SCIENCE CHINA
Information Sciences

• LETTER •

December 2021, Vol. 64 229501:1–229501:2
https://doi.org/10.1007/s11432-020-3037-1

Quantum illumination with post-processing of displacement and anti-displacement operations

Shengli ZHANG

School of Physics, Beijing Institute of Technology, Beijing 100081*, China*

Received 27 February 2020/Revised 29 May 2020/Accepted 3 August 2020/Published online 26 May 2021

Citation Zhang S L. Quantum illumination with post-processing of displacement and anti-displacement operations. Sci China Inf Sci, 2021, 64(12): 229501, https://doi.org/10.1007/s11432-020-3037-1

Dear editor,

Quantum illumination [1] is a new quantum information technique for detecting quantum targets with low reflectivity via quantum entanglement and correlation measurements. Compared with its classical variant, quantum illumination offers a higher signal-to-noise ratio and lower energy consumption. Thus far, quantum illumination has attracted considerable research attention; these studies have focused on three main objectives: (1) realizing more powerful quantum sources for target detection [2,3]; (2) determining efficient measurement schemes [4,5]; and (3) identifying potential applications in quantum communication [6] and for reading data in semiconductor devices [7]. This study focuses on the second objective. We demonstrate that quantum illumination performance can be further improved through a series of displacement and anti-displacement operations. This work proposes a deterministic method for improving the performance of quantum illumination schemes.

Scheme description. The scheme of quantum illumination is presented in Figure 1(a). A two-mode squeezed state $\rho_{AB} = |\psi\rangle_{\rm TMSS}\langle\psi|$ serves as the source with $|\psi\rangle_{\rm TMSS} = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle$, $\lambda = \tanh(r)$. A and B are the idle and signal modes, respectively. The target with reflectivity κ is modeled with a beam splitter of transmittance $T = 1-\kappa$. The E mode is a single-mode thermal state $\rho_{\rm th} = (1-\mu)\sum_{n=0}^{\infty}\mu^n|n\rangle\langle n|$ (average number of photons: $N_B = \frac{\mu}{1-\mu}$) for simulating thermal noise. The outgoing mode E' after the coupling of the beam-splitter is collected using a detector; E' and A are measured jointly to determine the existence of the target. If the target is present, $\rho_{E'A}$ is obtained at the receiver. However, if the target is absent, the product state $\rho_E\otimes\rho_A = \rho_{E'A}$ $(\kappa = 0)$ is obtained, where $\rho_A = {\rm Tr}_B[\rho_{AB}]$ is the reduced density matrix in the optical mode A. Generally, joint quantum measurements are more effective for quantum state discrimination; this explains the higher efficiency of quantum illumination, as compared with that of classical state illumination.

Here, we propose using the sequential displacement operation $D(\beta)$ and anti-displacement operation $D(-\beta)$ before and after the interaction of B with the target, respectively, in order to improve the performance of target detection (Figure 1(b)). The interaction of B with the target can be considered as a quantum channel $\mathcal{E}$, such that $\mathcal{E}: \rho_B \to {\rm Tr}[U_{EB}(\rho_E\otimes\rho_B)U_{EB}^{\dagger}]$. A plot of the equivalence to the channel is shown in Figure 1(c). By setting $\beta = 0$, the original quantum illumination can be recovered, as shown in Figure 1(a). This displacement operation can be easily implemented in a deterministic manner using a beam splitter and ancillary coherent states (Figure 1(d)).

Quantitatively, the performance of target detection is evaluated based on the probability of erroneous inferences of the existence of the target. This is equivalent to the discrimination between two mixed states, i.e., $\rho_{E'A}^{(0)} = \rho_{E'A}$ $(\kappa = 0)$ and $\rho_{E'A}^{(1)} = \rho_{E'A}$ $(\kappa = \kappa_t)$, where κ_t is the reflectivity of the target. In the absence of prior information regarding the target, the a priori probability can be assumed as $P(\kappa = 0) = P(\kappa = \kappa_t) = \frac{1}{2}$. According to the theory of optimal quantum state discrimination, the minimal error probability is [8] $P_{{\rm err},M} = \frac{1}{2}(1-\frac{1}{2}||(\rho_{E'A}^{(0)})^{\otimes M} - (\rho_{E'A}^{(1)})^{\otimes M}||)$ with $||\gamma|| = {\rm Tr}(\sqrt{\gamma^{\dagger}\gamma}) = \sum_i s_i(\gamma)$; $s_i(\gamma)$ is the absolute value of the eigenvalue [8], and M is the number of copies of $\rho_{E'A}$. When M is considerably large, the precise evaluation of $P_{\rm err}$ is upper-bounded by the quantum Chernoff bound (QCB) [4]: $P_{{\rm err},M} \leqslant P_{\rm QCB}^{M} = \frac{1}{2}Q_{E'A}^{M}$, where $Q_{E'A} = \min_{0\leqslant s\leqslant 1}{\rm Tr}[\rho_{E'A}^{(1)s}\rho_{E'A}^{(0)1-s}]$.

Performance analyses. Here, we consider the scheme in Figure 1(b) as an example; additional information regarding the analysis of Figure 1(d) can be found in the supplementary material. As the displacement and anti-displacement operations are Gaussian, the states collected by the receiver $(\rho_{E'A})$ are two-mode Gaussian states, which can be conveniently described by their covariance matrices and the average values of quadrature operators [9].

By setting $\kappa = 0$ and $\kappa = \kappa_t$, the quadrature average and covariance matrix of the Gaussian state for both the absence $(\rho_{E'A}^{(0)})$ and presence $(\rho_{E'A}^{(1)})$ of the target can be obtained as follows:

$$V_{EA}^{(0)} = \left(N_B + \frac{1}{2}\right)I_2 \oplus \frac{\cosh(2r)}{2}I_2, \quad (1)$$

Email: zhangshengli@bit.edu.cn

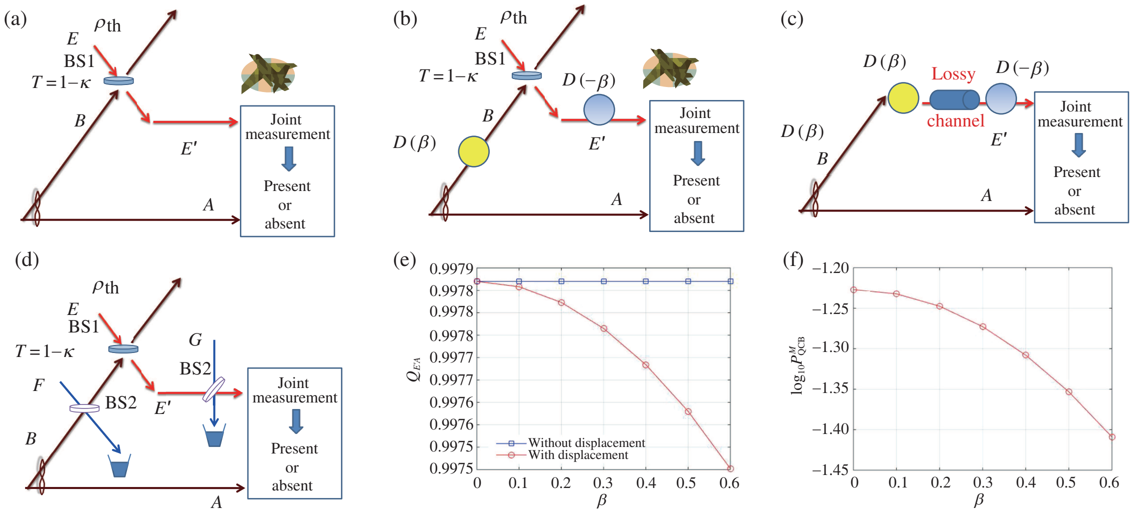

Figure 1 (Color online) (a) Target detection in the two-mode squeezed state. (b) Enhanced target detection via displacement and anti-displacement operations. On considering a displacement amplitude of zero, this scheme is reduced to the traditional quantum illumination scheme. (c) Channel model of the interaction with the target. (d) Practical implementation of displacement operation using a beam splitter with transmittance T_0, ancillary coherent states $|\beta_0\rangle$, and $|-\beta_0\rangle$ in F and G modes. BS1 is a beam splitter with transmittance $T = 1 - \kappa$; it is used to model the target. Trash boxes denote the discarding operation. (e) Value of $Q_{E'A}$ for a low-reflectivity target in a noisy environment, such that $\kappa_t = 0.05$, $N_s = 0.5$, and $N_B = 5$. (f) Its corresponding $P^M_{\rm QCB}$ for $M = 1000$.

$$R^{(0)}_{EA} = (-\sqrt{2}\beta, 0, 0, 0), \tag{2}$$

$$V^{(1)}_{EA} = \begin{pmatrix} a_1 & & c_1 & \\ & a_1 & & -c_1 \\ c_1 & & \frac{\cosh(2r)}{2} & \\ & -c_1 & & \frac{\cosh(2r)}{2} \end{pmatrix}, \tag{3}$$

$$R^{(1)}_{EA} = (\sqrt{2}\beta(\sqrt{\kappa_t} - 1), 0, 0, 0), \tag{4}$$

where I_2 is a 2D identity matrix, and $c_1 = \frac{\sqrt{\kappa}\sinh(2r)}{2}$, $a_1 = \frac{\kappa\cosh(2r)}{2} + (1-\kappa)(N_B + \frac{1}{2})$.

From Eqs. (1) and (4), using a computable bound for the discrimination between two arbitrary Gaussian states (see Appendix B), the values of $Q_{E'A}$ and the corresponding $P^M_{\rm QCB}$ can be easily determined.

Figure 1(e) shows the decrease in $Q_{E'A}$ and that in the corresponding $P^M_{\rm QCB}$ when the target is hidden in a significantly noisy environment. We consider a case where the average number of photons in the noisy environment is $N_B = 5$, and a two-mode squeezed state (TMSS) with an average photon number of $N_s = 0.50$ is employed for target detection. We note that the weak coherent state is enhanced on increasing β (= 0, 0.10, 0.20, 0.30, 0.40, 0.50, and 0.60). Furthermore, $Q_{E'A}$ is strictly dependent on the value of β, and the QCB decreases as the amplitude of the displacement operation increases. To clarify how these displacement and anti-displacement operations improve quantum illumination, Figure 1(f) illustrates the dependence of error probability on displacement for $M = 1000$. As shown in Figure 1(f), at $\beta = 0$, the curves correspond to the early scheme in Figure 1(a). The error probability (in the logarithmic scale) decreases monotonously as β increases. This implies that the displacement and anti-displacement operations aid in decreasing errors in target detection.

Conclusion. Displacement and anti-displacement operations are essentially unitary operations that can be deterministically implemented. In this study, we employ these operations before and after interactions with a potential target to achieve a significant reduction in the error probability of target detection. Our method is implemented deterministically, without wasting quantum entanglement.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 11574400, 11204379, 11981240356).

Supporting information Appendixes A–C. Appendix A is about the scheme of quantum illumination and basic formalism. Appendix B describes the quantum state evolution in illumination. Appendix C shows the physical implementation of on-line displacement operation. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Lloyd S. Enhanced sensitivity of photodetection via quantum illumination. Science, 2008, 321: 1463–1465
2 Barzanjeh S, Guha S, Weedbrook C, et al. Microwave quantum illumination. Phys Rev Lett, 2015, 114: 080503
3 Nair R, Yen B J. Optimal quantum states for image sensing in loss. Phys Rev Lett, 2011, 107: 193602
4 Guha S, Erkmen B I. Gaussian-state quantum-illumination receivers for target detection. Phys Rev A, 2009, 80: 052310
5 Sanz M, Heras U L, García-Ripoll J J, et al. Quantum estimation methods for quantum illumination. Phys Rev Lett, 2017, 118: 070803
6 Shapiro J H. Defeating passive eavesdropping with quantum illumination. Phys Rev A, 2009, 80: 022320
7 Pirandola S. Quantum reading of a classical digital memory. Phys Rev Lett, 2011, 106: 090504
8 Sacchi M F. Optimal discrimination of quantum operations. Phys Rev A, 2005, 71: 062340
9 Braunstein S L, van Loock P. Quantum information with continuous variables. Rev Mod Phys, 2005, 77: 513–577