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Stability and stabilization of a class of switched stochastic systems with saturation control

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Abstract A switching system always comprises of several subsystems and a rule supervising the switching between the subsystems. A major problem that is often inherent to all dynamical systems is actuator saturation. Saturation is a nonlinear property that nonlinearly maps small input signals to the output, which may affect the system properties and even destroy them. In this study, stability and stabilization of a class of switched stochastic systems with saturation control was investigated. First, the variation parameter method was used to present the integral form of switched stochastic systems. Second, to guarantee that the zero solution is globally exponentially stable in mean square, two sufficient conditions were obtained using direct computation with Gronwall inequality and indirect method with matrix theory, respectively. Further, another simple sufficient condition was obtained for the stability of the systems using the row norm, column norm, and Frobenius norm. Finally, two examples were used to illustrate the preciseness and effectiveness of the results. Moreover, various control designs were observed to stabilize the systems, which differ from the technique of linear matrix inequalities.

Keywords switched stochastic systems, matric theory, Gronwall inequality, stability

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1 Introduction

Switched systems are used to describe the switching law between various subsystems and have been studied considerably over the past decades because these systems have naturally physical applications and engineering applications [1–6]. The main challenge with switched systems, as usual systems [7], is their stability analysis and control signal. For instance, Aleksandrov et al. [8] investigated the stability and uniformly ultimate boundedness (UUB) control synthesis for nonlinear switched difference systems. Zong et al. [9–11] studied the finite-time H_∞ control for discrete-time switched nonlinear delay systems and switched linear parameter-varying systems with aero-engine applications. They also studied finite-time stability of interconnected impulsive switched systems.

In real world systems, such as stock prices or thermal fluctuations, stochastic perturbations are always present [12, 13]. Meanwhile, random behaviour exists in various phenomena. The theory of differential equations with stochastic perturbations and actuator saturation is also crucial in mathematical science. The authors [13–18] considered the stability and control of several stochastic differential systems. Zhang et al. [19, 20] introduced the design of highly nonlinear substitution boxes and graph convolutional broad network and its application in emotion recognition.

Stochastic switched systems are a class of switched systems containing numerous subsystems and a random switching signal showing the switching rules among the subsystems. Stochastic switched systems can effectively model the dynamical process impacted by abrupt changes. Hence, they have

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been used in many fields, such as biological models, physics, population science, and other engineering sciences [21–25]. For example, the authors [22, 24, 25] discussed the problem of stability for several switching semi-Markovian/Markovian jump systems. Further, the authors [23] investigated controller design for singular switching semi-Markovian jump systems with generally uncertain transition rates. Wu et al. [26, 27] considered the stabilization of Boolean control networks with stochastic switched signals and stochastic Boolean networks, respectively.

In practice, physical systems are always bounded by actuator constraints, such as speed, voltage, and torque. Saturation is a nonlinear property that nonlinearly maps small input signals to the output [28–35]. If the signal affected by the controller exceeds the limits of the actuator, then the actuator reaches saturation and the functionality of the controller will be reduced owing to the restrictions of the facilities. Saturation may affect the system properties and even destroy them [26,36,37]. The authors [28] presented a controller design for Markov jump systems subject to actuator saturation. In [36, 37], the authors introduced a new saturation control approach for linear systems by designing the linear feedback in a convex hull of a set of given matrices. Following the ideas in [36, 37], the authors in [38] studied the saturation control problem of switched stochastic systems with semi-Markovian switching signals using the technique of linear matrix inequalitys (LMIs). Many stability analyses use the Lyapunov's second method and LMI technique, and most existing results are effective on switched systems with all stable modes or a part of subsystems that is unstable. However, these results on the saturation problem of switched systems are often conservative owing to the limitations of the technique of LMIs.

In this study, globally exponentially stable in mean square (GESIMS) of the following switched stochastic system was considered using Gronwall inequality and matrix theory, which is different from the Lyapunov's second method and the technique of LMIs.

$$\mathrm{d}x(t) = [A_{\sigma(t)}x(t) + B_{\sigma(t)}\mathrm{sat}(u(t))]\mathrm{d}t + \sum_{k=1}^{d}[C_{k,\sigma(t)}x(t) + D_{k,\sigma(t)}\mathrm{sat}(u(t))]\mathrm{d}w_k(t),\ t \geqslant 0, \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state, $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ is the control input, $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{k,\sigma(t)}$, $D_{k,\sigma(t)}$ are the known mode-dependent constant matrices with appropriate dimensions. The switching signal $\sigma(t)$ is a semi-Markov process with finite state set $S = \{1, 2, \ldots, N\}$, $w_k(t)$ is the one-dimension Brownian motion and they are both on a complete probability space $(\Omega, \mathscr{F}, P)$ with the probability measure P. The map $\mathrm{sat}(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is a standard vector-valued saturation function, where $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \ldots, \mathrm{sat}(u_m)]$, and $\mathrm{sat}(u_j) = \mathrm{sign}(u_j)\min\{1, |u_j|\}$, $j \in \{1, 2, \ldots, m\}$. $\{\theta(s), -h \leqslant s \leqslant 0\}$ is $C([-h, 0]; \mathbb{R}^n)$-valued continuous function with a norm $\|\theta\| = \sup_{-h\leqslant\varsigma\leqslant0} |\theta(\varsigma)|$ and $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. $\mathrm{E}\{\}$ is the operator of the mathematical expectation about P. $\mathbb{R}$ is the real number set, A^{-1} is the inverse of a matrix A, $\rho(A)$ is the spectral radius of matrix A. If A is the symmetric matrix, $A \geqslant 0$ means that A is positive semi-definite. This study assumes that $w_k(t)$ and $\sigma(t)$ are independent.

Remark 1. Note that if the amount of time that the process requires in each state before making a transition is identically 1, then the semi-Markov process is just a Markov chain. Therefore, the semi-Markov process $\sigma(t)$ is an actual stochastic process that evolves over time t. Semi-Markov processes were introduced by Levy and Smith in 1950s and provide a model for many processes in queueing theory and reliability theory. In analytic issues, the study of semi-Markov processes reduces to a problem of integral equations.

To stabilize the switched stochastic system (1), the candidate mode-dependent controllers are designed as $u(t) = H_{\sigma(t)}x(t)$. Then, the closed-loop switched stochastic system (1) can be written as follows:

$$\mathrm{d}x(t) = [A_{\sigma(t)}x(t) + B_{\sigma(t)}\mathrm{sat}(H_{\sigma(t)}x(t))]\mathrm{d}t + \sum_{k=1}^{d}[C_{k,\sigma(t)}x(t) + D_{k,\sigma(t)}\mathrm{sat}(H_{\sigma(t)}x(t))]\mathrm{d}w_k(t),\ t \geqslant 0. \tag{2}$$

The initial value condition for system (2) are $x_i(t) = \theta_i(t) \in C([-\tau, 0], \mathbb{R})$.

Lemma 1 ([36, 37])**.** For given matrices $H_r, G_r \in \mathbb{R}^{m\times n}$, if $x(t) \in \ell(G_r)$, then $\mathrm{sat}(H_r x(t))$ can be represented as

$$\mathrm{sat}(H_r x(t)) = \sum_{p=1}^{2^m} \eta_p(E_p H_r + (I - E_p)G_r)x(t), \quad \sum_{p=1}^{2^m} \eta_p = 1, \quad 0 \leqslant \eta_p \leqslant 1,$$

where E_p is $m \times m$ diagonal matrix whose diagonal elements are either 1 or 0, and $\ell(G_r) = \{x(t) \in \mathbb{R}^n : |g_{rj}x(t)| \leqslant 1\}$, where g_{rj} is the j-th row of matrix G_r.

Lemma 2 ([36, 38, 39])**.** If matrices $M_i \in \mathbb{R}^{m\times n}$ $(i = 1, 2, \ldots, r)$ and a positive semi-definite matrix $P \in \mathbb{R}^{m\times m}$ are given. Suppose that $\sum_{i=1}^{r} \eta_i = 1,\ 0 \leqslant \eta_i \leqslant 1$, then

$$\left(\sum_{i=1}^{r} \eta_i M_i\right)^{\mathrm{T}} P \left(\sum_{i=1}^{r} \eta_i M_i\right) \leqslant \sum_{i=1}^{r} \eta_i M_i^{\mathrm{T}} P M_i.$$

Definition 1. The system (2) is said to be GESIMS if for any solution

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$$

of (2), there are constants $\lambda > 0$ and $m \geqslant 1$ such that

$$\mathrm{E}|x(t)|^2 \leqslant m\mathrm{E}\|\theta\|^2 \mathrm{e}^{-\lambda t},\ t \geqslant 0 \tag{3}$$

holds for any initial value $\theta(t) = (\theta_1(t), \theta_2(t), \ldots, \theta_n(t)) \in \mathcal{D} \subset \mathbb{R}^n$ and initial mode $r_0 \in S$, where $\mathcal{D}$ is the domain of attraction in GESIMS sense of the system (2).

The following result is well known:

Lemma 3. Let $\Lambda = (\mu_{ij})_{n\times n} \geqslant 0$. If $\rho(\Lambda) < 1$, then $(I - \Lambda)^{-1} \geqslant 0$, where I is the $n \times n$ identity matrix.

The following notations are also normal. $A_r = (a_{ij}^r)$, $B_r = (b_{ij}^r)$, $C_{k,r} = (c_{kij}^r)$, $D_{k,r} = (d_{kij}^r)$, $H_r = (h_{ij}^r)$, $E_r = (e_{ij}^r)$, $I - E_r = (e_{ij}'^r)$ with appropriate dimensions, where $r \in S$.

The main theorems about exponential stability in mean square (ESIMS) are explained in Section 2, and Section 3 presents two examples to prove that this study results are appealing.

2 Stability analysis

By the above analysis, Eq. (2) has the following form:

$$\begin{aligned} \mathrm{d}x(t) = &\left[A_{\sigma(t)} + B_{\sigma(t)} \sum_{p=1}^{2^m} \eta_p (E_p H_{\sigma(t)} + (I - E_p) G_{\sigma(t)})\right] x(t)\mathrm{d}t \\ &+ \sum_{k=1}^{d} \left[C_{k,\sigma(t)} + D_{k,\sigma(t)} \sum_{p=1}^{2^m} \eta_p (E_p H_{\sigma(t)} + (I - E_p) G_{\sigma(t)})\right] x(t)\mathrm{d}w_k(t), \end{aligned} \tag{4}$$

or

$$\begin{aligned} \mathrm{d}x_i(t) = &\sum_{p=1}^{2^m} \eta_p \left[\sum_{j=1}^{n} a_{ij}^r + \sum_{j=1}^{n} \left(\sum_{l=1}^{m}\sum_{v=1}^{m} b_{il}^r e_{lv}^p h_{vj}^r + \sum_{l=1}^{m}\sum_{v=1}^{m} b_{il}^r e_{lv}'^p g_{vj}^r\right)\right] x_j(t)\mathrm{d}t \\ &+ \sum_{p=1}^{2^m} \eta_p \sum_{k=1}^{d} \left[\sum_{j=1}^{n} c_{kij}^r + \sum_{j=1}^{n} \left(\sum_{l=1}^{m}\sum_{v=1}^{m} d_{kil}^r e_{lv}^p h_{vj}^r + \sum_{l=1}^{m}\sum_{v=1}^{m} d_{kil}^r e_{lv}'^p g_{vj}^r\right)\right] x_j(t)\mathrm{d}w_k(t) \\ = &\sum_{j=1}^{n} \alpha_{ij}^r x_j(t)\mathrm{d}t + \sum_{k=1}^{d}\sum_{j=1}^{n} \beta_{kij}^r x_j(t)\mathrm{d}w_k(t), \end{aligned} \tag{5}$$

where

$$\begin{aligned} \alpha_{ij}^r &= \sum_{p=1}^{2^m} \eta_p \left[a_{ij}^r + \sum_{l=1}^{m}\sum_{v=1}^{m} \left(b_{il}^r e_{lv}^p h_{vj}^r + b_{il}^r e_{lv}'^p g_{vj}^r\right)\right], \\ \beta_{kij}^r &= \sum_{p=1}^{2^m} \eta_p \left[c_{kij}^r + \sum_{l=1}^{m}\sum_{v=1}^{m} \left(d_{kil}^r e_{lv}^p h_{vj}^r + d_{kil}^r e_{lv}'^p g_{vj}^r\right)\right]. \end{aligned}$$

Theorem 1. If there are positive numbers $a_1, a_2, \ldots, a_n, \lambda$ and real numbers h_{lj}^r, g_{lr}^r such that

$$\min_{1\leqslant i\leqslant n}\{a_i\} > \lambda > 3\sum_{i=1}^{n}\left\{\frac{1}{a_i-\lambda}\left[\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^r h_{lj}^r+b_{il}^r g_{lj}^r\right)\right|\right]^2 + \left[\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^r h_{lj}^r+d_{kil}^r g_{lj}^r\right)\right|\right]^2\right\}. \tag{6}$$

Then the close-loop system (1) or (2) is GESIMS, where $\overline{a}_{ij}^r = a_{ij}^r (i\neq j)$, $\overline{a}_{ii}^r = a_{ij}^r + a_i$.

Proof. We rewrite (2) as

$$\mathrm{d}x_i(t) = \left[-a_i x_i(t)+\sum_{j=1}^{n}\overline{\alpha}_{ij}^r x_j(t)\right]\mathrm{d}t+\sum_{k=1}^{d}\sum_{j=1}^{n}\beta_{kij}^r x_j(t)\mathrm{d}w_k(t),$$

where $\overline{\alpha}_{ij}^r = \sum_{p=1}^{2^m}\eta_p[\overline{a}_{ij}^r+\sum_{l=1}^{m}\sum_{v=1}^{m}(b_{il}^r e_{lv}^p h_{vj}^r + b_{il}^r e_{lv}^{\prime p} g_{vj}^r)]$. For $t\geqslant 0$, $i=1,2,\ldots,n$, using the technique of variation, we have

$$x_i(t) = x_i(0)\mathrm{e}^{-a_i t}+\int_0^t \mathrm{e}^{-a_i(t-s)}\sum_{j=1}^{n}\overline{\alpha}_{ij}^r x_j(s)\mathrm{d}s+\int_0^t \mathrm{e}^{-a_i(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}\beta_{kij}^r x_j(s)\mathrm{d}w_k(s). \tag{7}$$

Hence

$$|x_i(t)| \leqslant |x_i(0)|\mathrm{e}^{-a_i t}+\int_0^t \mathrm{e}^{-a_i(t-s)}\sum_{j=1}^{n}|\overline{\alpha}_{ij}^r x_j(s)|\mathrm{d}s+\int_0^t \mathrm{e}^{-a_i(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}|\beta_{kij}^r x_j(s)|\mathrm{d}w_k(s). \tag{8}$$

Therefore,

$$\begin{aligned}
&\sup_{-\tau\leqslant u\leqslant t}|x_i(u)| \\
&\leqslant |x_i(0)|\mathrm{e}^{-a_i t}+\int_0^t\left(\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right)\left(\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{j=1}^{n}|\overline{\alpha}_{ij}^r|\right)\mathrm{d}s \\
&\quad+\int_0^t\left(\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right)\left(\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}|\beta_{kij}^r|\right)\mathrm{d}w_k(s) \\
&\leqslant |x_i(0)|\mathrm{e}^{-a_i t}+\int_0^t\left(\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right)\left(\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^r h_{lj}^r+b_{il}^r g_{lj}^r\right)\right|\right)\mathrm{d}s \\
&\quad+\int_0^t\left(\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right)\left[\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^r h_{lj}^r+d_{kil}^r g_{lj}^r\right)\right|\right]\mathrm{d}w_k(s).
\end{aligned} \tag{9}$$

As a result

$$\begin{aligned}
&\sup_{-\tau\leqslant u\leqslant t}\|x(u)\|^2 \\
&\leqslant 3\sum_{i=1}^{n}|x_i(0)|^2\mathrm{e}^{-\lambda t} \\
&\quad+3\sum_{i=1}^{n}\int_0^t\left[\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right]^2\mathrm{d}s\cdot\int_0^t\left[\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^r h_{lj}^r+b_{il}^r g_{lj}^r\right)\right|\right]^2\mathrm{d}s
\end{aligned}$$

$$+3\sum_{i=1}^{n}\int_0^t\left[\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right]^2\left[\mathrm{e}^{-(a_i-\lambda)(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^rh_{lj}^r+d_{kil}^rg_{lj}^r\right)\right|\right]^2\mathrm{d}s$$

$$\leqslant 3\sum_{i=1}^{n}|x_i(0)|^2\mathrm{e}^{-\lambda t}+\sum_{i=1}^{n}\left\{\frac{3}{a_i-\lambda}\left[\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^rh_{lj}^r+b_{il}^rg_{lj}^r\right)\right|\right]^2\right.$$

$$\left.+3\left[\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^rh_{lj}^r+d_{kil}^rg_{lj}^r\right)\right|\right]^2\right\}\int_0^t\left[\mathrm{e}^{-\lambda(t-s)}\sup_{-\tau\leqslant u\leqslant s}\|x(u)\|\right]^2\mathrm{d}s. \tag{10}$$

The substitution

$$x(t)=\mathrm{e}^{-\lambda t}y(t) \tag{11}$$

reduces (10) to the form

$$\sup_{-\tau\leqslant u\leqslant t}\mathrm{E}\|y(u)\|^2\leqslant 3\sum_{i=1}^{n}\mathrm{E}|\theta_i(0)|^2+\sum_{i=1}^{n}\left\{\frac{3}{a_i-\lambda}\left[\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^rh_{lj}^r+b_{il}^rg_{lj}^r\right)\right|\right]^2\right.$$

$$\left.+3\left[\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^rh_{lj}^r+d_{kil}^rg_{lj}^r\right)\right|\right]^2\right\}\int_0^t\sup_{-\tau\leqslant u\leqslant s}\|y(u)\|^2\mathrm{d}s. \tag{12}$$

Let

$$\eta:=3\sum_{i=1}^{n}\left\{\frac{1}{a_i-\lambda}\left[\sum_{j=1}^{n}\left|\overline{a}_{ij}^r+\sum_{l=1}^{m}\left(b_{il}^rh_{lj}^r+b_{il}^rg_{lj}^r\right)\right|\right]^2+\left[\sum_{k=1}^{d}\sum_{j=1}^{n}\left|c_{kij}^r+\sum_{l=1}^{m}\left(d_{kil}^rh_{lj}^r+d_{kil}^rg_{lj}^r\right)\right|\right]^2\right\}.$$

By the Gronwall's inequality, one gets

$$\sup_{-\tau\leqslant u\leqslant t}\mathrm{E}|y(u)|^2\leqslant 3\mathrm{E}\|\theta\|^2\mathrm{e}^{\eta t},$$

which along with (11) implies

$$\mathrm{E}|x(t)|^2\leqslant M\mathrm{E}\|\theta\|^2\mathrm{e}^{-(\lambda-\eta)t}.$$

The proof is completed.

Next, we will give another results of mean square exponential stability of (1) by a different view. Define

$$\overline{\alpha}_i^r=\sum_{j=1}^{n}(\overline{\alpha}_{ij}^r)^2,\quad \overline{\beta}_{ij}^r=\sum_{k=1}^{d}(\beta_{kij}^r)^2,\quad i=1,2,\ldots,n,\quad F=\mathrm{diag}(a_1,a_2,\ldots,a_n),$$

$$F_1^r=\mathrm{diag}(3\overline{\alpha}_1a_1^{-1},3\overline{\alpha}_2a_2^{-1},\ldots,3\overline{\alpha}_na_n^{-1}),\quad F_2^r=\left(\frac{3}{2}\overline{\beta}_{ij}^r\right)_{n\times n},\quad K=(k_{ij})_{n\times n},\quad k_{ij}=1,\quad 1\leqslant i,j\leqslant n,$$

where $\overline{a}_{ij}^r=a_{ij}^r(i\neq j)$, $\overline{a}_{ii}^r=a_{ii}^r+a_i$.

Theorem 2. If there are positive numbers $a_1,a_2,\ldots,a_n$ and real numbers h_{lj}^r,g_{lr}^r such that $\rho(F^{-1}(F_1^rK+F_2^r))<1$. Then the close-loop system (1) is GESIMS.

Proof. We rewrite (2) as

$$\mathrm{d}x_i(t)=\left[-a_ix_i(t)+\sum_{j=1}^{n}\overline{\alpha}_{ij}^rx_j(t)\right]\mathrm{d}t+\sum_{k=1}^{d}\sum_{j=1}^{n}\beta_{kij}^rx_j(t)\mathrm{d}w_k(t).$$

For $t\geqslant 0$, $i=1,2,\ldots,n$, using the technique of variation, we have

$$x_i(t)=x_i(0)\mathrm{e}^{-a_it}+\int_0^t\mathrm{e}^{-a_i(t-s)}\sum_{j=1}^{n}\overline{\alpha}_{ij}^rx_j(s)\mathrm{d}s+\int_0^t\mathrm{e}^{-a_i(t-s)}\sum_{k=1}^{d}\sum_{j=1}^{n}\beta_{kij}^rx_j(s)\mathrm{d}w_k(s).$$

Hence

$$|x_i(t)| \leqslant |x_i(0)|\mathrm{e}^{-a_it} + \int_0^t \mathrm{e}^{-a_i(t-s)} \sum_{j=1}^n |\overline{\alpha}_{ij}^r x_j(s)|\mathrm{d}s + \int_0^t \mathrm{e}^{-a_i(t-s)} \sum_{k=1}^d \sum_{j=1}^n |\beta_{kij}^r x_j(s)|\mathrm{d}w_k(s)$$
$$:= I_{i1} + I_{i2} + I_{i3}, \quad i = 1, 2, \ldots, n,$$

where, for convenience, we denote $|x_i(0)|\mathrm{e}^{-a_it} = I_{i1}$, $\int_0^t \mathrm{e}^{-a_i(t-s)} \sum_{j=1}^n |\overline{\alpha}_{ij}^r x_j(s)|\mathrm{d}s = I_{i2}$, and $\int_0^t \mathrm{e}^{-a_i(t-s)} \cdot \sum_{k=1}^d \sum_{j=1}^n |\beta_{kij}^r x_j(s)|\mathrm{d}w_k(s) = I_{i3}$. Multiplying $\mathrm{e}^{\lambda t}$ on both sides and taking expectations, for $\forall t \geqslant 0$, we obtain

$$\mathrm{e}^{\lambda t}\mathrm{E}[x_i(t)]^2 = \mathrm{e}^{\lambda t}\mathrm{E}(I_{i1} + I_{i2} + I_{i3})^2 \leqslant 3\mathrm{e}^{\lambda t}\mathrm{E}(I_{i1}^2 + I_{i2}^2 + I_{i3}^2).$$

Denote $G_i(t) = \sup_{-h\leqslant s\leqslant t} \mathrm{E}x_i^2(s)\mathrm{e}^{\lambda s}$. For some sufficiently small constant

$$\lambda \in \left(0, \min_i\{a_i : 1 \leqslant i \leqslant n\}\right),$$

we have

$$\begin{aligned}
\mathrm{e}^{\lambda t}\mathrm{E}I_{i2}^2 &\leqslant \mathrm{e}^{\lambda t}\mathrm{E}\left[\int_0^t \mathrm{e}^{-a_i(t-s)} \sum_{j=1}^n \overline{\alpha}_{ij}^r x_j(s)\mathrm{d}s\right]^2 = \mathrm{e}^{\lambda t}\mathrm{E}\left[\int_0^t \mathrm{e}^{-\frac{1}{2}a_i(t-s)}\mathrm{e}^{-\frac{1}{2}a_i(t-s)} \sum_{j=1}^n \overline{\alpha}_{ij}^r x_j(s)\mathrm{d}s\right]^2 \\
&\leqslant \mathrm{e}^{\lambda t}\mathrm{E}\left\{\left[\int_0^t \mathrm{e}^{-a_i(t-s)}\mathrm{d}s\right]\left[\int_0^t \mathrm{e}^{-a_i(t-s)}\left(\sum_{j=1}^n \overline{\alpha}_{ij}^r x_j(s)\right)^2\mathrm{d}s\right]\right\} \\
&\leqslant \frac{1}{a_i}\mathrm{e}^{\lambda t}\mathrm{E}\left[\int_0^t \mathrm{e}^{-a_i(t-s)}\left(\sum_{j=1}^n \overline{\alpha}_{ij}^r x_j(s)\right)^2\mathrm{d}s\right] \\
&\leqslant \frac{1}{a_i}\mathrm{e}^{\lambda t}\mathrm{E}\left[\int_0^t \mathrm{e}^{-a_i(t-s)}\left(\sum_{j=1}^n (\overline{\alpha}_{ij}^r)^2\right)\left(\sum_{j=1}^n x_j^2(s)\right)\mathrm{d}s\right] \\
&= \frac{1}{a_i}\left(\sum_{j=1}^n (\overline{\alpha}_{ij}^r)^2\right)\left[\int_0^t \mathrm{e}^{-(a_i-\lambda)(t-s)}\left(\sum_{j=1}^n \mathrm{e}^{\lambda s}\mathrm{E}x_j^2(s)\right)\mathrm{d}s\right] \\
&\leqslant \frac{1}{a_i(a_i-\lambda)}\left(\sum_{j=1}^n (\overline{\alpha}_{ij}^r)^2\right)\left[\sum_{j=1}^n G_j(t)\right].
\end{aligned}$$

Furthermore, due to $w(t) = (w_1(t), \ldots, w_n(t))^{\mathrm{T}}$ is the Brownian motion, we have $d\langle w_i, w_j\rangle_t = \delta_{ij}t, 1 \leqslant i, j \leqslant n$, where

$$\delta_{ij} = \begin{cases} 0, & i \neq j; \\ 1, & i = j. \end{cases}$$

Moreover, for any $f_i, f_j \in C[R]$, we have

$$\mathrm{E}\left(\int_0^t f_i(s)\mathrm{d}w_i(s) \int_0^t f_j(s)\mathrm{d}w_j(s)\right) = \mathrm{E}\int_0^t f_i(s)f_j(s)\mathrm{d}\langle w_i, w_j\rangle_s.$$

Then, we obtain

$$\begin{aligned}
\mathrm{e}^{\lambda t}\mathrm{E}I_{i3}^2 &\leqslant \mathrm{e}^{\lambda t}\mathrm{E}\left[\left(\int_0^t \mathrm{e}^{-a_i(t-s)} \sum_{k=1}^d \sum_{j=1}^n |\beta_{kij}^r x_j(s)|\mathrm{d}w_k(s)\right)\right]^2 \\
&= \sum_{j=1}^n \sum_{k=1}^d \left[\int_0^t \mathrm{e}^{-(2a_i-\lambda)(t-s)}(\mathrm{E}(\beta_{kij}^r \mathrm{e}^{\lambda s} x_j(s))^2\mathrm{d}s\right] \leqslant \frac{1}{2a_i-\lambda}\left(\sum_{j=1}^n \sum_{k=1}^d (\beta_{kij}^r)^2 G_j(t)\right).
\end{aligned}$$

Then, for any $t \geqslant 0$,

$$G_i(t) \leqslant 3\left\{\mathrm{E}x_i^2(0) + \frac{1}{a_i-\lambda}\left[\frac{1}{a_i}\left(\sum_{j=1}^{n}(\overline{\alpha}_{ij}^r)^2\right)\sum_{j=1}^{n}G_j(t) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{d}(\beta_{kij}^r)^2 G_j(t)\right]\right\}.$$

That is,

$$G(t) \leqslant 3\mathrm{E}x^2(0) + (F-\lambda I)^{-1}(F_1K+F_2)G(t),$$

where $G(t) = (G_1(t), G_2(t), \dots, G_n(t))^{\mathrm{T}}$, $\mathrm{E}x^2(0) = (\mathrm{E}x_1^2(0), \mathrm{E}x_2^2(0), \dots, \mathrm{E}x_n^2(0))^{\mathrm{T}}$. Since $\rho(F^{-1}(F_1K + F_2)) < 1$, $F^{-1}(F_1K + F_2)) \geqslant 0$, by using Lemma 3, we have $(I - F^{-1}(F_1K + F_2))^{-1} \geqslant 0$. So there is a positive constant $\alpha(< \lambda)$ satisfying

$$(I - (F-\alpha I)^{-1}(F_1K+F_2))^{-1} \geqslant 0.$$

Let

$$L(\alpha) = (L_{ij}(\alpha))_{n\times n} := (I - (F-\alpha I)^{-1}(F_1K+F_2))^{-1}.$$

We have

$$\mathrm{E}x^2(t) \leqslant 3L(\alpha)\mathrm{E}x^2(0)\mathrm{e}^{-\lambda t}.$$

Then, for $1 \leqslant i \leqslant n$,

$$\mathrm{E}x_i^2(t) \leqslant 3\sum_{j=1}^{n}L_{ij}(\alpha)\mathrm{E}x_j^2(0)\mathrm{e}^{-\lambda t} \leqslant 3\mathrm{e}^{-\lambda t}\sum_{j=1}^{n}L_{ij}(\alpha)\sum_{j=1}^{n}\mathrm{E}x_j^2(0).$$

As a result

$$\sum_{i=1}^{n}\mathrm{E}x_i^2(t) \leqslant 3\mathrm{e}^{-\lambda t}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}L_{ij}(\alpha)\right)\left(\sum_{j=1}^{n}\mathrm{E}x_j^2(0)\right),$$

that is,

$$\mathrm{E}|x(t)|^2 \leqslant 3\mathrm{e}^{-\lambda t}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}L_{ij}(\alpha)\right)\mathrm{E}\|\theta\|^2.$$

The proof is complete.

Corollary 1. The close-loop system (2) is GESIMS if one of the following case is satisfied: there are $a_1, a_2, \dots, a_n, p_1, p_2, \dots, p_n$ and and real numbers h_{lj}^r, g_{lr}^r such that

$$(1)\ \sum_{j=1}^{n}\left[\frac{p_i}{p_j}\left(\frac{\overline{\alpha}_i^r}{a_i^2} + \frac{\overline{\beta}_{ij}^r}{2a_i}\right)\right] < \frac{1}{3},\quad 1 \leqslant i \leqslant n;$$

$$(2)\ \sum_{i=1}^{n}\left[\frac{p_i}{p_j}\left(\frac{\overline{\alpha}_i^r}{a_i^2} + \frac{\overline{\beta}_{ij}^r}{2a_i}\right)\right] < \frac{1}{3},\quad 1 \leqslant j \leqslant n;$$

$$(3)\ \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{p_i}{p_j}\left(\frac{\overline{\alpha}_i^r}{a_i^2} + \frac{\overline{\beta}_{ij}^r}{2a_i}\right)\right]^2 < \frac{1}{9}.$$

Proof. Since $\rho(A) \leqslant \|A\|$, and $\|A\| = \|P^{-1}AP\|$ for any non-singular $P = \mathrm{diag}\{p_1, p_2, \dots, p_n\} > 0$, we can obtain the above results by Theorem 2, when the corresponding ones are the row norm, column norm and Frobenius norm.

Remark 2. Notice that $\overline{a}_{ii} = a_{ii} + a_i$ and the real number α depends on the choose of a_i, λ, and $V(x(t)) = x^{\mathrm{T}}(t)Px(t)$ may be chosen as a Lyapunov function of system (1), where $P = \mathrm{diag}(a_1, a_2, \dots, a_n)$.

Remark 3. The result of Theorem 1 is obtained by the direct computation in analytic terms, whereas the result of Theorem 2 is an algebraic expression by the non-direct method for matrix theory.

Remark 4. From Theorems 1 and 2, we see that there are many control designs to stabilize the systems. However, the results [38] in terms of the technique of LMIs provided only a possible method of control design. The following examples in Section 3 will show these issues.

3 Examples

Example 1. Consider a stochastic differential system (1) with $\sigma(t)$ taking values in $S=\{1,2,3\}$ and

$$A_1=\begin{pmatrix}-2 & 1\\ 1 & -2\end{pmatrix},\ A_2=\begin{pmatrix}0.3 & 0.2\\ 0.3 & -0.4\end{pmatrix},\ A_3=\begin{pmatrix}-0.4 & -0.1\\ 0.2 & -0.5\end{pmatrix},\ B_1=\begin{pmatrix}-4 & 0\\ 0 & 6\end{pmatrix},\ B_2=\begin{pmatrix}-6 & 0\\ 0 & -6\end{pmatrix},$$
$$B_3=\begin{pmatrix}-3 & 0\\ 0 & 2\end{pmatrix},\ C_1=\begin{pmatrix}3 & 0.2\\ 0 & 1\end{pmatrix},\ C_2=\begin{pmatrix}-0.1 & 0.5\\ 0.5 & -0.1\end{pmatrix},\ C_3=\begin{pmatrix}0.5 & 0.1\\ 0.1 & 0.5\end{pmatrix},\ D_1=\begin{pmatrix}-2 & 0\\ 0 & 3\end{pmatrix},$$
$$D_2=\begin{pmatrix}0.2 & 0\\ 0 & -1\end{pmatrix},\ D_3=\begin{pmatrix}0.5 & 0\\ 0 & 1\end{pmatrix}.$$

If we choose $a_1=a_2=100,\ \lambda=35$, and

$$H_1=\begin{pmatrix}0.03 & 0.2\\ 0 & 0.2\end{pmatrix},\ H_2=\begin{pmatrix}-0.02 & -\frac{1}{6}\\ 0.5 & -\frac{1}{15}\end{pmatrix},\ H_3=\begin{pmatrix}0.01 & \frac{1}{3}\\ -0.1 & 1\end{pmatrix}, \tag{13}$$

$$G_1=\begin{pmatrix}0.02 & -\frac{1}{6}\\ 0 & -\frac{4}{15}\end{pmatrix},\ G_2=\begin{pmatrix}0.07 & 0.2\\ -0.5 & 0\end{pmatrix},\ G_3=\begin{pmatrix}0.04 & -0.3\\ 0.1 & -\frac{16}{15}\end{pmatrix}, \tag{14}$$

For H_1 and G_1, we have

$$3\left\{\frac{1}{a_1-\lambda}(|-2-4h_{11}-4g_{11}|+|1-4h_{12}-4g_{12}|)^2+(|3-2h_{11}-2g_{11}|+|0.2-2h_{12}-2g_{12}|)^2\right\}$$
$$+3\left\{\frac{1}{a_2-\lambda}(|1+6h_{21}+6g_{21}|+|-2+6h_{22}+6g_{22}|)^2+(|0+3h_{21}+3g_{21}|+|1+3h_{22}+3g_{22}|)^2\right\}\approx 12.0.$$

For H_2 and G_2,

$$3\left\{\frac{(|0.3-6(h_{11}+g_{11})|+|0.2-6(h_{12}+g_{12})|)^2}{a_1-\lambda}+(|-0.1+0.2(h_{11}+g_{11})|+|0.5+0.2(h_{12}+g_{12})|)^2\right\}$$
$$+3\left\{\frac{(|0.3-6(h_{21}+g_{21})|+|-0.4-6(h_{22}+g_{22})|)^2}{a_2-\lambda}+(|0.5-h_{21}-g_{21}|+|-0.1-h_{22}-g_{22}|)^2\right\}\approx 1.991.$$

Also, for H_3 and G_3,

$$3\left\{\frac{(|0.4-3h_{11}-3g_{11}|+|-0.1-3h_{12}-3g_{12}|)^2}{a_1-\lambda}+(|0.5+0.5h_{11}+0.5g_{11}|+|0.1+0.5h_{12}+0.5g_{12}|)^2\right\}$$
$$+3\left\{\frac{(|0.2+2h_{21}+2g_{21}|+|-0.5+2h_{22}+2g_{22}|)^2}{a_2-\lambda}+(|0.1+h_{21}+g_{21}|+|0.5+h_{22}+g_{22}|)^2\right\}\approx 2.133.$$

So there are positive numbers a_1,a_2,λ and h_{lj}^r, g_{lr}^r such that $\min\{a_1,a_2\}>\lambda>\max\{11.99,1.991,2.133\}$, by Theorem 1, the above system (1) having the above parameters is GESIMS, which can be seen from the following Figure 1(a). The corresponding switching and control signals of the system are shown in Figures 1(b) and (c).

Moreover, if we choose $a_1=a_2=100,\ \lambda=35$, and

$$H_1=\begin{pmatrix}0.06 & 0.5\\ 0.3 & -\frac{4}{15}\end{pmatrix},\ H_2=\begin{pmatrix}0.03 & -\frac{1}{6}\\ 0.7 & 0\end{pmatrix},\ H_3=\begin{pmatrix}-0.03 & \frac{1}{3}\\ 1 & -\frac{1}{6}\end{pmatrix}, \tag{15}$$

$$G_1=\begin{pmatrix}-0.01 & -\frac{7}{15}\\ -0.3 & 0.2\end{pmatrix},\ G_2=\begin{pmatrix}0.02 & 0.2\\ -0.7 & -\frac{1}{15}\end{pmatrix},\ G_3=\begin{pmatrix}0.08 & -0.3\\ -1 & 0.1\end{pmatrix}. \tag{16}$$

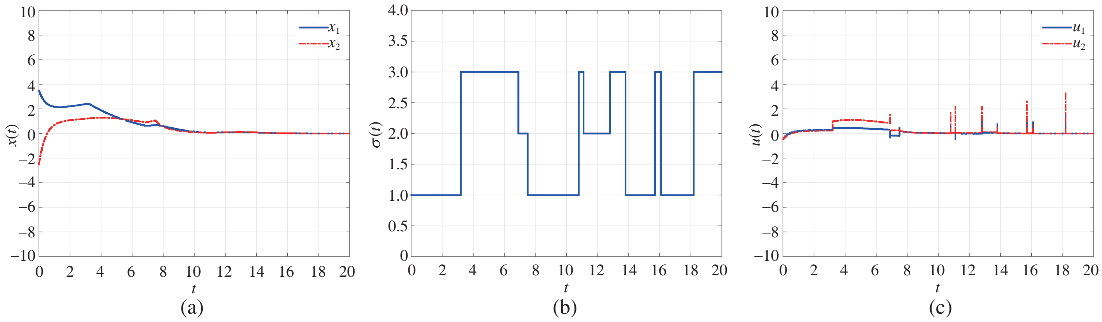

Figure 1 (Color online) (a) The solution trajectory, (b) the switching signal, and (c) the control signal in Example 1 with Eqs. (13) and (14).

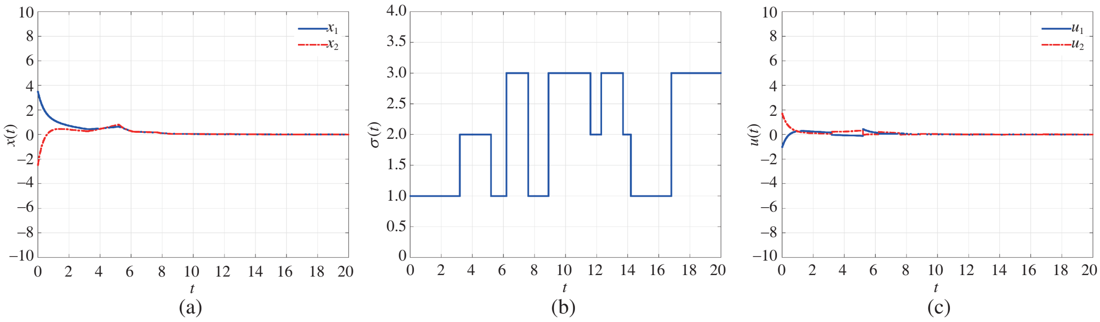

Figure 2 (Color online) (a) The solution trajectory, (b) the switching signal, and (c) the control signal in Example 1 with Eqs. (15) and (16).

In this case, we know that (6) is satisfied. Then the above system (1) is GESIMS by Theorem 1, which can be seen from the following Figure 2(a). The corresponding switching and control signals of the system are shown in Figures 2(b) and (c).

Remark 5. The above example is Example 3.1 in [38]. From our figures above and its part in the simulation of [38], we find that our control signals and the solution trajectories of system (1) in the above example are more practical than the corresponding parts in [38]. As a matter of fact, the control signals of switched system (1) are always sustainable in its whole during time as ours, not just having huge impact that may destroy a piece of equipment in the first during time as in [38].

Example 1. Consider a stochastic differential system (1) with $\sigma(t)$ taking values in $S=\{1,2,3\}$ and

$$A_1=\begin{pmatrix}-2 & 1\\ 0.6 & -1.5\end{pmatrix},\ A_2=\begin{pmatrix}0.3 & 0.2\\ 0.3 & -2.4\end{pmatrix},\ A_3=\begin{pmatrix}-1 & -0.1\\ 0.2 & -2.5\end{pmatrix},\ B_1=\begin{pmatrix}-3 & 0\\ 0 & 2\end{pmatrix},\ B_2=\begin{pmatrix}-5 & 0\\ 0 & -3\end{pmatrix},$$
$$B_3=\begin{pmatrix}-4 & 0\\ 0 & 5\end{pmatrix},\ C_1=\begin{pmatrix}2.3 & 0.2\\ 0 & 1\end{pmatrix},\ C_2=\begin{pmatrix}-2 & 1.4\\ 0.5 & -0.2\end{pmatrix},\ C_3=\begin{pmatrix}0.3 & 0.2\\ 0.4 & 0.5\end{pmatrix},\ D_1=\begin{pmatrix}-2 & 0\\ 0 & 5\end{pmatrix},$$
$$D_2=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ D_3=\begin{pmatrix}0.7 & 0\\ 0 & 0.7\end{pmatrix}.$$

Then, for H_1 and G_1, we have

$$(\overline{\alpha}^1_{ij})=\begin{pmatrix}-2+a_1-3(h_{11}+g_{11}) & 1-3(h_{12}+g_{12})\\ 0.6+2(h_{21}+g_{21}) & -1.5+a_2+2(h_{22}+g_{22})\end{pmatrix},$$
$$(\beta^1_{ij})=\begin{pmatrix}2.3-2(h_{11}+g_{11}) & 0.2-2(h_{12}+g_{12})\\ 5(h_{21}+g_{21}) & 0.6+5(h_{22}+g_{22})\end{pmatrix}.$$

For H_2 and G_2, we have

$$(\overline{\alpha}_{ij}^2) = \begin{pmatrix} 0.3 + a_1 - 5(h_{11} + g_{11}) & 0.2 - 5(h_{12} + g_{12}) \\ 0.3 - 3(h_{21} + g_{21}) & -2.4 + a_2 - 3(h_{22} + g_{22}) \end{pmatrix},$$
$$(\beta_{ij}^2) = \begin{pmatrix} -2 + (h_{11} + g_{11}) & 1.4 + (h_{12} + g_{12}) \\ 0.5 - (h_{21} + g_{21}) & -0.2 - (h_{22} + g_{22}) \end{pmatrix}.$$

For H_3 and G_3, we have

$$(\overline{\alpha}_{ij}^3) = \begin{pmatrix} -1 + a_1 - 4(h_{11} + g_{11}) & -0.1 - 4(h_{12} + g_{12}) \\ 0.2 + 5(h_{21} + g_{21}) & -2.5 + a_2 + 5(h_{22} + g_{22}) \end{pmatrix},$$
$$(\beta_{ij}^3) = \begin{pmatrix} 0.3 + 0.7(h_{11} + g_{11}) & 0.2 + 0.7(h_{12} + g_{12}) \\ 0.4 + 0.8(h_{21} + g_{21}) & 0.5 + 0.8(h_{22} + g_{22}) \end{pmatrix}.$$

Let

$$F_1^r = \begin{pmatrix} f_{11}^r & 0 \\ 0 & f_{22}^r \end{pmatrix}, \quad r = 1, 2, 3,$$

where, for H_1 and G_1,

$$f_{11}^1 = \frac{3}{a_1}\{[-2 + a_1 - 3(h_{11} + g_{11})]^2 + [1 - 3(h_{12} + g_{12})]^2\},$$
$$f_{22}^1 = \frac{3}{a_2}\{[0.6 + 2(h_{21} + g_{21})]^2 + [-1.5 + a_2 + 2(h_{22} + g_{22})]^2\};$$

for H_2 and G_2,

$$f_{11}^2 = \frac{3}{a_1}\{[0.3 + a_1 - 5(h_{11} + g_{11})]^2 + [0.2 - 5(h_{12} + g_{12})]^2\},$$
$$f_{22}^2 = \frac{3}{a_2}\{[0.3 - 3(h_{21} + g_{21})]^2 + [-2.4 + a_2 - 3(h_{22} + g_{22})]^2\};$$

and for H_3 and G_3,

$$f_{11}^3 = \frac{3}{a_1}\{[-1 + a_1 - 4(h_{11} + g_{11})]^2 + [-0.1 - 4(h_{12} + g_{12})]^2\},$$
$$f_{22}^3 = \frac{3}{a_2}\{[0.2 + 5(h_{21} + g_{21})]^2 + [-2.5 + a_2 + 5(h_{22} + g_{22})]^2\}.$$

Then there must be, for H_1 and G_1,

$$F_1^1 K + F_2^1 = \begin{pmatrix} f_{11}^1 + \frac{3}{a_2} \times [2.3 - 2(h_{11} + g_{11})]^2 & f_{11}^1 + \frac{3}{a_2} \times [0.2 - 2(h_{11} + g_{11})]^2 \\ f_{22}^1 + \frac{3}{a_2} \times [0 - 5(h_{21} + g_{21})]^2 & f_{22}^1 + \frac{3}{a_2} \times [0.6 - 5(h_{22} + g_{22})]^2 \end{pmatrix},$$

for H_2 and G_2,

$$F_1^2 K + F_2^2 = \begin{pmatrix} f_{11}^1 + \frac{3}{a_2} \times [-2 + (h_{11} + g_{11})]^2 & f_{11}^1 + \frac{3}{a_2} \times [1.4 + (h_{11} + g_{11})]^2 \\ f_{22}^1 + \frac{3}{a_2} \times [0.5 - (h_{21} + g_{21})]^2 & f_{22}^1 + \frac{3}{a_2} \times [-0.2 - (h_{22} + g_{22})]^2 \end{pmatrix},$$

for H_3 and G_3,

$$F_1^3 K + F_2^3 = \begin{pmatrix} f_{11}^1 + \frac{3}{a_2} \times [0.3 + 0.7(h_{11} + g_{11})]^2 & f_{11}^1 + \frac{3}{a_2} \times [0.2 + 0.7(h_{11} + g_{11})]^2 \\ f_{22}^1 + \frac{3}{a_2} \times [0.4 + 0.8(h_{21} + g_{21})]^2 & f_{22}^1 + \frac{3}{a_2} \times [0.5 + 0.8(h_{22} + g_{22})]^2 \end{pmatrix}.$$

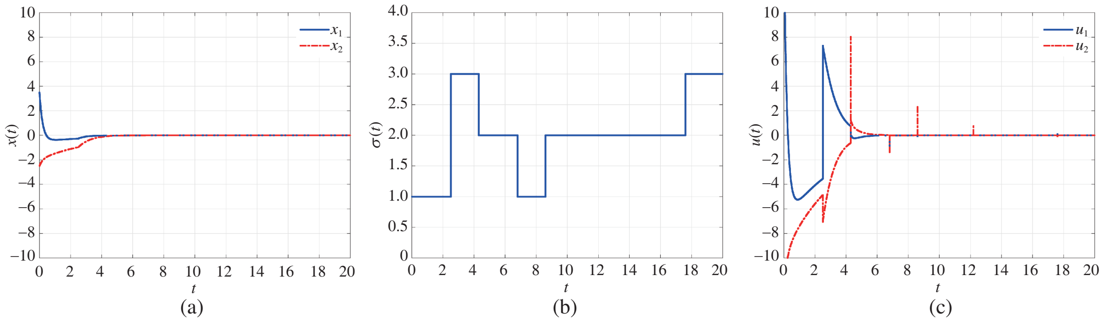

Figure 3 (Color online) (a) The solution trajectory, (b) the switching signal, and (c) the control signal in Example 2 with Eqs. (17) and (18).

If we choose $a_1 = 5,\ a_2 = 2$ and

$$H_1 = \begin{pmatrix} 0.6 & 0.2 \\ 0 & 0.5 \end{pmatrix},\ H_2 = \begin{pmatrix} 1.3 & -0.3 \\ 0.5 & -1.5 \end{pmatrix},\ H_3 = \begin{pmatrix} 0.9 & -1 \\ 0.1 & 0.7 \end{pmatrix}, \tag{17}$$

$$G_1 = \begin{pmatrix} 0.4 & -0.2 \\ 0 & -0.5 \end{pmatrix},\ G_2 = \begin{pmatrix} -0.3 & 0.3 \\ -0.5 & 1.5 \end{pmatrix},\ G_3 = \begin{pmatrix} 0.1 & 1 \\ -0.1 & -0.7 \end{pmatrix}, \tag{18}$$

then we have

$$F^{-1}(F_1^1K + F_2^1) = \begin{pmatrix} 0.147 & 0.132 \\ 0.4575 & 0.7275 \end{pmatrix},\ F^{-1}(F_1^2K + F_2^2) = \begin{pmatrix} 0.3156 & 0.6036 \\ 0.3075 & 0.2175 \end{pmatrix},$$
$$F^{-1}(F_1^3K + F_2^3) = \begin{pmatrix} 0.3012 & 0.0132 \\ 0.3375 & 0.4050 \end{pmatrix}.$$

Thus

$$\rho(F^{-1}(F_1^rK + F_2^r)) < 0.9585 < 1, \quad r \in \{1, 2, 3\}.$$

Then the above system (1) is GESIMS by Theorem 2, which can be seen from the following Figure 3(a). The corresponding switching and control signals of the system are shown in Figures 3(b) and (c).

What is more, if we choose $a_1 = 5,\ a_2 = 2$ and

$$H_1 = \begin{pmatrix} 1.2 & -0.7 \\ 0.8 & 1.1 \end{pmatrix},\ H_2 = \begin{pmatrix} 0.3 & -0.3 \\ 0.5 & -0.6 \end{pmatrix},\ H_3 = \begin{pmatrix} 0.5 & -0.3 \\ 1.5 & 0.7 \end{pmatrix}, \tag{19}$$

$$G_1 = \begin{pmatrix} -0.2 & 0.7 \\ -0.8 & -1.1 \end{pmatrix},\ G_2 = \begin{pmatrix} 0.7 & 0.3 \\ -0.5 & 0.6 \end{pmatrix},\ G_3 = \begin{pmatrix} 0.5 & 0.3 \\ -1.5 & -0.7 \end{pmatrix}. \tag{20}$$

In this case, we also get

$$\rho(F^{-1}(F_1^rK + F_2^r)) < 1, \quad r \in \{1, 2, 3\}.$$

Then the above system (1) is GESIMS by Theorem 2, which can be seen from the following Figure 4(a). The corresponding switching and control signals of the system are shown in Figures 4(b) and (c).

Remark 6. The estimation of the corresponding domain of attractions of Examples 1 and 2 are $\{x \in R^2 : x^{\mathrm{T}}P_1x < 1\}$ and $\{x \in R^2 : x^{\mathrm{T}}P_2x < 1\}$, respectively, where $P_1 = \mathrm{diag}(100, 100), P_2 = \mathrm{diag}(5, 2)$. Therefore, Theorem 2 appears to be more non-conservative compared with Theorem 1 from the above examples. Moreover, we have carefully studied the references [40, 41] involving the domain of attractions and revealed that the domain of attractions of the closed-loop systems is a very compelling problem that might be a subject for another research in the future.

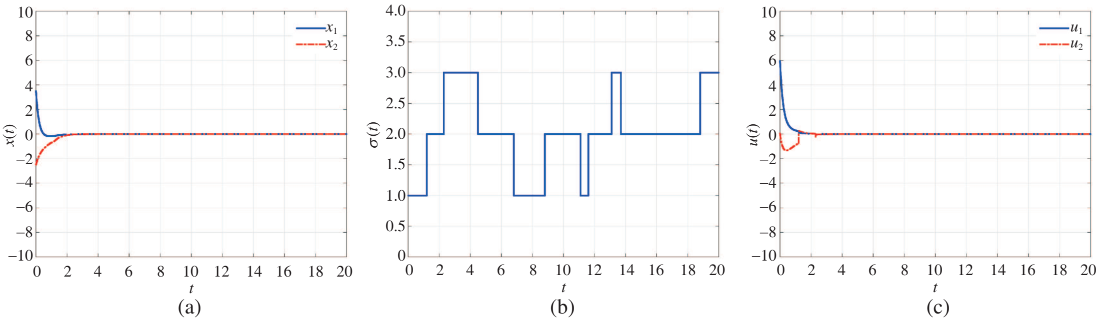

Figure 4 (Color online) (a) The solution trajectory, (b) the switching signal, and (c) the control signal in Example 2 with Eqs. (19) and (20).

4 Conclusion

In this study, by choosing different inequality and technique, we obtain and prove the two theorems about GESIMS for a class of stochastic switched differential systems. Finally, we present two examples to illustrate our results.

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