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A correlation-breaking interleaving of polar codes in concatenated systems

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Dear editor,

Polar codes, along with a low complexity successive cancellation (SC) decoding, were discovered in [1] by Arıkan. It is shown in [2] that the bit errors with the SC decoding are correlated. To improve the performance, other decoding techniques [3, 4] and concatenation schemes [5, 6] were studied. In this study, the theoretical aspects of the error correlation are investigated to improve the decoding performance of polar codes. This is the first attempt to utilize the error correlation to improve the performance of polar codes.

In this study, two blind interleaving (BI) schemes (all encoded bits in the outer block being divided into different polar blocks) are presented to decorrelate the possible correlated bit errors completely. The BI scheme is also called a direct product (DP) of the inner and outer code, termed as BI-DP. An improved BI scheme, called 'quasi' cyclically shifted direct product BI (BI-CDP), is introduced to improve the BI-DP scheme. This BI-CDP scheme takes into consideration of the different levels of protection experienced by the information bits in one polar encoding block.

From the error correlation pattern presented in the study, a new interleaving scheme, the correlation-breaking interleaving (CBI), is provided to better balance performance, memory consumption, and the decoding delay. The proposed CBI scheme divides the information bits into two groups: the group of the correlated bits and the group of the uncorrelated bits. Theoretical foundation for procedures to assign elements into these two groups is provided. Although any outer code works in the CBI scheme, LDPC (low-density parity check) and BCH (Bose, Chaudhuri, and Hocquenghem) codes are chosen in this study as examples: the former requiring an iterative soft decoding process while the latter only requiring a simpler syndrome decoder.

Error correlation pattern. The generator matrix $G = F^{\otimes n}$ is considered. The basic details of polar codes can be found in Appendix A. Define set $\mathcal{A}$ with indices of the information bit channels. The set $\mathcal{A}$ can be constructed as in [7]. The complementary set $\bar{\mathcal{A}}$ contains the indices of the frozen bit channels (these bits are known to the receiver).

Corollary 1. The matrix $G_{\bar{\mathcal{A}}\mathcal{A}} = \mathbf{0}$.

The proof of this corollary can be found in [8]. Now let us define the set $\mathcal{A}_j$ containing the non-zero positions of column j of G as

$$\mathcal{A}_j = \{\ell \mid 1 \leqslant \ell \leqslant N \text{ and } G_{\ell,j} = 1\}. \tag{1}$$

Proposition 1. Let $\mathcal{A}_i$ be defined as in (1). Define $\hat{u}_{\mathcal{A}_i}$ as the estimated bits with indices from the set $\mathcal{A}_i$. Then, the errors of $\hat{u}_{\mathcal{A}_i}$ are dependent (or coupled).

The proof of this proposition and an example are provided in Appendix B. From Proposition 1, an error correlation pattern among the errors in the estimated information bits can be deduced. The information bits with indices from the set $\mathcal{A}_i$ are the correlated information bits. This shows that statistically, these errors are coupled.

The correlation-breaking interleaving schemes. The interleaving schemes are considered to break the correlated bit errors of polar codes. Here polar codes are inner codes and LDPC codes are outer codes. Note that the introduced schemes work for all types of outer codes. The interleaving scheme can be designed so that the correlated information bits are taken from different LDPC blocks in the transmitter side.

The BI-DP scheme is introduced in Appendix C.1, which breaks all bits in one LDPC block into different polar code blocks. The improved BI-CDP scheme is introduced in Appendix C.2. From Proposition 1, it is shown that scattering all bits from the outer LDPC block into different inner polar blocks is not necessary, because there are both correlated bits and uncorrelated bits in one polar block. The CBI scheme that only breaks the correlated bits is introduced below.

The set $\mathcal{A}_i$ contains the indices of the non-zero entries of column $i \in \mathcal{A}$. First, the $K = |\mathcal{A}|$ columns of G are

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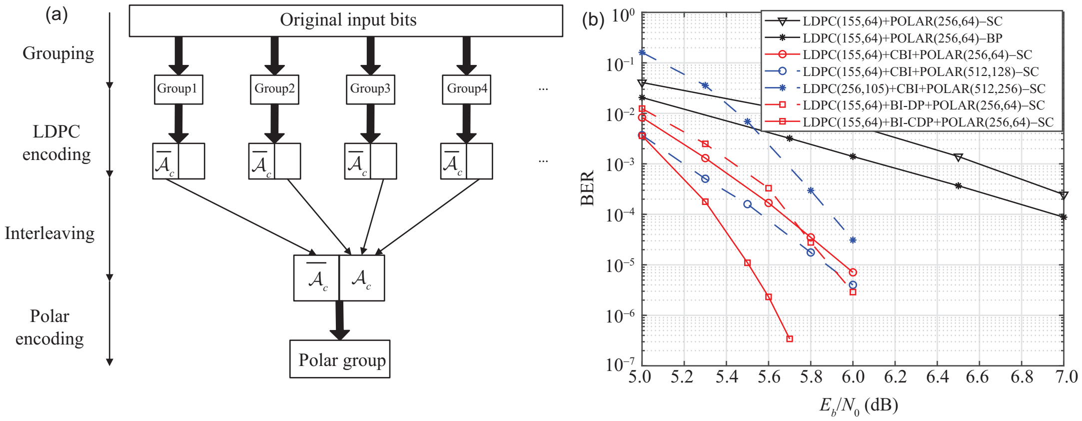

Figure 1 (Color online) (a) A general CBI scheme; (b) the BER performance of the proposed schemes.

extracted, forming a submatrix $G(:,\mathcal{A})$. Divide this submatrix further as $G(:,\mathcal{A}) = [G_{\bar{\mathcal{A}}\mathcal{A}} \;\; G_{\mathcal{A}\mathcal{A}}]$. Since the submatrix $G_{\bar{\mathcal{A}}\mathcal{A}} = \mathbf{0}$ from Corollary 1, it is only necessary to analyze the submatrix $G_{\mathcal{A}\mathcal{A}}$. The indices of information bits are divided into two groups: $\mathcal{A}_c$ with indices of correlated bits and $\bar{\mathcal{A}}_c$ with indices of the uncorrelated bits.

Let ω_i denote the Hamming weight of row i of $G_{\mathcal{A}\mathcal{A}}$. The sets $\mathcal{A}_c$ and $\bar{\mathcal{A}}_c$ can be obtained from Proposition 2.

Proposition 2. For the submatrix $G_{\mathcal{A}\mathcal{A}}$, define $\mathcal{A}_{cs} = \{i\,|\,1 \leqslant i \leqslant K \text{ and } \omega_i > 1\}$, and $\bar{\mathcal{A}}_{cs} = \{j\,|\,1 \leqslant j \leqslant K \text{ and } \omega_j = 1\}$. The corresponding sets of $\mathcal{A}_{cs}$ and $\bar{\mathcal{A}}_{cs}$ with respect to the matrix G are the sets $\mathcal{A}_c$ and $\bar{\mathcal{A}}_c$, respectively.

The proof of this proposition and an example are provided in Appendix D.

Figure 1(a) is a general CBI scheme. The assignment of LDPC coded bits to the polar encoding blocks are similarly done as the BI-CDP scheme, except that there are coded LDPC bits which are put into the uncorrelated positions of the same polar block. The general rules and design examples are specified in Appendix E. The complexity analysis of the CBI scheme is provided in Appendix F.

Simulation results. The LDPC codes are the (155, 64) and (256, 105) MacKay codes [9]. The additive white Gaussian noise (AWGN) channel is employed in the simulations. The construction of polar code is from [7]. Some details of the correlated and uncorrelated bits can be found in Appendix G.

Figure 1(b) shows that at a BER $= 10^{-5}$, the improved BI-CDP scheme has a 0.4 dB advantage over the BI-DP scheme. At a BER $= 10^{-4}$, the LDPC(155, 64) + CBI + POLAR(256, 64) $-$ SC system achieves 1.4 dB and 1.2 dB gains over the direct concatenation systems LDPC(155, 64) + POLAR(256, 64) $-$ SC and LDPC(155, 64) + POLAR(256, 64) $-$ BP, respectively. Here BP is the belief propagation decoding.

A longer outer code with a better performance is shown by the LDPC(155, 64) + CBI + POLAR(512, 128) $-$ SC. The code rates and the code lengths of the outer and inner codes are increased in the LDPC(256, 105) + CBI + POLAR(512, 256) $-$ SC. As a consequence, it requires an additional 0.2 dB to achieve the BER of 10^{-4}.

Compared with the BI-DP scheme, the CBI scheme requires only an additional 0.05 dB of E_b/N_0 to achieve the BER at 10^{-5}. Also, the CBI scheme requires a memory size $N_l/n_p = 1.5$ times smaller than that of the BI-DP scheme.

The BLER (block error rate) performance and results with BCH code are shown in Appendix F.

Conclusion. The bit error correlation of polar codes is studied in this study. Based on the studies, BI-DP, BI-CDP, and CBI schemes are proposed to de-correlate the coupled bit errors. The proposed CBI scheme yields a better performance than the existing direct concatenation schemes. Simulation results validate the theories and the proposed schemes in the study.

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Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Arıkan E. Channel polarization: a method for constructing capacity-achieving codes for symmetric binary-input memoryless channels. IEEE Trans Inform Theor, 2009, 55: 3051–3073
2 Meng Y, Li L P, Hu Y J. A novel interleaving scheme for polar codes. In: Proceedings of the 84th Vehicular Technology Conference (VTC 2016), 2016
3 Hussami N, Korada S B, Urbanke R. Performance of polar codes for channel and source coding. In: Proceedings of IEEE International Symposium on Information Theory, 2009. 1488–1492
4 Chen K, Niu K, Lin J R. Improved successive cancellation decoding of polar codes. IEEE Trans Commun, 2013, 61: 3100–3107
5 Guo J, Qin M H, Fabregas A G I, et al. Enhanced belief propagation decoding of polar codes through concatenation. In: Proceedings of IEEE International Symposium on Information Theory, 2014. 2987–2991
6 Wang T, Qu D, Jiang T. Parity-check-concatenated polar codes. IEEE Commun Lett, 2016, 20: 2342–2345
7 Tal I, Vardy A. How to construct polar codes. IEEE Trans Inform Theor, 2013, 59: 6562–6582
8 Li L P, Xu Z Z, Hu Y J. Channel estimation with systematic polar codes. IEEE Trans Veh Technol, 2018, 67: 4880–4889
9 Mackay D J. Good error-correcting codes based on very sparse matrices. 2002. http://www.inference.phy.cam.ac.uk/mackay/CodesGallager.html