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Optimal replacement of degrading components: a control-limit policy

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Dear editor,

Components such as gyros, turbines, and bearings are vital in aircraft and spacecraft, and must run safely during their entire life cycle. However, gradual damage inevitably occurs resulting in component failure. Therefore, necessary maintenance such as component replacement should be scheduled. Traditional maintenance models are generally time-based, and in such models, maintenance decisions are made to minimize the expected cost rate, regardless of the actual degradation condition. Therefore, traditional models are inappropriate for individual servicing components. Advances in sensor technologies have enabled the monitoring of critical components for detecting degradation signals, which can enhance condition-based maintenance (CBM) [1]. In CBM, maintenance activities are scheduled only if necessary based on the collected degradation signals. In contrast to time-based maintenance, CBM can potentially improve operating reliability and reduce economic costs.

Generally, component degradation is assumed to be either monotonic [2] or associated degradation model parameters are fixed [3]. However, degradation signals collected by sensors are generally not monotonic owing to noise. In addition, fixed model parameters lower the adaptive ability of the degradation model in predicting the future degradation state. In cases where nonmonotonic processes are used, some additional assumptions are imposed on the distribution of the predicted degradation state [4] or only current degradation observations are used to update model parameters [5]. As such, historical data of similar components are ignored causing the updated parameters to fluctuate. Therefore, developing a new optimal condition-based replacement policy is necessary to overcome the abovementioned limitations.

A control-limit replacement policy for degrading components is proposed based on degradation process modeling wherein the replacement problem is constructed as a Markov decision process (MDP). The optimal solution to the proposed policy is found to be a monotonic control-limit replacement policy. Finally, a case study illustrates the proposed method.

Problem formulation. Let $\{L(t), t \geqslant 0\}$ denote the component degradation process with an increasing but not necessarily monotonic trend. A sensor is used to monitor the degradation state in discrete real time t_k, $k = 0, 1, \ldots$. Here, we consider that the monitoring interval is Δt, and it is equally spaced. Then, $t_k = k\Delta t$, which denotes the current time. Further, let $L_k \in \mathbb{R}$ be the stochastic degradation state of the component at t_k, i.e., $L_k = L(t_k)$, and the realization of L_k is expressed as l_k, denoting the measured degradation quantity. Then, the optimal replacement policy for the degrading component can be obtained by solving the following MDP:

$$V(k,l_k)=\begin{cases}c_2+V(0,l_0), & l_k>\xi',\\ \min\{c_1+V(0,l_0), & \\ \qquad \lambda(c_3+\mathrm{E}[V(k+1,L)])\}, & l_k\leqslant\xi',\end{cases} \tag{1}$$

where $V(k, l_k)$ is the value function that represents the total discounted cost with the initial state (k, l_k); l_0 denotes the initial observation; $0 < \lambda < 1$ denotes the discount factor; L is defined as $L = L_{k+1}$, which represents the degradation state at t_{k+1}; ξ' is the failure threshold; and $\mathrm{E}[V(k+1, L)]$ is an expected value function after the transition from (k, l_k) to $(k+1, L)$. The basic principle of the proposed policy (1) can be formulated as follows: if $l_k > \xi'$, a corrective replacement takes place with a cost c_2 and the component is restored to the state $(0, l_0)$; if $l_k \leqslant \xi'$, either a preventive replacement or no action is selected according to the minimal cost. Here, a preventive replacement incurs a cost c_1 ($c_2 > c_1$) while making the component restore to $(0, l_0)$. Otherwise, an expected discount cost continues according to $\mathrm{E}[V(k+1, L)]$, incurring a monitoring cost c_3. The case when the degradation level exceeds the failure threshold between inspection epochs but turns back below the threshold at the inspections is not considered as a failure in this study.

To solve (1), $\mathrm{E}[V(k+1, L)]$ should be evaluated based on the predicted L at t_{k+1}.

Degradation model. Considering extensive applications of the Wiener process in degradation modeling, this study

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considers a linear Wiener process with a drift to model $\{L(t), t \geqslant 0\}$, formulated as follows:

$$L(t) = l_0 + \beta t + \sigma B(t), \tag{2}$$

where l_0 denotes the initial degradation, β is a drift coefficient expressing the degradation rate, σ is a diffusion parameter, and $B(t)$ is the standard Brownian motion (BM) process distributed as $B(t) \sim N(0, t)$ and expressing stochastic dynamics. This model is nonmonotonic due to the effect of $B(t)$. In practice, each component may encounter different operating environments, and thus, has an individual degradation rate even for the same type of components. Therefore, the parameter β is generally considered as a random variable representing the individual variability, while σ is considered as a deterministic parameter representing the common characteristic shared by all components. In this study, β is assumed to be normally distributed as $\beta \sim N(\mu_0, \sigma_0^2)$. Furthermore, it is assumed that β and $B(t)$ are statistically independent. The maximum likelihood estimation for the model parameters in (1) can be obtained if the historical degradation data of the components in the considered population are available.

According to (2), the degradation trend is governed by β. Thus, one should consider how to update β based on the monitored degradation signals up to t_k for the considered component. To do so, the degradation data $l_0, l_1, \ldots, l_k$ up to an epoch k is denoted as $l_{0:k} = \{l_0, l_1, \ldots, l_k\}$. Then, the joint probability density function (PDF) of $l_{0:k}$ (i.e., likelihood function) is normal according to the independent increment and Markov properties of BM. In this case, the prior distribution for β falls into the conjugate family of the likelihood function $p(l_0, l_1, \ldots, l_k|\beta')$. As a result, the posterior distribution of β is still normal with the mean and variance being as follows:

$$\begin{aligned} &\mu_{\beta,k} = (\mu_0\sigma^2 + l_k\sigma_0^2)/(k\Delta t\sigma_0^2 + \sigma^2), \\ &\sigma_{\beta,k}^2 = \sigma^2\sigma_0^2/(k\Delta t\sigma_0^2 + \sigma^2). \end{aligned} \tag{3}$$

Then, we can obtain the estimate for β (denoted as $\hat{\beta}_{\mathrm{Bayes}}$) using the Bayesian decision theory. To do so, we apply the loss function defined as $h(\beta, \hat{\beta}_{\mathrm{Bayes}}) = (\beta - \hat{\beta}_{\mathrm{Bayes}})^2$, and the Bayesian estimate $\hat{\beta}_{\mathrm{Bayes}}$ for β can be obtained by minimizing the expected loss function $h(\beta, \hat{\beta}_{\mathrm{Bayes}})$. The estimation result is provided in Proposition 1 as follows.

Proposition 1. According to model (2) and based on $l_{0:k}$, minimizing the expected loss function $h(\beta, \hat{\beta}_{\mathrm{Bayes}})$ yields the following Bayesian estimate $\hat{\beta}_{\mathrm{Bayes}}$ for β at t_k:

$$\begin{aligned} \hat{\beta}_{\mathrm{Bayes}}(k, l_k) &= \arg\min_{\hat{\beta}_{\mathrm{Bayes}}} \left[h(\beta, \hat{\beta}_{\mathrm{Bayes}}(k, l_k)) \middle| l_{0:k} \right] \\ &= \mu_{\beta,k}, \end{aligned} \tag{4}$$

where $\mu_{\beta,k}$ can be obtained according to (3).

The proof of (4) can be easily achieved and is thus omitted here. According to (3) and (4), the Bayesian estimate $\hat{\beta}_{\mathrm{Bayes}}(k, l_k)$ can combine the historical degradation data of other components and degradation signals of the component in service. Here, the historical degradation data of other components are used to estimate σ, μ_0, and σ_0, while the degradation signals of the component in service are utilized to update the posterior estimation of β.

Providing the Bayesian estimate $\hat{\beta}_{\mathrm{Bayes}}(k, l_k)$ and $l_{0:k}$, L at t_{k+1} can be formulated as

$$L = l_k + \hat{\beta}_{\mathrm{Bayes}}(k, l_k)\Delta t + B(t_{k+1}) - B(t_k). \tag{5}$$

Then, L at t_{k+1} is normally distributed with the cumulative distribution function (CDF) $F_L(x)$, defined as

$$F_L(x) = \Phi\left(\frac{x - \tilde{\mu}(k, l_k)}{\tilde{\sigma}}\right), \tag{6}$$

where $\Phi(\cdot)$ is the CDF of the normally distributed random variable with a mean of 1 and a variance of 1, and

$$\tilde{\mu}(k, l_k) = l_k + \mu_{\beta,k}\Delta t, \tag{7}$$

$$\tilde{\sigma}^2 = \sigma^2\Delta t. \tag{8}$$

Based on (6)–(8), the following Proposition 2 can be formulated for the predicted degradation state L. The proof can be achieved using the definition of the stochastic order and thus is omitted here.

Proposition 2. The predicted degradation state L increases in l_k and decreases in k with the increase of the stochastic order.

Structural properties and solution policy. Based on Proposition 2, the structural properties of the value function $V(k, l_k)$ are summarized in the following theorems.

Theorem 1. Given k, $V(k, l_k)$ is an increasing function with regards to l_k.

Theorem 2. Given l_k, $V(k, l_k)$ is a decreasing function with regards to k.

Providing Theorems 1 and 2, the optimal solution to (1) can be found according to Theorem 3.

Theorem 3. The optimal replacement policy is a monotonic nondecreasing control-limit policy with regards to k, and the optimal decision is to conduct the preventive replacement once $l_k \geqslant l_k^*$, where l_k^* is the control limit.

Proofs of Theorems 1–3 can be found in Appendixes A–C.

Case study. Gyros are a vital component for inertial navigation systems used in various aircraft and missiles. In this case study, we use the degradation data of three gyros, as illustrated in Figure E1(a). Here, the regular inspection interval is $\Delta t = 2.5$ h, $\xi' = 0.37°$/h, and $l_0 = 0$.

To initialize the degradation model, the degradation data of the gyros 2 and 3 are considered as the historical data, while the model parameters can be determined offline. As such, the estimated results for (2) are $\mu_0 = 0.02586$, $\sigma_0^2 = 8.562 \times 10^{-4}$, and $\sigma^2 = 2.1062 \times 10^{-4}$. Then, the data of the gyro 1 can be used to sequentially update β and obtain the Bayesian estimate $\hat{\beta}_{\mathrm{Bayes}}(k, l_k)$ at each t_k. Based on $\hat{\beta}_{\mathrm{Bayes}}(k, l_k)$, the predicted mean degradation level of L can be obtained according to (7). Figure E1(b) shows the predicted mean degradation signal $\tilde{\mu}(k, l_k)$ and the actual degradation signal, indicating a good fitting. To show the optimal control-limit policy, we set $c_1 = 6000$ RMB, $c_3 = 30$ RMB, and the discount factor $\lambda = 0.99$. To conduct sensitivity analysis on the optimal policies, we consider three different settings for c_2. Then, the value iteration algorithm in Appendix D is used to solve the MDP and find the optimal control limit. As shown in Figure E1(c), l_k^* has a monotonic nondecreasing feature in t for all considered cases. In addition, the control limits decrease monotonically with the increase of c_2. The decreasing limit lowers the failure risk during the component operation because a large threshold would delay the replacement and may lead to a high risk of failure in the case of a costly corrective replacement. In contrast, the decrease in c_2 results in l_k^* approaching ξ' and the increase of the consumption of the useful life of the considered component as much as possible. These results for the considered case study verify the proposed policy and demonstrate its application.

Conclusion. A condition-based control-limit replacement policy for degrading components formulated as an MDP problem was proposed. Using degradation modeling and analyzing the structural properties of the formulated MDP, it was proved that the associated optimal solution is a monotonic nondecreasing control-limit replacement policy. The application of the proposed method was demonstrated with a case study. The method can be extended to multi-unit systems based on a recent promising work reported in [6].

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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