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OFF-grid full-dimension channel estimation for mmWave/THz systems with angular block prior

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Dear editor,

The acquirement of the channel state information (CSI) is indispensable for designing efficient beamformers [1]. Unfortunately, estimating CSI is extremely challenging in the mmWave/THz systems where the number of antennas is super-large and the receive signal-to-noise ratio (SNR) is relatively low. Most of the existing estimators are based on the direct applications of inherent channel sparsity and generic compressive sensing (CS) techniques, which cannot lead to fine enough performances in channel estimation because it puts strict requirements on the channel sparsity or the data measurements [2–7].

In the mmWave/THz systems, the channels exhibit strong line-of-sight (LoS), hence it is reasonable to obtain the ranges of angles of departure/arrival (AoDs/AoAs) in advance. In this letter, we propose a super-resolution channel estimator that can incorporate the prior knowledge of angular blocks to enhance the performance of recovering channel with continuous-valued angles, based on weighted atomic norm minimization.

We assume a full-dimension (FD) mmWave/THz communication system, where the base station (BS) and the receiver pack uniform planar array (UPA) with $N = N_1N_2$ transmit antennas and $M = N_3N_4$ receive antennas, respectively. The downlink channel between the BS and the receiver can be modeled according to a geometric channel model composed of L multi-paths [5]. Then the channel matrix $\boldsymbol{H}$ during a coherent time block of multiple time slots can be expressed as

$$\boldsymbol{H} = \sum_{l=1}^{L} \alpha_l \boldsymbol{b}_{R,l} \boldsymbol{b}_{T,l}^{\mathrm{H}} \in \mathbb{C}^{M\times N}, \tag{1}$$

where $\alpha_l \sim \mathcal{CN}(0, \sigma_l^2) \in \mathbb{C}$ denotes the complex gain of the l-th multi-path, $l = 1, \ldots, L$, and the set of vectors $\{\boldsymbol{b}_{T,l}\}_{l=1}^{L}$ denotes the steering responses of the transmit arrays with half-wavelength antenna element separation along the elevation-and-azimuth-axis. Then the UPA transmit array response can be expressed as

$$\boldsymbol{b}_{T,l} = \boldsymbol{e}_{N_1}(g_{l1}) \otimes \boldsymbol{e}_{N_2}(g_{l2}), \tag{2}$$

where $g_{l1} = \sin(\theta_l)\cos(\phi_l)$, $g_{l2} = \cos(\theta_l)$, θ_l and ϕ_l are the elevation- and azimuth- AoDs, respectively, $\otimes$ denots the Kronecker product, and the array response of an n-dimension uniform linear array (ULA) with half-wavelength separation between adjacent antenna elements, is in the form with frequency $g \in [-1, 1)$:

$$\boldsymbol{e}_n(g) = \frac{1}{\sqrt{n}}[1, \mathrm{e}^{\mathrm{j}\pi g}, \ldots, \mathrm{e}^{\mathrm{j}\pi(n-1)g}]^{\mathrm{T}} \in \mathbb{C}^n, \tag{3}$$

where j is the imginary unit. Here, let N_1 and N_2 denote the numbers of elevation- and azimuth- transmit antennas, respectively; i.e., the total number of transmit antennas is $N = N_1N_2$. Owing to the strong LoS property of the channels at both the mmWave/THz bands, we assume the ranges of the AoDs are obtained as a priori, i.e., $\forall l$, $g_{l1} \in \mathcal{I}_1$, $g_{l2} \in \mathcal{I}_2$, with the angular blocks $\mathcal{I}_1, \mathcal{I}_2 \subset [-1, 1)$, respectively. Similarly, the antenna array response $\boldsymbol{b}_{R,l}$ is specified as $\boldsymbol{e}_{N_3}(g_{l3}) \otimes \boldsymbol{e}_{N_4}(g_{l4})$, $g_{l3} \in \mathcal{I}_3$, $g_{l4} \in \mathcal{I}_4$, with the angular blocks $\mathcal{I}_3, \mathcal{I}_4 \subset [-1, 1)$, respectively.

For estimating the channel matrix $\boldsymbol{H}$, the BS should send K beams during K successive time slots within a coherent time block. More concretely, during the k-th time slot, the beamforming vector $\boldsymbol{F}_k \in \mathbb{C}^N$ is set as a unitary vector which is selected from a predefined codebook, e.g., $\boldsymbol{f}_k = \boldsymbol{f}_{k,1} \otimes \boldsymbol{f}_{k,2}$ where $\boldsymbol{f}_{k,1} \in \mathbb{C}^{N_1}$ and $\boldsymbol{f}_{k,2} \in \mathbb{C}^{N_2}$ are selected from two discrete Fourier transform (DFT) codebooks of dimensions N_1 and N_2, respectively [8]. Then the k-th signal vector at the receiver can be expressed as

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{f}_k s_k + \boldsymbol{w}_k, \tag{4}$$

where $\boldsymbol{w}_k \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2\mathbf{I}_M)$ is the additive white Gaussian noise (AWGN) with $\mathbf{I}_M$ denoting the $M \times M$ identity matrix and σ^2 being the average power gain of the AWGN,

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and s_k denotes the pilot symbol transmitted during the k-th time slot. After receiving K pilots during K time slots, we have the signal at the receiver,

$$\boldsymbol{Y} = \boldsymbol{HFS} + \boldsymbol{W} \in \mathbb{C}^{M\times K}, \tag{5}$$

where $\boldsymbol{Y} = [\boldsymbol{y}_1,\ldots,\boldsymbol{y}_K] \in \mathbb{C}^{M\times K}$, $\boldsymbol{F} = [\boldsymbol{f}_1,\ldots,\boldsymbol{f}_K] \in \mathbb{C}^{N\times K}$, $\boldsymbol{S} = \mathrm{diag}([s_1,\ldots,s_K]) = \sqrt{P}\mathbf{I}_K \in \mathbb{C}^{K\times K}$, P is the power of the pilot symbol, $\mathbf{I}_K$ is the $K\times K$ identity matrix, and $\boldsymbol{W} = [\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K] \in \mathbb{C}^{M\times K}$.

Before describing our proposed estimator, we vectorize the measurements $\boldsymbol{Y}$ in (5) to obtain

$$\boldsymbol{y} = \mathrm{vec}(\boldsymbol{Y}) = \sqrt{P}(\boldsymbol{F}^{\mathrm{T}}\otimes \mathbf{I}_M)\boldsymbol{h} + \boldsymbol{w} \tag{6}$$

with $\mathbf{I}_M$ being the $M\times M$ identity matrix and the vectorized representation of the channel matrix $\boldsymbol{H}$, i.e.,

$$\begin{aligned}\boldsymbol{h} = \mathrm{vec}(\boldsymbol{H}) &= \sum_{l=1}^{L}\alpha_l \boldsymbol{b}_{T,l}\otimes \boldsymbol{b}_{R,l}\\ &= \sum_{l=1}^{L}\alpha_l \boldsymbol{e}_{N_1}(g_{l1})\otimes \boldsymbol{e}_{N_2}(g_{l2})\otimes \boldsymbol{e}_{N_3}(g_{l3})\otimes \boldsymbol{e}_{N_4}(g_{l4}),\end{aligned} \tag{7}$$

where $\boldsymbol{F}^{\mathrm{T}}\otimes \mathbf{I}_M \in \mathbb{C}^{MK\times MN}$, $\boldsymbol{y}\in\mathbb{C}^{MK}$, $\boldsymbol{h}\in\mathbb{C}^{MN}$ and $\boldsymbol{w} = \mathrm{vec}(\boldsymbol{W})\in\mathbb{C}^{MK}$.

Sparsity enhancement via weighted atomic norm. Let $\boldsymbol{g} = \{g_d\}_{d=1}^4$ and $\boldsymbol{b}(\boldsymbol{g}) = \boldsymbol{e}_{N_1}(g_1)\otimes \boldsymbol{e}_{N_2}(g_2)\otimes \boldsymbol{e}_{N_3}(g_3)\otimes \boldsymbol{e}_{N_4}(g_4)$. Then the set of atoms $\boldsymbol{b}(\boldsymbol{g})$, $g_d\in[-1,1)$, $d = 1,\ldots,4$, is defined as

$$\mathcal{A} = \{\boldsymbol{b}(\boldsymbol{g}), g_d\in[-1,1)\}. \tag{8}$$

Assume the four independent intervals $\mathcal{I}_d$ equal probable, then the probability density function (pdf) of $\boldsymbol{g}$ in (8) is

$$p(\boldsymbol{g}) = \prod_{d=1}^{4} p(g_d) = p(g_1)\times p(g_2)\times p(g_3)\times p(g_4), \tag{9}$$

with

$$p(g_d) = \frac{1}{(g_d^U - g_d^L)}\mathbb{I}(g_d\in\mathcal{I}_d), \tag{10}$$

where

$$\mathbb{I}(g_d\in\mathcal{I}_{gd}) = \begin{cases}1, & g_d\in\mathcal{I}_d,\\ 0, & \text{else},\end{cases}$$

is an indicator function.

Based on $p(\boldsymbol{g})$ in (9), we define

$$\begin{aligned}\boldsymbol{C} &= \mathrm{E}[\boldsymbol{b}(\boldsymbol{g})\boldsymbol{b}^{\mathrm{H}}(\boldsymbol{g})] = \int_{-1}^{1}\boldsymbol{b}(\boldsymbol{g})\boldsymbol{b}^{\mathrm{H}}(\boldsymbol{g})p(\boldsymbol{g})\mathrm{d}\boldsymbol{g}\\ &= \boldsymbol{c}_{g1}\otimes\boldsymbol{c}_{g2}\otimes\boldsymbol{c}_{g3}\otimes\boldsymbol{c}_{g4},\end{aligned} \tag{11}$$

where $\boldsymbol{c}_{gd}$ is a symmetric Toeplitz matrix whose first row is given by

$$\begin{cases} c_{11} = 1,\\ c_{1n} = \frac{\mathrm{j}}{\pi(n-1)(g_d^U-g_d^L)}(\mathrm{e}^{-\mathrm{j}\pi(n-1)g_d^U} - \mathrm{e}^{-\mathrm{j}\pi(n-1)g_d^L}),\\ \qquad\qquad n = 2,\ldots,N_d.\end{cases} \tag{12}$$

Using $\boldsymbol{C}$, we define a weighting function $w(\boldsymbol{g})$ as

$$w(\boldsymbol{g}) = [\boldsymbol{b}^{\mathrm{H}}(\boldsymbol{g})\boldsymbol{C}^{-1}\boldsymbol{b}(\boldsymbol{g})]^{-1/2} \tag{13}$$

and a set of corresponding weighted atoms [9]:

$$\mathcal{A}^w = \{w(\boldsymbol{g})\boldsymbol{b}(\boldsymbol{g}) : \boldsymbol{g}_d\in\mathcal{I}_d, \forall d\}. \tag{14}$$

Then the weighted atomic norm for $\boldsymbol{h} = \sum_l \alpha_l \boldsymbol{b}(\boldsymbol{g}_l)$ is defined as

$$\left\|\sum_l \alpha_l\boldsymbol{b}(\boldsymbol{g}_l)\right\|_{\mathcal{A}^w} = \inf_{\boldsymbol{b}(\boldsymbol{g}_l)\in\mathcal{A}^w}\left\{\sum_l\frac{|\alpha_l|}{w(\boldsymbol{g})} : \sum_l\alpha_l\boldsymbol{b}(\boldsymbol{g}_l)\right\}. \tag{15}$$

On this basis, a convex format for (15) is equally formulated using the following form of the weighted atomic norm:

$$\begin{aligned}\|\boldsymbol{h}\|_{\mathcal{A}^w} = &\min_{\substack{\boldsymbol{T}^4(\boldsymbol{V})\in\mathbb{C}^{NM\times NM},\\ \boldsymbol{h}\in\mathbb{C}^{NM},\varepsilon\in\mathbb{R}}} \frac{1}{2}\mathrm{tr}(\boldsymbol{C}^{-1}\boldsymbol{T}^4(\boldsymbol{V})) + \frac{1}{2}\varepsilon\\ &\text{s.t.}\ \begin{bmatrix}\boldsymbol{T}^4(\boldsymbol{V}) & \boldsymbol{h}\\ \boldsymbol{h}^{\mathrm{H}} & \varepsilon\end{bmatrix}\succeq \mathbf{0},\end{aligned} \tag{16}$$

where $\succeq \mathbf{0}$ stands for a positive semidefinite matrix, $\varepsilon = \sum_l \frac{|\alpha_l|}{w(\boldsymbol{g})} > 0$, and $\boldsymbol{T}^4(\boldsymbol{V})$ is a 4-level block Toeplitz matrix with each block being a 3-level Toeplitz matrix.

Channel estimation using weighted atomic norm. Based on (6) and (16), the channel estimator can be formulated as the following optimization problem:

$$\begin{aligned}\hat{\boldsymbol{h}} = \arg&\min_{\substack{\boldsymbol{T}^4(\boldsymbol{V})\in\mathbb{C}^{NM\times NM},\\ \boldsymbol{h}\in\mathbb{C}^{NM},\varepsilon\in\mathbb{R}}}\frac{1}{2}\left\|\boldsymbol{y} - \sqrt{P}(\boldsymbol{F}^{\mathrm{T}}\otimes\mathbf{I}_M)\boldsymbol{h}\right\|_2^2\\ &+ \mu\|\boldsymbol{h}\|_{\mathcal{A}^w}.\end{aligned} \tag{17}$$

Note that the formulation in (17) is a positive semidefinite problem which can be solved by using a convex solver in CVX packet.

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