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• **Supplementary File** •

An Iterative BiGAMP-Based Receiver for Coded Massive MIMO Systems With Low-Resolution ADCs

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Appendix A   Iterative BiGAMP-based JCD and Decoding

In Algorithm A1, $N_{\rm JCD}$ and $N_{\rm outer}$ denote the maximum number of inner and outer iterations, respectively. Besides, $\mathcal{T}_p$ and $\mathcal{T}_d$ are used to distinguish the index intervals of pilots and data symbols. D_k refers to the decoding flag of the k-th user, which is determined by the CRC result after decoding.

Algorithm A1 Iterative BiGAMP-based JCD and Decoding

Require: Quantized observations $\mathbf{R}$, pilots $\mathbf{X}^P$.
Ensure: Channel estimates $\hat{\mathbf{H}}$, decoding results.

Initialize: $\forall m,k,t: \hat{s}_{mt}(0)=0, \hat{h}_{mk}(1)=10^{-5}, v^h_{mk}(1)=\frac{1}{K}$, $\hat{x}_{k,t\in\mathcal{T}_p}(1)=x^P_{kt}, v^x_{k,t\in\mathcal{T}_p}(1)=0, \hat{x}_{k,t\in\mathcal{T}_d}(1)=0, v^x_{k,t\in\mathcal{T}_d}(1)=1$, $D_k=0$; $n\leftarrow 1$.
while $n\leqslant N_{outer}$ **and** $\exists k: D_k=0$ **do**
  $\xi\leftarrow 1$.
  while $\xi\leqslant N_{JCD}$ **do**
    $\forall m,t: \bar{v}^p_{mt}(\xi)=\sum_k |\hat{h}_{mk}(\xi)|^2 v^x_{kt}(\xi)+v^h_{mk}(\xi)|\hat{x}_{mt}(\xi)|^2$.
    $\forall m,t: \bar{p}_{mt}(\xi)=\sum_k \hat{h}_{mk}(\xi)\hat{x}_{mt}(\xi)$.
    $\forall m,t: v^p_{mt}(\xi)=\bar{v}^p_{mt}(\xi)+\sum_k v^h_{mk}(\xi)v^x_{kt}(\xi)$.
    $\forall m,t: \hat{p}_{mt}(\xi)=\bar{p}_{mt}(\xi)-\hat{s}_{mt}(\xi-1)\bar{v}^p_{mt}(\xi)$.
    $\forall m,t: \hat{s}_{mt}(\xi)=\frac{r_{mt}/\rho-\hat{p}_{mt}(\xi)}{v^p_{mt}(\xi)+\gamma^2}, v^s_{mt}(\xi)=\frac{1}{v^p_{mt}(\xi)+\gamma^2}$.
    if $D_k=0$ **and** $t\in\mathcal{T}_d$ **then**
      $\forall k,t: v^r_{kt}(\xi)=[\sum_m |\hat{h}_{mk}(\xi)|^2 v^s_{mt}(\xi)]^{-1}$.
      $\forall k,t: \hat{r}_{kt}(\xi)=\hat{x}_{kt}(\xi)[1-v^r_{kt}(\xi)\sum_m v^h_{mk}(\xi)v^s_{mt}(\xi)]+v^r_{kt}(\xi)\sum_m \hat{h}^*_{mk}(\xi)\hat{s}_{mt}(\xi)$.
      $\forall k,t: \hat{x}_{kt}(\xi+1)=\mathbb{E}\{x_{kt}|\hat{r}_{kt}(\xi),v^r_{kt}(\xi)\}, v^x_{kt}(\xi+1)=\mathbb{V}\{x_{kt}|\hat{r}_{kt}(\xi),v^r_{kt}(\xi)\}$.
    else
      $\forall k,t: \hat{x}_{kt}(\xi+1)=\hat{x}_{kt}(\xi), v^x_{kt}(\xi+1)=v^x_{kt}(\xi)$.
    end if
    $\forall m,k: v^q_{mk}(\xi)=[\sum_t |\hat{x}_{kt}(\xi+1)|^2 v^s_{mt}(\xi)]^{-1}$.
    $\forall m,k: \hat{q}_{mk}(\xi)=\hat{h}_{mk}(\xi)[1-v^q_{mk}(\xi)\sum_t v^x_{kt}(\xi+1)v^s_{mt}(\xi)]+v^q_{mk}(\xi)\sum_t \hat{x}^*_{kt}(\xi+1)\hat{s}_{mt}(\xi)$.
    $\forall m,k: \hat{h}_{mk}(\xi+1)=\frac{\hat{q}_{mk}(\xi)}{1+Kv^q_{mk}(\xi)}, v^h_{mk}(\xi+1)=\frac{v^q_{mk}(\xi)}{1+Kv^q_{mk}(\xi)}$.
    $\xi\leftarrow\xi+1$.
  end while
  Compute the extrinsic LLRs $\mathbf{\Lambda}$ as the input of the decoder via $\Lambda(x^q_{kt})=\ln\frac{\sum_{x_{kt}\in\Omega^0_q}\mathcal{P}(x_{kt})}{\sum_{x_{kt}\in\Omega^1_q}\mathcal{P}(x_{kt})}-\Pi(x^q_{kt})$.
  for $k=1\rightarrow K$ **do**
    if $D_k=0$ **then**
      Conduct decoding to obtain the extrinsic soft outputs $\mathbf{\Pi}$, if the corresponding hard decisions pass the CRC detection, then let $D_k=1$;
      if $D_k=1$ **then**
        Reconstruct $\hat{x}_{kt}(\xi)$ by re-encoding and modulating the decoded bits and let $v^x_{kt}(\xi)=0$;
      else
        Conduct soft modulation with $\mathbf{\Pi}$ to obtain $\hat{x}_{kt}(\xi)$ and $v^x_{kt}(\xi)$.
      end if
    end if
  end for
  $\forall m,k,t: \hat{s}_{mt}(0)=\hat{s}_{mt}(\xi-1), \hat{h}_{mk}(1)=\hat{h}_{mk}(\xi), v^h_{mk}(1)=v^h_{mk}(\xi), \hat{x}_{kt}(1)=\hat{x}_{kt}(\xi), v^x_{kt}(1)=v^x_{kt}(\xi)$.
  $n\leftarrow n+1$.
end while

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Appendix B MMSE detection with CRC-aided hard interference cancellation

1. Recover the original data symbols of the k-th user, denoted as $\mathbf{x}_k = [x_{k1}, x_{k2}, ..., x_{kt}, ...]$, by re-encoding and modulating the decoded bits that has passed the CRC detection.

2. Subtract their contributions from the observed signals $\mathbf{R}^{\text{left}}$, which corresponds to the left undetected symbols $\mathbf{X}^{\text{left}}$. As a result, the remainder can be expressed as

$$\mathbf{R}_c^{\text{left}} = \mathbf{R}^{\text{left}} - \rho \sum\nolimits_{k\in\{k|D_{\mathrm{k}}=1\}} \hat{\mathbf{h}}_k \mathbf{x}_k^{\text{left}}, \tag{B1}$$

   where $\hat{\mathbf{h}}_k = [h_{1k}, h_{2k}, ..., h_{Mk}]^T$ refers to the estimated channel vector of the k-th user.

3. Compute the MMSE equalization matrix according to

$$\hat{\mathbf{W}} = \rho \hat{\mathbf{H}}_c^H (\rho^2 \hat{\mathbf{H}}_c \hat{\mathbf{H}}_c^H + \gamma^2 \mathbf{I}_M)^{-1}, \tag{B2}$$

   where $\hat{\mathbf{H}}_c$ is the submatrix of $\hat{\mathbf{H}}$ obtained by removing the column vectors with the subscript k satisfying $D_k = 1$, and the noise variance is substituted by γ^2 in Eq.(3). Thus, the estimate of the data symbol matrix is given by

$$\hat{\mathbf{X}}_c^{\text{left}} = \hat{\mathbf{W}} \mathbf{R}_c^{\text{left}}. \tag{B3}$$

4. Calculate the LLRs $\mathbf{\Lambda}_c^{\text{left}}$ required for channel decoding via (the superscript $left$ is omitted for the sake of clarity)

$$\Lambda_c(x_{kt}^q) = \ln \frac{\sum_{x_{kt}\in\Omega_q^1} \exp(-|\hat{x}_{kt} - \mu_k x_{kt}|^2/\varepsilon_k^2)}{\sum_{x_{kt}\in\Omega_q^0} \exp(-|\hat{x}_{kt} - \mu_k x_{kt}|^2/\varepsilon_k^2)}, \tag{B4}$$

   where $\mu_k = (\rho \hat{\mathbf{W}} \hat{\mathbf{H}}_c)_{k,k}$ and $\varepsilon_k^2 = \mu_k - \mu_k^2$. Hence, by combining the LLRs obtained from the above two stages as a complete input of the channel decoder, the iterative receiver is enabled.