{bmatrix} \sqrt{\sigma_{1i}}A_i^{\mathrm{T}}P_{1i} & \sqrt{\sigma_{1i}}\bar{\beta}_i C_i^{\mathrm{T}}W_i^{\mathrm{T}} \\ \mathbf{0} & \sqrt{\sigma_{1i}}A_i^{\mathrm{T}}P_{2i} - \sqrt{\sigma_{1i}}C_i^{\mathrm{T}}W_i^{\mathrm{T}} \end{bmatrix}$$

hold for all $i, j \in \mathcal{V}$, the estimation error dynamics (12) with the adaptive event-triggered transmission scheme (6)–(9) and the gain parameters $K_i = P_{2i}^{-1}W_i$ is input-to-state stable in the mean-square sense. Furthermore, the upper bound of estimation error is obtained via (17).

Finally, the developed result can be easily degenerated into the case without deception attacks, and the corresponding result can be easily obtained by setting $\bar{\beta}_i = 0$ and $\xi = 0$.

Corollary 1. Consider the large-scale discrete-time system (1) without deception attacks. For given scalars δ_m, δ_M, a, b_i, $\varsigma_s > 0$ ($s = 1, 2, 3$) and $0 < \mu_1 < 1$ if there exist positive-definite matrices $\bar{P}_i$, matrices $\bar{Q}_i$ for $i \in \mathcal{V}$ such that the following inequalities:

$$\begin{bmatrix} (\mu_1 - 1)\bar{P}_i + \varsigma_1 \bar{\Phi}_{2i} & \sqrt{\varsigma_2}(A_i^{\mathrm{T}}\bar{P}_i^{\mathrm{T}} - C_i^{\mathrm{T}}\bar{Q}_i^{\mathrm{T}}) \\ * & -\bar{P}_i \end{bmatrix} < 0, \tag{39}$$

$$\begin{bmatrix} -\Psi_j & (\bar{P}_i A_{ij})^{\mathrm{T}} \\ * & -P_i \end{bmatrix} < 0 \tag{40}$$

hold for all $i, j \in \mathcal{V}$ where $\varsigma_1 = (1 + \varrho_1^{-1} + \varrho_3)$, $\varsigma_2 = \sqrt{1 + \varrho_1 + \varrho_2}$ and $\bar{\Phi}_{2i} = m_j \sum_{j \in \mathbb{S}_i} A_{ji}^{\mathrm{T}} \bar{P}_i A_{ji}$, the estimation error dynamics (12) with the adaptive event-triggered transmission scheme (6)–(9) and the gain parameters $K_i = \bar{P}_i^{-1}\bar{Q}_i$ is input-to-state stable in the mean-square sense. Furthermore, the upper bound of estimation error is $\theta_1(\mu_1 \min_{i \in \mathbb{N}} \lambda_{\min}(\bar{P}_i))^{-1}$ with $\varsigma_3 = t_{1i}(1 + \varrho_2^{-1} + \varrho_3^{-1})$ and

$$\theta_1 = \sum_{i=1}^{N} \mathrm{tr}\big(B_i^{\mathrm{T}}\bar{P}_i B_i R_{i,k} + D_i^{\mathrm{T}} K_i^{\mathrm{T}} \bar{P}_i K_i D_i Q_{i,k}\big) + \sum_{i=1}^{N} \left\{ \varsigma_3 \sum_{j \in \mathbb{S}_i} \delta_M \right\}.$$

Remark 2. In Theorem 2, the state estimation issue has been addressed for large-scale discrete-time systems (1) with measurement outputs (4) subject to deception attacks. Some elaborate operations on coupling terms have been implemented to realize the scalability of the developed method. In other words, the calculation burden cannot be increased as the increasing number of subsystems, which has been verified via the developed theorem. Specifically, the developed inequality conditions only depend on the parameters of the subsystem itself and the coupling parameters, instead of the global parameters. On the other hand, the estimation error dynamics is essentially a switching system governed by a set of stochastic switching signals $\beta_i(k)$. Usually, the observability of switching systems can be regarded as joint observability, which is jointly determined by that of all subsystems combined with the switching feature. Note that the observability of addressed systems in this paper could not be guaranteed owing to deception attacks. Furthermore, the accurate range of the allowable probability of randomly occurring deception attacks cannot be easily obtained owing mainly to the complexity of the addressed problem. Assuming that the occurring probabilities are the same for all subsystems (denoted as $\bar{\beta}_i = \bar{\beta}$ for $i \in \mathcal{V}$), the linear search approach (starting from $\bar{\beta} = 0$) can be utilized to maximize the occurring probability of deception attacks under the case of guaranteeing the solvability of the proposed matrix inequalities.

4 Illustrative example

In this section, a simulation example is presented to illustrate the effectiveness of the proposed state estimation algorithm for discrete-time large-scale systems. The test system employed in this paper is a power system consisting of four coupled power generation areas, which is a test benchmark proposed in the Hycon2 Project [35]. The system parameters and their physical connection are the same with the ones in the scenario 1 in [29,35]. Specifically, the physical connections are $(1,2)$, $(2,1)$, $(2,3)$, $(3,2)$, $(3,4)$ and $(4,3)$, and the corresponding system structure is shown in Figure 1. The dynamics of each power generation area is modeled by the following linear continuous-time model:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j\in\mathbb{S}_i} A_{ij} x_i(t) + B_i' u_i + L_i' \Delta P_{L_i} \tag{41}$$

with the subsystem state $x_i = (\,\Delta\theta_i\ \Delta\omega_i\ \Delta P_{m_i}\ \Delta P_{v_i})^{\mathrm{T}}$ where $\Delta\theta_i$ stands for the deviation of the angular displacement of the rotor with respect to the stationary reference axis on the stator, Δw_i means the speed deviation of the rotating mass, ΔP_{m_i} reflects the deviation of the mechanical power and ΔP_{v_i} is the deviation of the steam valve position. u_i is the control input and ΔP_{L_i} is the local power load. The system parameters are given as follows:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{\sum_{j\in\mathbb{S}_i} P_{ij}}{2H_i} & -\frac{D_i}{2H_i} & \frac{1}{2H_i} & 0 \\ 0 & 0 & -\frac{1}{T_{t_i}} & \frac{1}{T_{t_i}} \\ 0 & -\frac{1}{R_i T_{g_i}} & 0 & -\frac{1}{T_{g_i}} \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & 0\ 0\ 0 \\ \frac{P_{ij}}{2H_i} & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0.25 & 0 & 0.6 \\ 0.5 & 1 & 0.2 & 0.3 \\ 0.2 & 0 & 1 & 0.5 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0.2\ 0\ 0.6 \\ 0.5 & 1\ 0\ 0.3 \\ 0.2 & 0\ 1\ 0.5 \end{bmatrix}, \quad C_3 = C_4 = \begin{bmatrix} 1\ 0\ 0\ 0.6 \\ 0\ 1\ 0\ 0.3 \\ 0\ 0\ 1\ 0.5 \end{bmatrix}, \quad L_i' = \begin{bmatrix} 0 \\ -\frac{1}{2H_i} \\ 0 \\ 0 \end{bmatrix}, \quad B_i' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{g_i}} \end{bmatrix}.$$

Then, the definitions of system variables are the same with the ones in [29]. The exploited values of model parameters are given in Table 1 and the slopes of the power angle curves are $P_{12} = P_{21} = 4$ and $P_{23} = P_{32} = P_{34} = P_{43} = 2$. The values of other parameters, except the parameter H_2, are the same with the ones in [29]. Finally, its corresponding discrete-time model can be easily obtained by selecting the sampling period $T = 1$ s.

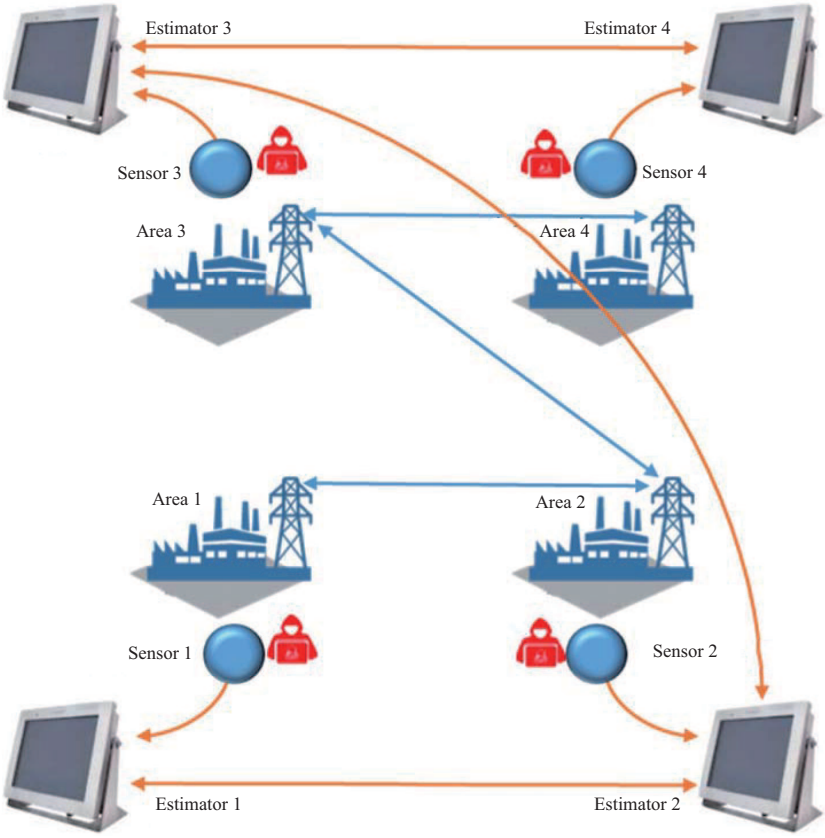

Figure 1 (Color online) Framework of distributed estimation for power systems.

Table 1 Parameter values

	T_{t_i}	T_{g_i}	R_i	H_i	D_i
Area 1	0.65	0.1	0.05	12	0.7
Area 2	0.4	0.1	0.0625	6	0.8
Area 3	0.3	0.1	0.08	8	0.9
Area 4	0.6	0.1	0.08	8	0.7

4.1 Simulation test

For the purpose of simulation, the initial values are selected as follows:

$$x_{1,0} = [1.39,\ 1.62,\ 1.28,\ 1.39]^{\mathrm{T}}, \quad x_{2,0} = [1.09,\ 1.19,\ 1.81,\ 1.29]^{\mathrm{T}},$$
$$x_{3,0} = [1.18,\ 2.79,\ 2.82,\ 1.58]^{\mathrm{T}}, \quad x_{4,0} = [1.29,\ 1.31,\ 1.09,\ 1.42]^{\mathrm{T}}$$

for each subsystem and $\hat{x}_{1,0} = \hat{x}_{2,0} = \hat{x}_{3,0} = \hat{x}_{4,0} = 0$ for each sub-estimator. The occurring probabilities of deception attacks are supposed to be $\bar{\beta}_1 = 0.15$, $\bar{\beta}_2 = 0.12$, $\bar{\beta}_3 = 0.11$ as well as $\bar{\beta}_4 = 0.13$ and then the malicious signals are selected as $\xi_1(k) = [0.02\sin(k),\ 0.01,\ 0.02]^{\mathrm{T}}$, $\xi_2(k) = [0.2\sin(k),\ 0.2,\ 0.2]^{\mathrm{T}}$, $\xi_3(k) = [0.2,\ 0.2\sin(k),\ 0.2]^{\mathrm{T}}$, and $\xi_4(k) = [0.1,\ 0.1,\ 0.2\sin(k)]^{\mathrm{T}}$. Then, the covariance matrices of Gaussian noise sequences $\omega_i(k)$ and $v_i(k)$ are selected as $Q_{1,k} = Q_{3,k} = Q_{4,k} = 0.01I$, $Q_{2,k} = 0.02I$ and $R_{1,k} = R_{3,k} = R_{4,k} = 0.01I$, $R_{2,k} = 0.02I$. The adaptive event-triggered thresholds and the adjustable parameters are given as $\delta_M = 0.3$, $\delta_m = 0.15$, $a = 0.25$, $b_1 = 0.57$, $b_2 = 0.59$, $b_3 = 0.8$ and $b_4 = 1.1$. Furthermore, the event-triggered weight matrices are given as $\Psi_1 = 0.31I$, $\Psi_2 = 0.25I$, $\Psi_3 = 0.26I$ and $\Psi_4 = 0.26I$.

According to the conditions "$\Phi_{1i}+\Phi_{2i}+(\mu-1)P_i < 0$" and "$\Phi_{4i} < \lambda_i I$" in Theorem 1, we can find that the predetermined parameters $\varepsilon_i > 0$ and $0 < \mu < 1$ should guarantee the solvability of the developed matrix inequalities as far as possible. Specificality, ε_1, ε_2, ε_3 and μ should be some small positive scalars. Without loss of generality, we adopt the parameters $\varepsilon_1 = 0.1, \varepsilon_2 = 0.011, \varepsilon_3 = 0.25, \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0.01$

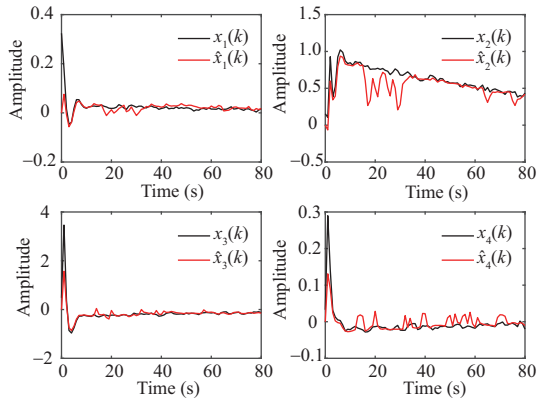

Figure 2 (Color online) The state $\Delta\theta_i$ and its estimate of each area.

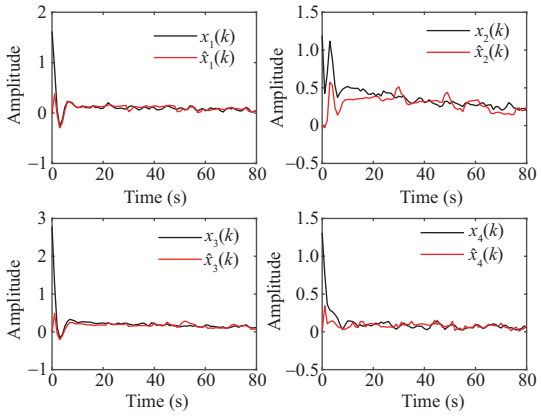

Figure 3 (Color online) The state $\Delta\omega_i$ and its estimate of each area.

and $\mu = 0.001$. Using the Matlab software with the well-known LMI toolbox, the desired estimator gains dependent on the solutions of (36)–(38) are

$$K_1 = \begin{bmatrix} 0.0017 & -0.1015 & 0.0001 \\ -0.0010 & 0.0048 & -0.0011 \\ 0.1040 & -0.0773 & 0.0202 \\ 0.0199 & -0.1009 & 0.0229 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.1803 & 0.4369 & -0.1373 \\ -0.1929 & 0.3784 & 0.0147 \\ 2.9090 & -8.6099 & 0.2736 \\ 3.1919 & -7.0826 & -0.1067 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.2046 & 0.4617 & -0.1713 \\ -0.0511 & 0.3737 & -0.0302 \\ 0.6438 & -6.4158 & 0.6611 \\ 0.6621 & -5.2949 & 0.4645 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 0.1241 & 0.2291 & -0.1093 \\ -0.0907 & 0.3312 & -0.0176 \\ 1.0930 & -5.4958 & 0.6227 \\ 1.9239 & -7.5287 & 0.4997 \end{bmatrix}.$$

The simulation results are shown in Figures 2–12. Specifically, Figures 2–5 respectively plot the trajectories of $\Delta\theta_i$, Δw_i, ΔP_{m_i} and ΔP_{v_i} and their estimates, and correspondingly, Figures 6–9 depict the evolution of estimation errors of each area. Furthermore, to reflect the influence from event-triggering protocols and deception attacks, the release time (i.e., the triggering instants) and the release intervals (i.e., the interval between the last triggering and current triggering instants) are, respectively, shown in Figure 10, and the instants of deception attacks are plotted in Figure 11. Compared to these three sets of figures, we can find that the bad fluctuations of state estimation mainly occur at the instants of deception attacks. Finally, the adaptive dynamics $\delta_i(k)$ is shown in Figure 12, which clearly discloses that all $\delta_i(k)$ are involved in the given range $[\delta_m,\ \delta_M]$ and can be dynamically adjusted according to the real-time gap $f_i(k)$.

4.2 Comparison analysis

In this subsection, we will further verify the effectiveness of the developed event-triggered protocol. To this end, the performance comparison is performed to the proposed adaptive event-triggered scheme (denoted as AETM) with other communication schemes: (a) traditional event-triggering mechanisms with a fixed threshold (ETM-FT) and (b) periodic communication mechanisms (PCM). To be fair, the threshold in ETM-FT is chosen as the initial value $\delta_i(0)$ (i.e., δ_m). The experiment results are shown in Table 2. We can find from this table that all data in the whole simulation duration must be transmitted for the case under PCM, and the trigger number of our approach AETM is lower than that of ETM-FT, which means that the proposed adaptive scheme is definitely effective.

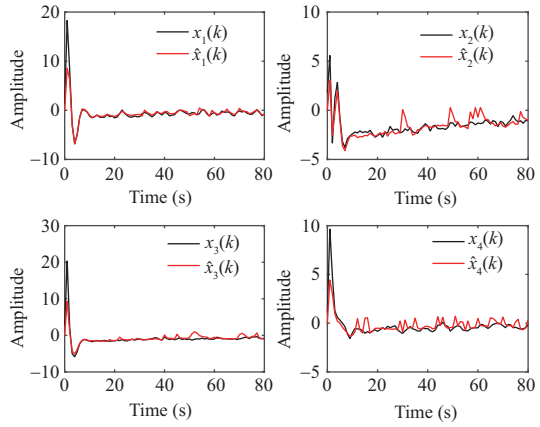

Figure 4 (Color online) The state ΔP_{m_i} and its estimate of each area.

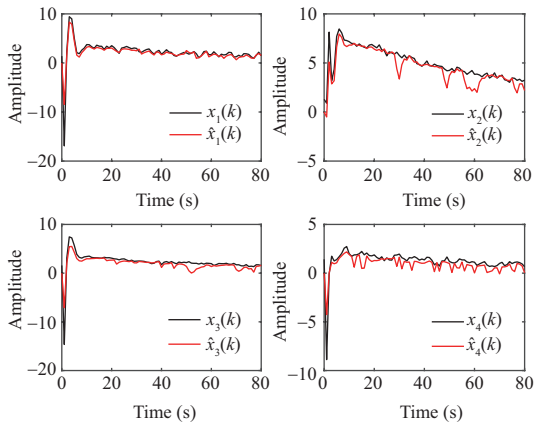

Figure 5 (Color online) The state ΔP_{v_i} and its estimate of each area.

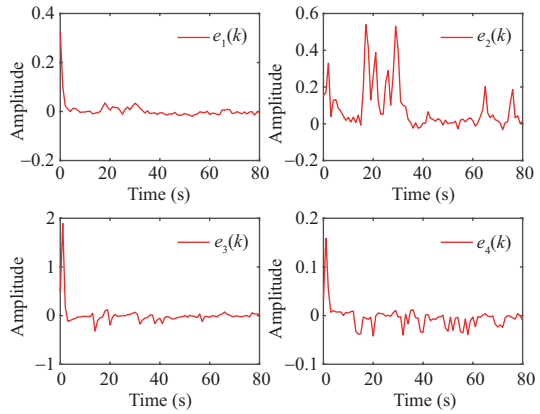

Figure 6 (Color online) The evolution of errors of $\Delta\theta_i$.

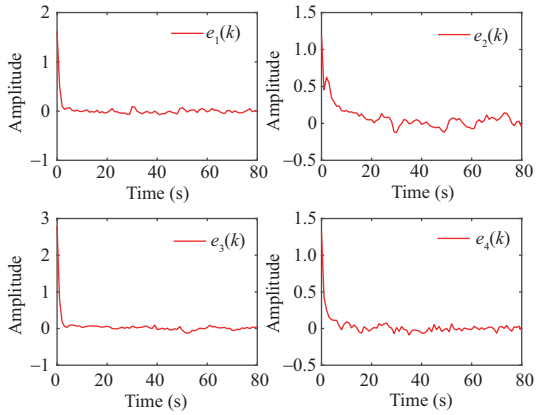

Figure 7 (Color online) The evolution of errors of $\Delta\omega_i$.

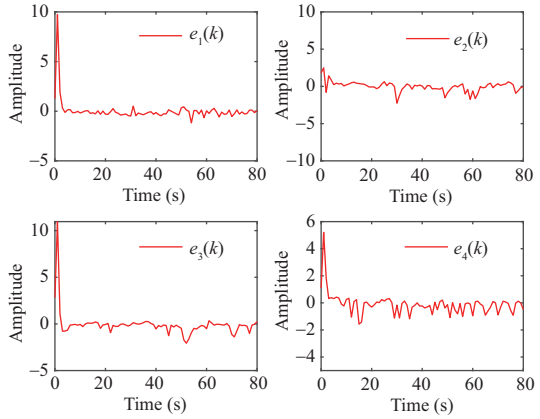

Figure 8 (Color online) The evolution of errors of ΔP_{m_i}.

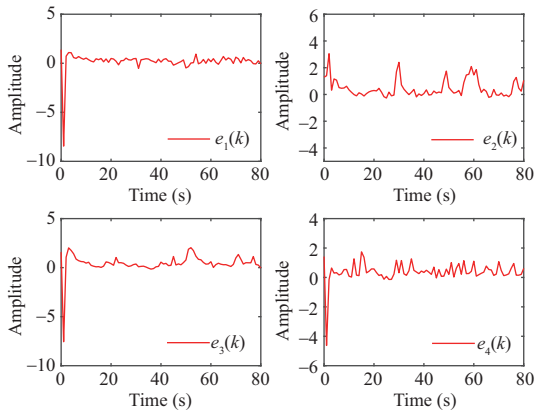

Figure 9 (Color online) The evolution of errors of ΔP_{v_i}.

5 Conclusion

In this paper, we investigate the state estimation problem for large-scale discrete-time systems with measurements subject to deception attacks, which are described by a set of Bernoulli distributed random variables. To reduce the burden of information transmission, a new adaptive event-triggered scheme is designed to determine whether the estimated state is delivered or not. The adopted adaptive rule,

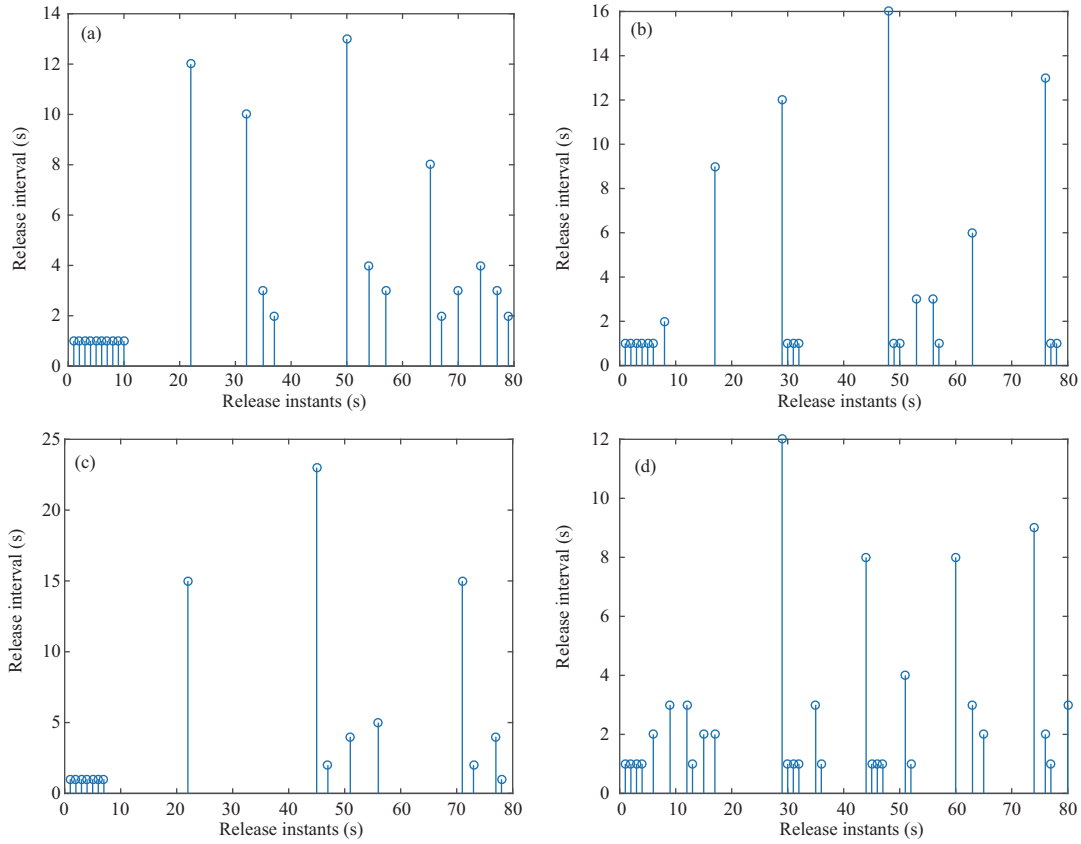

Figure 10 (Color online) Release instants and release intervals on (a) Area 1, (b) Area 2, (c) Area 3, (d) Area 4.

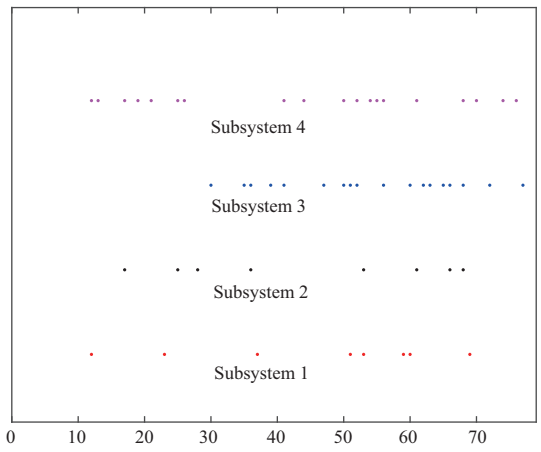

Figure 11 (Color online) Deception attack sequences.

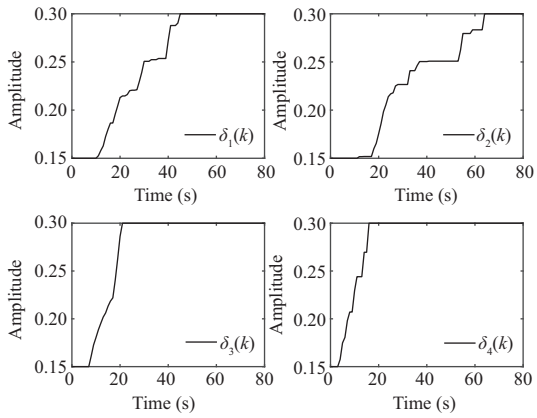

Figure 12 Adaptive parameters $\delta_i(k)$.

Table 2 The number of triggers under different communication mechanisms

	Area 1	Area 2	Area 3	Area 4
PCM	80	80	80	80
ETM-FT	41	55	33	34
AETM	24	31	18	20

dependent on four parameters, can adjust the threshold dynamically, thus governing the transmission frequency. Some sufficient conditions are derived to guarantee the input-to-state stability with the upper

bound of the estimation error dynamics. Furthermore, the desired estimator parameters are obtained via the solution of a set of matrix inequalities. The developed scheme depends on the local information of subsystems and thus satisfies the requirement of scalability. Finally, a simulation example on power systems is utilized to illustrate the usefulness and effectiveness of the proposed design scheme.

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