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Exponential stability of stochastic Markovian jump systems with time-varying and distributed delays

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Dear editor,

In the field of applied mathematics and control theory, the research on stochastic Markovian jump systems has attracted increasing attention in recent years. A few studies have been conducted on stochastic Markovian jump systems [1–6]. Most previously conducted studies on stochastic Markovian jump systems investigate the discrete time-varying delay [4–6], whereas the distributed delay has not been given much attention owing to the difficulties involved in applying mathematical analysis.

It is our purpose, in this study, to investigate the exponential stability of stochastic Markovian jump systems with the discrete time-varying and distributed delays. Herein, all the delays under discourse having various weights contribute to the stability criterion of the corresponding system compared to the previously conducted studies. Our results are more applicable than the stability results of the previously conducted studies, in which only the maximum bound of all the delays exists [4–6]. Furthermore, our results are more general than the previous results obtained in [1–6].

Let $\{r(t), t \geqslant t_0\}$ be a right-continuous Markov chain on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant t_0}, P)$ taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = \{\pi_{ij}\}$.

Herein, we consider the following stochastic Markovian jump systems with time-varying discrete delays and distributed delays:

$$\begin{cases} \mathrm{d}x(t) = [A_0(r(t))x(t) \\ \qquad + \sum_{j=1}^{m} A_j(r(t))x(t-\tau_j(t)) \\ \qquad + \int_{t-\tau_0(t)}^{t} B(r(s))x(s)\mathrm{d}s]\mathrm{d}t \\ \qquad + C(r(t))x(t)\mathrm{d}W(t), \ t \geqslant t_0, \\ x_{t_0}(\theta) = \xi(\theta), \ -\tau \leqslant \theta \leqslant 0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state; $\xi(\cdot) \in C([-\tau, 0], \mathbb{R}^n)$ is an initial value; $A_j(i), B(i), C(i) \in \mathbb{R}^{n\times n}$, $0 \leqslant j \leqslant m$, $i \in \mathcal{S}$; $\tau_j(t)$ $(0 \leqslant j \leqslant m)$ are time-varying delays satisfying $\tau_0(t) \leqslant \tau_0, 0 \leqslant \tau_j(t) \leqslant \tau_j$ $(1 \leqslant j \leqslant m), \dot{\tau}_j(t) \leqslant d_j < 1$ $(1 \leqslant j \leqslant m)$, $\tau = \max_{0\leqslant j\leqslant m} \tau_j$; and $W(t)$ is a scalar Wiener process defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant t_0}, P)$. We assume that Wiener process $W(t)$ is independent of Markov chain $r(t)$. It is known that system (1) has a unique global solution, which will be denoted by $x(t) = x(t, \xi)$ [3].

Theorem 1. Assume that $P(i)$ is a positive-definite matrix, $V \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n \times \mathcal{S}, \mathbb{R}^+)$ is a Lyapunov function, c_1, c_2, λ, and ε_j $(0 \leqslant j \leqslant m)$ are positive constants, β, τ_j $(0 \leqslant j \leqslant m), \alpha_j$, and $0 \leqslant d_j < 1$ $(1 \leqslant j \leqslant m)$ are non-negative constants such that

$$c_1|x|^2 \leqslant V(t, x, i) \leqslant c_2|x|^2,$$

$$\lambda > \sum_{j=1}^{m} \frac{\alpha_j}{\varepsilon_j(1-d_j)} + \frac{\beta\tau_0}{\varepsilon_0},$$

$$-\lambda \geqslant \lambda_{\max}\Bigg[A_0^{\mathrm{T}}(i)P(i) + P(i)A_0(i) + \sum_{j=1}^{N} \pi_{ij}P(j) + C(i)^{\mathrm{T}}P(i)C(i) + \Bigg(\sum_{j=1}^{m} \varepsilon_j + \varepsilon_0\tau_0\Bigg)P(i)\Bigg]$$

for all $i \in \mathcal{S}$. Then, the solution of system (1) has the property $\limsup_{t\to\infty} \frac{1}{t}\log(\mathrm{E}|x(t,\xi)|^2) \leqslant -\alpha < 0$ for any initial data ξ, where the positive number α is the unique root of $\alpha c_2 - \lambda + \sum_{j=1}^{m} \frac{\alpha_j}{\varepsilon_j(1-d_j)}\mathrm{e}^{\alpha\tau_j} + \frac{\beta\tau_0}{\varepsilon_0}\mathrm{e}^{\alpha\tau_0} = 0$, $\alpha_j = \max_{1\leqslant i\leqslant N} \lambda_{\max}(A_j^{\mathrm{T}}(i)P(i)A_j(i))$, and $\beta = \max_{1\leqslant i,j\leqslant N} \lambda_{\max}(B^{\mathrm{T}}(j)P(i)B(j))$.

Proof. Let $V = x^{\mathrm{T}}(t)P(r(t))x(t)$, where $P(r(t))$ be a positive-definite matrix,

$$c_1 = \min_{1\leqslant i\leqslant N} \lambda_{\min}(P(i)) \text{ and } c_2 = \max_{1\leqslant i\leqslant N} \lambda_{\max}(P(i)).$$

Then, we have

$$\mathcal{L}V(t, x(t), i) = 2x^{\mathrm{T}}(t)P(i)\Bigg[A_0(i)x(t) + \sum_{j=1}^{m} A_j(i)x(t-\tau_j(t))$$

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$$
\begin{aligned}
&+\int_{t-\tau_0(t)}^{t} B(r(s))x(s)\mathrm{d}s\Bigg] + \sum_{j=1}^{N}\pi_{ij}V(t,x(t),j)\\
&+x(t)^{\mathrm{T}}C(i)^{\mathrm{T}}P(i)C(i)x(t)\\
=\ & x^{\mathrm{T}}(t)\Bigg[A_0^{\mathrm{T}}(i)P(i)+P(i)A_0(i)+C(i)^{\mathrm{T}}P(i)C(i)\\
&+\sum_{j=1}^{N}\pi_{ij}P(j)\Bigg]x(t)+2\sum_{j=1}^{m}x^{\mathrm{T}}(t)P(i)A_j(i)\\
&\times x(t-\tau_j(t))+2x^{\mathrm{T}}(t)P(i)\int_{t-\tau_0(t)}^{t}B(r(s))x(s)\mathrm{d}s.
\end{aligned}
$$

Using the inequality $2x^{\mathrm{T}}PA_jy \leqslant \varepsilon_j x^{\mathrm{T}}Px + \frac{1}{\varepsilon_j}y^{\mathrm{T}}A_j^{\mathrm{T}}PA_jy$, and our assumptions, we obtain that

$$
\begin{aligned}
&\mathcal{L}V(t,x(t),i)\\
\leqslant\ & x^{\mathrm{T}}(t)\Bigg[A_0^{\mathrm{T}}(i)P(i)+P(i)A_0(i)+C(i)^{\mathrm{T}}P(i)C(i)\\
&+\left(\sum_{j=1}^{m}\varepsilon_j+\varepsilon_0\tau_0\right)P(i)+\sum_{j=1}^{N}\pi_{ij}P(j)\Bigg]x(t)\\
&+\sum_{j=1}^{m}\frac{1}{\varepsilon_j}x^{\mathrm{T}}(t-\tau_j(t))A_j^{\mathrm{T}}(i)P(i)A_j(i)x(t-\tau_j(t))\\
&+\frac{1}{\varepsilon_0}\int_{t-\tau_0(t)}^{t}x^{\mathrm{T}}(s)B^{\mathrm{T}}(r(s))P(i)B(r(s))x(s)\mathrm{d}s\\
\leqslant\ & -\lambda x^{\mathrm{T}}(t)x(t)+\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j}x^{\mathrm{T}}(t-\tau_j(t))x(t-\tau_j(t))\\
&+\frac{\beta}{\varepsilon_0}\int_{t-\tau_0(t)}^{t}x^{\mathrm{T}}(s)x(s)\mathrm{d}s.
\end{aligned}
$$

By applying the Itô's rule to $\mathrm{e}^{\alpha t}V(t,x(t),r(t))$, we have

$$
\begin{aligned}
&\mathrm{E}(\mathrm{e}^{\alpha t}V(t,x(t),r(t)))\\
\leqslant\ & \mathrm{E}(\mathrm{e}^{\alpha t_0}V(t_0,x(t_0),r_0))\\
&+(\alpha c_2-\lambda)\mathrm{E}\int_{t_0}^{t}\mathrm{e}^{\alpha s}x^{\mathrm{T}}(s)x(s)\mathrm{d}s\\
&+\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j}\mathrm{E}\int_{t_0}^{t}\mathrm{e}^{\alpha s}x^{\mathrm{T}}(s-\tau_j(s))x(s-\tau_j(s))\mathrm{d}s\\
&+\frac{\beta}{\varepsilon_0}\mathrm{E}\int_{t_0}^{t}\mathrm{e}^{\alpha s}\int_{s-\tau_0(s)}^{s}x^{\mathrm{T}}(u)x(u)\mathrm{d}u\mathrm{d}s\\
\leqslant\ & \mathrm{E}(\mathrm{e}^{\alpha t_0}V(t_0,x(t_0),r_0))\\
&+\left\{\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)}\mathrm{e}^{\alpha\tau_j}+\frac{\beta\tau_0}{\varepsilon_0}\mathrm{e}^{\alpha\tau_0}\right\}\\
&\times\mathrm{E}\int_{t_0-\tau}^{t_0}\mathrm{e}^{\alpha s}x^{\mathrm{T}}(s)x(s)\mathrm{d}s\\
&+\left\{\alpha c_2-\lambda+\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)}\mathrm{e}^{\alpha\tau_j}\right.\\
&\left.+\frac{\beta\tau_0}{\varepsilon_0}\mathrm{e}^{\alpha\tau_0}\right\}\mathrm{E}\int_{t_0}^{t}\mathrm{e}^{\alpha s}x^{\mathrm{T}}(s)x(s)\mathrm{d}s\\
\leqslant\ & \mathrm{E}(\mathrm{e}^{\alpha t_0}V(t_0,x(t_0),r_0))\\
&+\left\{\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)}\mathrm{e}^{\alpha\tau_j}+\frac{\beta\tau_0}{\varepsilon_0}\mathrm{e}^{\alpha\tau_0}\right\}\\
&\times\mathrm{E}\int_{t_0-\tau}^{t_0}\mathrm{e}^{\alpha s}x^{\mathrm{T}}(s)x(s)\mathrm{d}s\triangleq\mathrm{E}U(||\xi||^2).
\end{aligned}
$$

Let $f(\alpha)=\alpha c_2-\lambda+\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)}\mathrm{e}^{\alpha\tau_j}+\frac{\beta\tau_0}{\varepsilon_0}\mathrm{e}^{\alpha\tau_0}$, and then $f'(\alpha)>0$ and $f(0)=-\lambda+\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)}+\frac{\beta\tau_0}{\varepsilon_0}<0$ by our assumptions. Therefore, we obtain that

$$
\limsup_{t\to\infty}\frac{1}{t}\log(\mathrm{E}|x(t,\xi)|^2)\leqslant-\alpha<0.
$$

Remark 1. In Theorem 1, we can easily see that all the delays in system (1) play key roles in the establishment of the stability theorem with different weights. Herein, the exponential decay rate α in the study is relevant to all the time-varying delays instead of the maximum value of time delays in [5, 6]; this implies that the corresponding systems can admit a greater maximum value of all the time delays by choosing different weights under the same exponential decay rate. Therefore, Theorem 1 is less conservative. Moreover, based on the same conditions with time-varying delays, the method established in Theorem 1 can increase the exponential decay rate α of the corresponding systems, as will be shown using an example below.

Now, we can set $\varepsilon_j=1$, for $0\leqslant j\leqslant m$, or $m=1$ in Theorem 1 to obtain some neat corollaries. However, Theorem 1 is more general. Moreover, if we set $\tau_0(t)=0$, then we obtain the following corollary.

Corollary 1. Assume that $P(i)$ is a positive-definite matrix, $V\in C^{1,2}(\mathbb{R}^+\times\mathbb{R}^n\times\mathcal{S},\mathbb{R}^+)$ is a Lyapunov function, c_1,c_2,λ, and ε_j $(1\leqslant j\leqslant m)$ are positive constants, and $\beta,\alpha_j,0\leqslant d_j<1$ $(1\leqslant j\leqslant m)$ are non-negative constants such that

$$
\begin{aligned}
&c_1|x|^2\leqslant V(t,x,i)\leqslant c_2|x|^2,\ \lambda>\sum_{j=1}^{m}\frac{\alpha_j}{\varepsilon_j(1-d_j)},\\
&-\lambda\geqslant\lambda_{\max}\Bigg[A_0^{\mathrm{T}}(i)P(i)+P(i)A_0(i)\\
&\qquad+C(i)^{\mathrm{T}}P(i)C(i)+mP(i)+\sum_{j=1}^{N}\pi_{ij}P(j)\Bigg]
\end{aligned}
$$

for all $i\in\mathcal{S}$. Then, the solution of system (1) has the property $\limsup_{t\to\infty}\frac{1}{t}\log(\mathrm{E}|x(t,\xi)|^2)\leqslant-\alpha<0$ for any initial data ξ, where the positive number α is the unique root of $\alpha c_2-\lambda+\sum_{j=1}^{m}\frac{\alpha_j}{1-d_j}\mathrm{e}^{\alpha\tau_j}=0$, and $\alpha_j=\max_{1\leqslant i\leqslant N}\lambda_{\max}(A_j^{\mathrm{T}}(i)P(i)A_j(i))$.

Example 1. Consider a two-dimensional system (1) with two modes $N=2$ and $m=2$ with the corresponding parameters as follows: $\mathcal{S}=\{1,2\},\pi_{11}=\pi_{22}=-1,\pi_{12}=\pi_{21}=1$, $A_0(1)=\left[\begin{smallmatrix}-9/2&1/2\\0&-2\end{smallmatrix}\right]$, $A_0(2)=\left[\begin{smallmatrix}-8&0\\1&-8\end{smallmatrix}\right]$, $A_1(1)=\left[\begin{smallmatrix}1/4&0\\1/2&1/4\end{smallmatrix}\right]$, $A_1(2)=\left[\begin{smallmatrix}1/3&1/2\\1/4&1/3\end{smallmatrix}\right]$, $A_2(1)=\left[\begin{smallmatrix}\sqrt{2}/4&0\\0&\sqrt{2}/4\end{smallmatrix}\right]$, $A_2(2)=\left[\begin{smallmatrix}\sqrt{2}/3&0\\0&\sqrt{2}/3\end{smallmatrix}\right]$, $B(1)=\left[\begin{smallmatrix}1/4&0\\0&1/4\end{smallmatrix}\right]$, $B(2)=\left[\begin{smallmatrix}1/3&0\\0&1/3\end{smallmatrix}\right]$, $C(1)=\left[\begin{smallmatrix}\sqrt{2}/2&0\\0&\sqrt{2}/2\end{smallmatrix}\right]$, and $C(2)=\left[\begin{smallmatrix}\sqrt{6}/3&0\\0&\sqrt{6}/3\end{smallmatrix}\right]$.

Time-varying delays $\tau_j(t)$ $(j=0,1,2)$ satisfy $0\leqslant\tau_0(t)\leqslant 1/5$, $0\leqslant\tau_1(t)\leqslant1/3$ with $\dot{\tau}_1(t)\leqslant1/3<1$ and $0\leqslant\tau_2(t)\leqslant1/2$ with $\dot{\tau}_2(t)\leqslant1/2<1$.

Let $V=x(t)^{\mathrm{T}}P(r(t))x(t)$, where $P(1)=\left[\begin{smallmatrix}2&0\\0&2\end{smallmatrix}\right]$ and $P(2)=\left[\begin{smallmatrix}3/2&0\\0&3/2\end{smallmatrix}\right]$. By choosing $\varepsilon_1=\varepsilon_2=1$, and $\varepsilon_0=1/2$, a direct computation yields $c_1=3/2$, $c_2=2$, and $1.957<\lambda_1\leqslant3.401$. If we take $\lambda=3.401$, then the transcendental equation has the following form: $2\alpha-3.401+1.2023\mathrm{e}^{\frac{\alpha}{3}}+0.6666\mathrm{e}^{\frac{\alpha}{2}}+0.0888\mathrm{e}^{\frac{\alpha}{5}}=0$. It follows that $\alpha=0.5091$. Hence, by applying Theorem 1, the trivial solution of system (1) is exponentially stable in mean square with the exponent α, as shown in Figure 1 with the initial value $\xi(\theta)=[\cos\theta,-\cos\theta]^{\mathrm{T}}$, where $\theta\in[-0.5,0]$ and $t_0=0$.

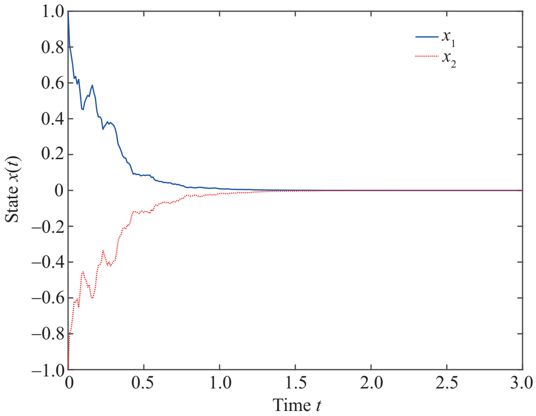

Figure 1 (Color online) Trajectory of x.

Based on the method in [5, 6], if we only take the maximum bound of all the time-varying delays, that is $\tau_0 = \tau_1 = \tau_2 = \max\{\tau_0, \tau_1, \tau_2\} = 0.5$, we can similarly obtain $\alpha = 0.2599$. Consequently, the exponential decay rate is increased by 48.95% in comparison with the value of α obtained in Theorem 1. This implies that our results are less conservative.

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