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Global practical tracking via disturbance rejection control for uncertain nonlinear systems with quantized input and output

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Dear editor,

With the rapid development of digital control systems and other fields, quantized control has attracted increasing attention [1]. Although considerable research has been conducted on the analysis of quantized control systems, most of the theoretical results are based on a single quantizer. In real life, when the distance between devices and controllers is relatively remote, sensors and controllers are connected through the communication network to exchange information [2]. Therefore, a single quantizer cannot meet the needs of remote control systems. It is necessary to quantify the input and output signals.

In modern industrial systems, external disturbances widely exist that may destroy equipment and prevent systems from working properly. If the disturbances are measurable, a simple feedforward compensation approach can be used to reduce the impact of the disturbances. However, in real industrial production, disturbances are not measurable, or the measurement costs are very high. To solve this problem, the method of estimating disturbances, proposed by [3], has been widely used. In [3], the generalized proportional integral observer (GPIO) used the estimation of external disturbance as the extended state. The control strategy in [3] used high gain to suppress nonlinear uncertainties in the system, and a GPIO-based disturbance compensation technique to mitigate the influence of external disturbances.

It is a well-known fact that tracking is one of the most important problems in control theory [4]. However, the global practical tracking problem of input and output quantization for nonlinear systems with time-varying external disturbance is still challenging. To overcome this challenge, we developed a novel combination of the static gain method, the GPIO-based disturbance compensation technique, and the quantized control scheme. The controller designed in this study not only has a simple structure, but also has the capability of anti-disturbance and saving communication resources.

Problem description. We consider a class of uncertain nonlinear systems with time-varying external disturbance, which are described as

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(t, x, q_1(u)), \ i = 1, \dots, n-1, \\ \dot{x}_n = q_1(u) + f_n(t, x, q_1(u)) + d(t), \\ y = x_1 - y_r, \end{cases} \tag{1}$$

where $x = [x_1, \dots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ and $y \in \mathbb{R}$ are the system state vector and the system output (tracking error), respectively; $q_1(u) \in \mathbb{R}$ represents the quantized input with $u \in \mathbb{R}$; $y_r \in \mathbb{R}$ is the reference signal for tracking; $f_i(\cdot)$, $i = 1, \dots, n-1$ are unmatched nonlinear uncertain terms and $f_n(\cdot)$ is matched nonlinear uncertain term; and $d(t) \in \mathbb{R}$ represents the time-varying external disturbance.

Based on the principle of the Taylor series expansion approximation function, $d(t)$ is assumed to be in the following form [3]:

$$d(t) = \sum_{i=0}^{p-1} a_i t^i + g(t), \tag{2}$$

where a_i, $i = 0, 1, \dots, p-1$ are unknown constant coefficients with integer $p \geqslant 1$, and $g(t)$ is an unknown residual term.

The uniform quantizer $q(\varphi)$ with quantization parameter μ is modeled as follows [5]:

$$q(\varphi) = \begin{cases} \sigma_j \mu, & \frac{2\sigma_j - 1}{2}\mu \leqslant \varphi < \frac{2\sigma_j + 1}{2}\mu, \\ 0, & -\frac{\mu}{2} \leqslant \varphi < \frac{\mu}{2}, \\ -\sigma_j \mu, & \frac{-2\sigma_j - 1}{2}\mu \leqslant \varphi < \frac{-2\sigma_j + 1}{2}\mu, \end{cases} \tag{3}$$

where $\sigma_1 = 1$, $\sigma_{j+1} = \sigma_j + 1$, $j \in \mathbb{N}^+$, and μ represents the quantization interval length. Obviously, based on (3), we have $|q(\varphi) - \varphi| \leqslant \frac{\mu}{2}$. In this study, the quantization parameters of input quantizer $q_1(u)$ and output quantizer $q_2(y)$ are expressed as μ_1 and μ_2, respectively.

Next, three assumptions are presented for system (1), which will play a critical role in the follow-up research.

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Assumption 1. The time-varying external disturbance $d(t)$ considered in this study satisfies: (i) at least the pth-order time derivatives of $d(t)$ exist; (ii) $\left|d^{(p)}(t)\right| \leqslant \tau$ for $t \in [0, +\infty)$, where τ is a positive constant.

Assumption 2. There exists a known constant $c \geqslant 0$ such that

$$|f_i(\cdot)| \leqslant c(|x_1| + \cdots + |x_i| + 1),\ i = 1, \ldots, n.$$

Assumption 3. The tracking reference signal y_r is continuously differentiable. In addition, there is a known constant $M > 0$ such that

$$|y_r| + |\dot{y}_r| \leqslant M, \quad \forall\, t \in [0, +\infty).$$

Output feedback disturbance rejection control design. First, to deal with the following problem easily, we introduce variable transformation. Let us define $\eta_1 = y$, $\eta_2 = x_2, \ldots, \eta_n = x_n$; then system (1) can be rewritten as

$$\begin{cases} \dot{\eta}_i = \eta_{i+1} + \tilde{f}_i(t, \eta, y_r, q_1(u)),\ i = 1, \ldots, n-1, \\ \dot{\eta}_n = q_1(u) + \tilde{f}_n(t, \eta, y_r, q_1(u)) + d(t), \\ y = \eta_1, \end{cases} \tag{4}$$

where $\tilde{f}_1(\cdot) = f_1(t, \eta_1 + y_r, \eta_2, \ldots, \eta_n, q_1(u)) - \dot{y}_r$, $\tilde{f}_i(\cdot) = f_i(t, \eta_1 + y_r, \eta_2, \ldots, \eta_n, q_1(u)),\ i = 2, \ldots, n$.

Combining with Assumptions 2 and 3, we have

$$\left|\tilde{f}_i(\cdot)\right| \leqslant \tilde{c}\left(\sum_{j=1}^{i} |\eta_j| + 1\right),\ i = 1, \ldots, n, \tag{5}$$

where $\tilde{c} = cM + c + M$.

Based on the first condition of Assumption 1, we can define the following auxiliary variables:

$$z_1(t) = d(t),\ z_2(t) = \dot{d}(t),\ \ldots,\ z_p(t) = d^{(p-1)}(t).$$

Next, we introduce the static gain $r \geqslant 1$, which will be determined later, to construct the following coordinate transformation [6]:

$$\bar{\eta}_i = \frac{\eta_i}{r^{i-1}},\ i = 1, \ldots, n,$$

$$\bar{z}_i = \frac{z_i}{r^{n+i-1}},\ i = 1, \ldots, p,$$

$$\bar{q}_1(\bar{u}) = \frac{q_1(u)}{r^n},\ \bar{u} = \frac{u}{r^n}.$$

With the help of denotation $\bar{\eta} = [\bar{\eta}_1, \ldots, \bar{\eta}_n]^{\mathrm{T}}$, system (4) is written as

$$\begin{cases} \dot{\bar{\eta}} = rA_0\bar{\eta} + rB_0(\bar{q}_1(\bar{u}) + \bar{z}_1) + \Phi, \\ y = C_0\bar{\eta}, \end{cases} \tag{6}$$

where $A_0 = \left[\begin{smallmatrix} \mathbf{0}_{n-1} & I_{n-1} \\ 0 & \mathbf{0}_{n-1}^{\mathrm{T}} \end{smallmatrix}\right], B_0 = \left[\begin{smallmatrix} \mathbf{0}_{n-1} \\ 1 \end{smallmatrix}\right]$, $\Phi = [\tilde{f}_1, \frac{1}{r}\tilde{f}_2, \ldots, \frac{1}{r^{n-1}}\tilde{f}_n]^{\mathrm{T}}$, and $C_0 = [\,1,\ \mathbf{0}_{n-1}^{\mathrm{T}}\,]$.

Define $\xi = \left[\bar{\eta}^{\mathrm{T}}, \bar{z}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\bar{z} = [\bar{z}_1, \ldots, \bar{z}_p]^{\mathrm{T}}$. System (6) can be rewritten as

$$\begin{cases} \dot{\xi} = rA\xi + rB\bar{q}_1(\bar{u}) + \Omega, \\ y = C\xi, \end{cases} \tag{7}$$

where $A = \left[\begin{smallmatrix} \mathbf{0}_{n+p-1} & I_{n+p-1} \\ 0 & \mathbf{0}_{n+p-1}^{\mathrm{T}} \end{smallmatrix}\right]$, $B = [\,\mathbf{0}_{n-1}^{\mathrm{T}}, 1, \mathbf{0}_p^{\mathrm{T}}\,]^{\mathrm{T}}$, $C = [\,1,\ \mathbf{0}_{n+p-1}^{\mathrm{T}}\,]$, and $\Omega = [\,\Phi^{\mathrm{T}}, \mathbf{0}_{p-1}^{\mathrm{T}}, \frac{g^{(p)}(t)}{r^{n+p-1}}\,]^{\mathrm{T}}$.

Following the idea of [7], the form of GPIO used in this study is

$$\dot{\hat{\xi}} = rA\hat{\xi} + rB\bar{q}_1(\bar{u}) + rH(q_2(y) - \widehat{\bar{\eta}}_1), \tag{8}$$

where $\hat{\xi} = \left[\widehat{\bar{\eta}}^{\mathrm{T}}, \widehat{\bar{z}}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\widehat{\bar{\eta}} = [\widehat{\bar{\eta}}_1, \ldots, \widehat{\bar{\eta}}_n]^{\mathrm{T}}$, $\widehat{\bar{z}} = [\widehat{\bar{z}}_1, \ldots, \widehat{\bar{z}}_p]^{\mathrm{T}}$, $H = [h_1, \ldots, h_{n+p}]^{\mathrm{T}}$, and $h_i,\ i = 1, \ldots, n+p$ are coefficients of the Hurwitz polynomial $f_1(l) = l^{n+p} + h_1 l^{n+p-1} + \cdots + h_{n+p-1}l + h_{n+p}$.

In this study, the controller based on GPIO is constructed as follows:

$$u = r^n\bar{u} = -r^n(K_0\widehat{\bar{\eta}} + \widehat{\bar{z}}_1) = -r^n K\hat{\xi}, \tag{9}$$

where $K_0 = [k_1, \ldots, k_n]$, $k_i,\ i = 1, \ldots, n$ are coefficients of the Hurwitz polynomial $f_2(l) = l^n + k_n l^{n-1} + \cdots + k_2 l + k_1$, and $K = \left[K_0, 1, \mathbf{0}_{p-1}^{\mathrm{T}}\right]$.

Let $e = \xi - \hat{\xi}$ denote the error vector. Based on system (7) and system (8), we obtain

$$\dot{e} = r(A - HC)e - rH(q_2(y) - y) + \Omega. \tag{10}$$

Combining system (6), controller (9) and error system (10), the extended dimension system is obtained as

$$\dot{Y} = r\Lambda Y + r\bar{B}(\bar{q}_1(\bar{u}) - \bar{u}) - r\bar{H}(q_2(y) - y) + \Psi, \tag{11}$$

where $Y = \left[\begin{smallmatrix} \bar{\eta} \\ e \end{smallmatrix}\right], \Lambda = \left[\begin{smallmatrix} A_0 - B_0K_0 & B_0K \\ 0_{(n+p)\times n} & A - HC \end{smallmatrix}\right]$, $\Psi = \left[\begin{smallmatrix} \Phi \\ \Omega \end{smallmatrix}\right]$, $\bar{B} = \left[\begin{smallmatrix} B_0 \\ \mathbf{0}_{n+p} \end{smallmatrix}\right]$, and $\bar{H} = \left[\begin{smallmatrix} \mathbf{0}_n \\ H \end{smallmatrix}\right]$.

Because $A_0 - B_0K_0$ and $A - HC$ are Hurwitz matrices, the conclusion that Λ is the Hurwitz matrix can be reached. Consequently, there exists a positive definite matrix $P \in \mathbb{R}^{(2n+p)^2}$ satisfying $P\Lambda + \Lambda^{\mathrm{T}}P \leqslant -I$.

Next, we give an important theorem to summarize the main result of this study.

Theorem 1. Consider system (1) under Assumptions 1–3. If the static gain r and quantization parameters μ_1, μ_2 satisfy the following inequities:

$$r\mu_i \leqslant \theta_i,\ i = 1, 2, \tag{12}$$

$$r = \max\{1, \delta + d_2\}, \tag{13}$$

where $\delta = \frac{\left(8d_1^2 + 2\tau^2 + \theta_1^2\|\bar{B}\|^2 + \theta_2^2\|\bar{H}\|^2\right)\|P\|^2}{2m}$, $d_1 = n(cM + c + M)$, $d_2 = 4d_1\|P\| + \gamma \cdot \lambda_{\max}(P)$, θ_1 and θ_2 are two known positive constants (for auxiliary inequality scaling), m and γ are adjustable positive constants, then the global practical tracking problem is solved by output feedback disturbance rejection controller (9).

Proof. Choose a Lyapunov function candidate $V = Y^{\mathrm{T}}PY$ for (11), where P is given above. Taking the derivative of V along system (11), we have

$$\begin{aligned} \dot{V} \leqslant & -r\|Y\|^2 + 2\|Y\|\|P\|\|\Psi\| + \frac{\mu_1}{r^{n-1}}\|Y\| \\ & \cdot \|P\|\left\|\bar{B}\right\| + r\mu_2\|Y\|\|P\|\left\|\bar{H}\right\|. \end{aligned} \tag{14}$$

Next, let us estimate the right end of (14) one by one. Because the static gain satisfies $r \geqslant 1$, we have the following inequalities with the help of (2), (5) and Assumption 1.

$$\left|\frac{g^{(p)}(t)}{r^{n+p-1}}\right| \leqslant \left|d^{(p)}(t)\right| \leqslant \tau, \tag{15}$$

$$\left|\frac{1}{r^{i-1}}\tilde{f}_i\right| \leqslant \tilde{c}\left(\sum_{j=1}^{i}|\bar{\eta}_j|+1\right),\ i=1,\ldots,n, \quad (16)$$

$$\|\Psi\| \leqslant 2d_1(\|Y\|+1)+\tau. \quad (17)$$

Substituting (12) and (15)–(17) into (14), we get the following inequality:

$$\dot{V} \leqslant -(r-4d_1\|P\|-\delta)\|Y\|^2+3m. \quad (18)$$

Because $\lambda_{\min}(P)\|Y\|^2 \leqslant V \leqslant \lambda_{\max}(P)\|Y\|^2$, if the static gain r is selected as shown in (13), then (18) can be rewritten as

$$\dot{V} \leqslant -\gamma V+3m. \quad (19)$$

Based on (19), we further get

$$\limsup_{t\longrightarrow\infty} V(t) \leqslant \frac{3m}{\gamma}. \quad (20)$$

Because $\|Y\| \leqslant \sqrt{\frac{V}{\lambda_{\min}(P)}}$, we can get that Y is well-defined on $t\in[0,\infty)$ and globally bounded.

In addition, based on (20), there exists a finite time $T>0$ such that for any initial condition and any given error accuracy $\varepsilon>0$, appropriate parameters m and γ can be selected to satisfy

$$|x_1-y_r| \leqslant \sqrt{\frac{3m}{\gamma\lambda_{\min}(P)}} \leqslant \varepsilon,\ \ \forall\, t\in[T,+\infty).$$

This completes the proof of Theorem 1, thereby solving the global practical tracking problem.

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