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A Bayesian belief-rule-based inference multivariate alarm system for nonlinear time-varying processes

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Abstract This study considers the multivariate alarm design problem of nonlinear time-varying systems by a Bayesian belief-rule-based (BRB) method. In the method, the series of belief rules are constructed to approximate the relationship between input and output variables. Hence, the method does not require an explicit model structure and is suitable for capturing nonlinear causal relationships between variables. For the purpose of online application, this study further introduces sequential Monte Carlo (SMC) sampling to update the BRB model parameters, which is a fast and efficient method for approximately inferring nonlinear sequence models. Using the model parameters obtained by SMC sampling, the series of output variable tracking errors can be estimated and employed for multivariate alarm design. The case study of a condensate pump verifies the effectiveness of the proposed method.

Keywords multivariate alarm design, belief-rule-based method, nonlinear time-varying process, sequential Monte Carlo

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1 Introduction

Industrial alarm systems are widely used in modern plants to ensure the safety of equipment and processes. In a typical alarm system, the measurement of a process variable is compared with a set point known as an alarm limit or a control limit. Once the measurement exceeds the alarm limit, an alarm is triggered to alert an operator so that corrective actions are performed. The application of alarm systems greatly reduces the risk of failure or accident; however, it also causes alarm overloading and the concomitant loss of productivity owing to the occurrence of numerous false or nuisance alarms [1,2]. This is especially true for large-scale systems such as power-plants and pyrometallurgical and petrochemical facilities in which numerous process variables are monitored.

To reduce false and nuisance alarms, researchers have developed different kinds of univariate alarm methods, including those designed for systems with deadbands and delay-timers [3–6]. However, it is found that univariate alarm systems may cause a significant number of false alarms as they do not consider relationships between process variables [1]. Therefore, the design of an effective multivariate alarm system has become an important research topic. Different methods have been developed, including multivariate statistical methods and probability density-based approaches. For example, Kondaveeti et al. [7] applied different multivariate statistical methods to design an efficient alarm system. Gupta et al. [8] integrated several statistical methods such as wavelet analysis, principal component analysis (PCA), and qualitative trend analysis to perform alarm design. Zhang and Li [9] performed kernel density estimation to optimize process alarm thresholds. Han et al. [10] determined an alarm propagation path based on

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causal relationships among variables and developed an alarm threshold optimization strategy based on a joint probability density analysis. Other methods are also considered, including the geometric process control technique [11], methods based on correlation directions [12], evidence updating methods [13], hybrid criterion combining intrinsic mode function and fractal wavelet package energy entropy [14], and fractional differential operator [15].

More recently, research attention has been directed to multivariate alarm design for more complex systems, such as nonlinear, dynamic and time-varying systems. Alrowaie et al. [16] considered the alarm design problem for nonlinear stochastic systems using a particle filter. Zhu et al. [17] developed a dynamic alarm management strategy for transition processes of chemical production. Yu et al. [18] designed dynamic alarm limits for multivariate alarm systems using a hyper-ellipsoid model and also considered the adjustment of manipulated variables for alarm design. Cheng et al. [19] designed a PCA and BRB heath monitoring method for high-speed train running gears. Xiong et al. [20] developed a multivariate alarm system for time-varying processes. The work in [20] was based on a linear input-output model, in which the time-varying parameters were estimated using Bayesian filters and the moving window approach. This method works well in the condensate pump application. However, while using a linear input-output model, it is inherently assumed that a process is linear time-varying, which may not be valid for many practical processes. Meanwhile, the application of the moving window approach requires the determination of a new parameter: window length, which may lead to a significant number of false or missing alarms if not appropriately determined.

Inspired by the work of Xiong et al. [20], this study considers the multivariate alarm design problem for time-varying process using the BRB method [21, 22]. It is a rule-based method based on belief structure and is widely used to capture nonlinear causal relationships and process uncertainties. Owing to its rule-based structure, it is capable of utilizing expert knowledge. In addition, its parameters and structures are trained based on historical data. Hence, the BRB method shares merits of both knowledge-based and data driven methods. To accommodate time-varying characteristics, this study further introduces an online Bayesian estimation method to estimate the time-varying parameters and a new alarm design method is developed. The benefits and contributions of this study can be summarized as follows: (i) the application of the BRB method improves the ability to handle process nonlinearities; (ii) the online sequential Monte Carlo (SMC) sampling-based Bayesian method is well-suited to estimating the time-varying parameters of multivariate systems; (iii) the BRB method does not require an explicit model structure incorporating input and output variables; hence, it is general.

2 Problem formulation

2.1 Model structure

Consider a multivariate system involving m inputs and n outputs. Assume a training data set consisting of N samples has been collected as $\{(\boldsymbol{x}(t), \boldsymbol{y}(t))|\boldsymbol{x}(t) \in \mathbb{R}^m, \boldsymbol{y}(t) \in \mathbb{R}^n, t = 1, \ldots, N\}$, where $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are the input and output sample at the tth time instance. The relationship between $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ can be described as follows:

$$\boldsymbol{y}(t) = f(\boldsymbol{x}(t), \boldsymbol{\Theta}(t)) + \boldsymbol{\epsilon}(t), \tag{1}$$

where $f(\cdot)$ is a nonlinear function with the time-varying parameter set $\boldsymbol{\Theta}(t)$ and $\boldsymbol{\epsilon}(t)$ is the error vector. In contrast to the work in [20], the model in (1) is more general and does not require an explicit expression, which is often not available in practical systems. The purpose now is to estimate the time-varying parameter $\boldsymbol{\Theta}(t)$ and the error vector $\boldsymbol{\epsilon}(t)$ from the dataset for the subsequent alarm design. The parameter estimation method used here is the BRB method.

2.2 Belief-rule-based inference method

The BRB is a rule-based inference method. It has been widely applied in fields such as fault diagnosis [23], safety assessment [24] and failure prognosis [25, 26]. Instead of relying on an explicit expression of model (1), the BRB method transfers the parameter estimation problem into learning of a series of belief rules, with the kth rule described as follows:

$$\begin{aligned} R_k: \quad & \text{If } x_1 \text{ is } A_1^k \ \wedge \ x_2 \text{ is } A_2^k \ \wedge \cdots \wedge \ x_{m_k} \text{ is } A_{m_k}^k \\ & \text{Then } \{(D_1, \beta_{1k}), (D_2, \beta_{2k}), \ldots, (D_M, \beta_{Mk})\}, \\ & \text{with the rule weight } \theta_k \\ & \text{and attribute weights } \delta_1, \delta_2, \ldots, \delta_{m_k}, \end{aligned} \tag{2}$$

where the input variable x_i ($i = 1, \ldots, m_k$) is called the antecedent attribute, A_i^k ($i = 1, \ldots, m_k; k = 1, \ldots, L$) is the referential value of the ith antecedent attribute, and m_k ($m_k \leqslant m$) and L are the number of attributes and rules respectively. D_j ($j = 1, \ldots, M$) is the jth referential value of the output or consequent, $\beta_{jk} \in \{0, 1\}$ is the belief degree to which D_j is the consequent of $(x_1, x_2, \ldots, x_{m_k}) = (A_1^k, A_2^k, \ldots, A_{m_k}^k)$. If $\sum_{j=1}^M \beta_{jk} = 1$, the kth rule is a complete rule; otherwise, it is incomplete. In addition, "$\wedge$" is the logical "AND" operator. Each rule has a rule weight θ_k and each antecedent attribute has an attribute weight δ_i. The final output is obtained by aggregating the contributions of all activated rules. For the sake of simplicity, the number of output variables is set as $n = 1$ here. Extending to multiple outputs can be easily achieved by expanding the number of output referential values.

Compared to the traditional IF-THEN rule whose consequent is either 100% true or 100% false, the belief rule has better flexibility and can be easily extended to deal with continuous variables. The parameter set of the BRB model can be defined as

$$\boldsymbol{\Theta} = \left\{\boldsymbol{\theta}, \boldsymbol{\delta}, \boldsymbol{\beta}^k, k = 1, \ldots, L\right\}, \tag{3}$$

where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_L)$ is the rule weight vector, $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_{m_k})$ is the attribute weight vector and $\boldsymbol{\beta}^k = (\beta_{1k}, \ldots, \beta_{Mk})$ is the belief degree vector.

To estimate the model parameters, evidence reasoning (ER) [27] is generally applied, which consists of an activation weight calculation step and a belief degree calculation step. The activation weight, referred to ω_k, is the degree of activation for each of the kth rule and can be calculated as

$$\omega_k = \frac{\theta_k \prod_{i=1}^{m_k} (\alpha_{ik})^{\bar{\delta}_i}}{\sum_{l=1}^{L} \left[\theta_l \prod_{i=1}^{m_l} (\alpha_{il})^{\bar{\delta}_i}\right]}, \quad \bar{\delta}_i = \frac{\delta_i}{\max_{i=1,2,\ldots,m_k} \{\delta_i\}}, \tag{4}$$

where α_{ik} ($\alpha_{ik} \geqslant 0, \sum_{i=1}^{m_k} \alpha_{ik} \leqslant 1$) is called the individual matching degree, which is the belief degree of the input x_i to the referential value A_i^k in the kth rule. Given a specific measurement of the ith input $x_i(t)$ and a set of referential values $\mathbb{A} = \{A_{i,1}, A_{i,2}, \ldots, A_{i,M_i}\}$ sorted in ascending order (with $A_i^k \in \mathbb{A}$, if $A_{i,q} \leqslant x_i(t) \leqslant A_{i,q+1}$), the individual matching degree α_{iq} of $x_i(t)$ to the qth reference value $A_{i,q}$ and the $(q+1)$th reference value $A_{i,q+1}$ can be calculated as

$$\alpha_{iq} = \frac{A_{i,q+1} - x_i(t)}{A_{i,q+1} - A_{i,q}}, \quad \alpha_{i(q+1)} = \frac{x_i(t) - A_{i,q}}{A_{i,q+1} - A_{i,q}}. \tag{5}$$

The individual matching degrees of $x_i(t)$ to other referential values are 0. On the other hand, if $x_i(t) \leqslant A_{i,1}$, the individual matching degree of $x_i(t)$ to $A_{i,1}$ is 1. Similarly, if $x_i(t) \geqslant A_{i,M_i}$, the individual matching degree of $x_i(t)$ to A_{i,M_i} is 1.

Once the activation weight is obtained, ER can be used to combine the rules and generate the output belief degree β_j as follows:

$$\beta_j = \frac{\mu[\prod_{k=1}^{L} (\omega_k \beta_{jk} + 1 - \omega_k \sum_{i=1}^{M} \beta_{ik}) - \prod_{k=1}^{L} (1 - \omega_k \sum_{i=1}^{M} \beta_{ik})]}{1 - \mu(\prod_{k=1}^{L} (1 - \omega_k))}, \tag{6}$$

where the parameter μ can be obtained as

$$\mu = \left[\sum_{j=1}^{M} \prod_{k=1}^{L} \left(\omega_k \beta_{jk} + 1 - \omega_k \sum_{i=1}^{M} \beta_{ik}\right) - (N-1) \prod_{k=1}^{L} \left(1 - \omega_k \sum_{i=1}^{M} \beta_{ik}\right)\right]^{-1}. \tag{7}$$

And the final output can be estimated as

$$\hat{y}(t) = \sum_{j=1}^{M} \beta_j D_j. \tag{8}$$

3 Online evaluating procedures using sequential Monte Carlo

3.1 Basic assumptions

The BRB inference introduced in Subsection 2.2 is for time-invariant system. For time-varying system whose parameter set $\boldsymbol{\Theta}(t)$ changes with time, this inference method is not appropriate anymore. Here a Bayesian online updating method is applied by estimating the posterior parameter distribution $p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$, with $\boldsymbol{S}(1:t) = \{(\boldsymbol{x}(1), y(1)), \ldots, (\boldsymbol{x}(t), y(t))\}$ being the dataset. In order to approximate the posterior distribution, the SMC sampling [28] is applied which introduces a transition probability $p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}^i(t-1))$ to propagate samples from $t-1$ to t as well as an observation probability to update the sample weight. The inherent assumptions for using SMC include the Markov assumption and the observation assumption.

The Markov assumption assumes the change of BRB parameters is a Markov process. That is to say, the distribution of parameters at time t only depends on those at time $t-1$. And the transition can be described as a random walk model as follows:

$$\boldsymbol{\Theta}(t) = \boldsymbol{\Theta}(t-1) + \boldsymbol{\eta}(t), \quad \boldsymbol{\eta}(t) \sim N(0, \nu_1^2), \tag{9}$$

where ν_1^2 is the variance of transition probability. For the observation assumption, it is assumed that the current output is only dependent on the current inputs and parameters, which can be described as follows:

$$p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(1), \ldots, \boldsymbol{\Theta}(t-1), \boldsymbol{\Theta}(t)) = p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t)), \tag{10}$$

where $p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t)) \sim N(0, \nu_2^2)$ is the observation probability, with ν_2^2 the variance of the observation probability. The variances of the transition probability and observation probability determine the sensitivity of the tracking algorithm. Smaller variances result in increased sensitivity of the algorithm to incipient or slow-varying faults; however, it may increase the risk of false alarms as small disturbances may be mistaken to be a fault. In contrast, greater variances cause reduced false alarms, while incipient or slow-varying faults may be easily adapted. In practice, a trade-off between the sensitivity and false alarms should be considered and this issue can be resolved by using a trial and error approach on the training dataset by considering the balance between detection sensitivity and the false alarm rate.

The BRB parameters are subject to the following constraints:

$$\begin{aligned}
&0 \leqslant \beta_{jk} \leqslant 1,\ j = 1, \ldots, M; k = 1, \ldots, L, \\
&\sum_{j=1}^{M} \beta_{jk} = 1, \\
&0 \leqslant \theta_k \leqslant 1,\ k = 1, \ldots, L, \\
&0 \leqslant \delta_i \leqslant 1,\ i = 1, \ldots, m_k, \\
&D_j < D_{j+1}.
\end{aligned} \tag{11}$$

3.2 Online updating using sequential Monte Carlo sampling

The model in (1) as well as the BRB rules in (2) are nonlinear, hence it is difficult to estimate the posterior parameter distribution $p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$ analytically. Here, the SMC sampling is used for approximation. The SMC sampling is a kind of importance sampling technique, which assumes certain values in the sampling procedures have more impact on the parameters to be estimated. In the BRB parameter estimation problem, instead of directly sampling from $p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$, a simpler importance distribution $q(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$ is introduced. The true posterior distribution of $p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$ is a weighted sum of the following form:

$$p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t)) \approx \sum_{s=1}^{N_s} \omega^s(t) \cdot \Delta(\boldsymbol{\Theta}(t) - \boldsymbol{\Theta}^s(t)), \tag{12}$$

where $\Delta(\cdot)$ is the Dirac function, $\boldsymbol{\Theta}^s(t)$ is the sth sampling of $\boldsymbol{\Theta}(t)$ from $q(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))$, N_s is the number of sampling, and $\omega^s(t)$ is the importance weight defined as

$$\omega^s(t) \propto \frac{p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))}{q(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t))}. \tag{13}$$

The weights are normalized and $\sum_{s=1}^{N_s} \omega^s(t) = 1$.

The importance distribution can be factorized as

$$q(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t)) = q(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}(t-1), \boldsymbol{S}(1:t))q(\boldsymbol{\Theta}(t-1)|\boldsymbol{S}(1:t-1)). \tag{14}$$

And the posterior distribution can be factorized according to Bayesian theory and Markov assumption as

$$\begin{aligned} p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t)) &= \frac{p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t), \boldsymbol{S}(1:t-1))p(\boldsymbol{\Theta}(t)|\boldsymbol{S}(1:t-1))}{p(\boldsymbol{S}(t)|\boldsymbol{S}(1:t-1))} \\ &= \frac{p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t), \boldsymbol{S}(1:t-1))p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}(t-1), \boldsymbol{S}(1:t-1))}{p(\boldsymbol{S}(t)|\boldsymbol{S}(1:t-1))} \\ &\quad \times p(\boldsymbol{\Theta}(t-1)|\boldsymbol{S}(1:t-1)) \\ &= \frac{p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t))p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}(t-1))}{p(\boldsymbol{S}(t)|\boldsymbol{S}(1:t-1))} p(\boldsymbol{\Theta}(t-1)|\boldsymbol{S}(1:t-1)) \\ &\propto p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t))p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}(t-1))p(\boldsymbol{\Theta}(t-1)|\boldsymbol{S}(1:t-1)). \end{aligned} \tag{15}$$

Substituting (14) and (15) into (13), $\omega^s(t)$ can be obtained as follows:

$$\begin{aligned} \omega^s(t) &\propto \frac{p(\boldsymbol{S}(t)|\boldsymbol{\Theta}^s(t))p(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1))p\left(\boldsymbol{\Theta}^s(t-1)|\boldsymbol{S}(1:t-1)\right)}{q\left(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1), \boldsymbol{S}(1:t-1)\right)} \\ &= \omega^s(t-1)\frac{p(\boldsymbol{S}(t)|\boldsymbol{\Theta}^s(t))p(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1))}{q\left(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1), \boldsymbol{S}(1:t-1)\right)}. \end{aligned} \tag{16}$$

For simplicity, the importance distribution is often set as [29]

$$q\left(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1), \boldsymbol{S}(1:t-1)\right) = p(\boldsymbol{\Theta}^s(t)|\boldsymbol{\Theta}^s(t-1)). \tag{17}$$

So that the importance weight can be updated as

$$\omega^s(t) \propto \omega^s(t-1)p(\boldsymbol{S}(t)|\boldsymbol{\Theta}^s(t)). \tag{18}$$

During the sampling procedures, the samples drawn from the transitional distribution in (9) may violate the constraints in (11). In such cases, the sample will be rejected and the parameter set will be resampled until it satisfies the constraints.

Another important problem commonly encountered during the sampling procedures is the degeneration problem. That is, after certain times of iterations, the weight of most samples may decay to almost zero while a few samples have significant great weights. In this case, resampling may be needed. In practice, it is the effective number of samples that determines whether to resample, which can be calculated as follows [29]:

$$\hat{N}_{\text{eff}} = \frac{1}{\sum_{s=1}^{N_s}(\omega^s(t))^2}. \tag{19}$$

If $\hat{N}_{\text{eff}}$ is smaller than a predefined threshold N_{thr}, then resampling should be performed. The updating algorithm of the SMC sampling is shown in Algorithm 1.

Algorithm 1 Updating algorithm for SMC sampling

Update $\{\boldsymbol{\Theta}^s(t), \omega^s(t)\}_{s=1}^{N_s}$, given $\{\boldsymbol{\Theta}^s(t-1), \omega^s(t-1)\}_{s=1}^{N_s}$ and the input and output data pair $\boldsymbol{S}(t)$. Here N_s is the number of sampling.

1. For $s = 1, \ldots, N_s$, sample $\boldsymbol{\Theta}^s(t) \sim p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}^s(t-1))$; check the constraints in (11); if $\boldsymbol{\Theta}^s(t)$ does not satisfy the constraints, perform resampling until the constraints are satisfied;
2. For $s = 1, \ldots, N_s$, calculate the importance weights using (13);
3. For $s = 1, \ldots, N_s$, normalize the importance weights;
4. Calculate $\hat{N}_{\text{eff}}$ using (19) and check whether resampling is needed; if $\hat{N}_{\text{eff}} < N_{\text{thr}}$, draw N_s samples from the current parameter set $\boldsymbol{\Theta}^s(t), s = 1, \ldots, N_s$ with probabilities proportional to their weights $\omega^s(t)$ and replace the current parameter set with the new one;
5. For $s = 1, \ldots, N_s$, set the new weights as $\frac{1}{N_s}$.

3.3 General procedures of BRB inference

In summary, the parameter estimation procedures for BRB model using SMC can be described as follows.

(1) Initialize BRB parameters. The parameters can be determined from (i) expert knowledge, (ii) extraction from historical data, (iii) utilizing existing rule base, and (iv) random generation [30].

(2) Determine the transition probability $p(\boldsymbol{\Theta}(t)|\boldsymbol{\Theta}(t-1))$ and observation probability $p(\boldsymbol{S}(t)|\boldsymbol{\Theta}(t))$.

(3) Parameter updating using Algorithm 1.

Once the BRB parameters are determined, the model output can be estimated in two ways. The first way is to use the estimated parameters to estimate the model output according to (8). The other way is to calculate directly as

$$\hat{y}(t) \approx \sum_{s=1}^{N_s} \omega^s(t) \times f(\boldsymbol{x}(t), \boldsymbol{\Theta}^s(t)). \tag{20}$$

4 Multivariate alarm system design

The Bayesian BRB updating can be used to obtain estimation of model parameters online. Once an anomaly arises, it is essential to detect it and trigger alarm to notify an operator. The difference between normal and abnormal situations can be reflected by the change in model parameters. For a time-varying system, if an abrupt change in the model parameters is observed, it indicates that the operational status of the system has changed. Consequently, the abrupt change in model parameters will result in significant increase in model error, which can be used to construct the multivariate alarm system.

For the purpose of alarm design, a historical normal dataset and a historical abnormal dataset are collected and the BRB method is applied to estimate the parameters online. The purpose of introducing a historical normal dataset and a historical abnormal dataset is to obtain a more appropriate alarm threshold [20]. The threshold is constructed using the model error between the current output and its estimation, which can be obtained from (8) using the current input and the model parameters of the previous time instance.

Let $\phi(t)$ be the model error defined as follows:

$$\phi(t) = |y(t) - \hat{y}(t)|. \tag{21}$$

Based on (21), two sets of model errors $\phi_1(t)$ and $\phi_2(t)$ can be obtained, corresponding to the historical normal dataset and the historical abnormal dataset respectively. Intuitively, the maximum of $\phi_2(t)$ will be greater than that of $\phi_1(t)$ as the anomaly causes greater tracking errors in faulty conditions. Hence, the alarm threshold can be defined as

$$\phi_{tp} = \frac{\max(\phi_1(t)) + \max(\phi_2(t))}{2}. \tag{22}$$

And the alarm system can be designed as

$$a(t) = \begin{cases} 1, & \phi(t) > \phi_{tp}, \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

With the alarm threshold set and alarm system designed, the BRB method can be used online to trigger alarms for abnormal conditions. The procedures can be summarized by the following steps.

(1) Set alarm threshold using (22).

(2) Determine whether to alarm. At the tth time instance, the model input and model parameters obtained at the last time instance are used to obtain the model error $\phi(t)$. Based on (23), the alarming variable $a(t)$ is calculated. If $a(t) = 1$, then trigger alarm.

(3) Decide whether to update model parameters. If $a(t) = 0$, the system is operating at the normal situation; then update the model parameters using the current input and output data. If $a(t) = 1$, the model parameters remain unchanged and $\boldsymbol{\Theta}(t) = \boldsymbol{\Theta}(t-1)$.

(4) Repeat Steps (2) and (3) whenever a new sample is available.

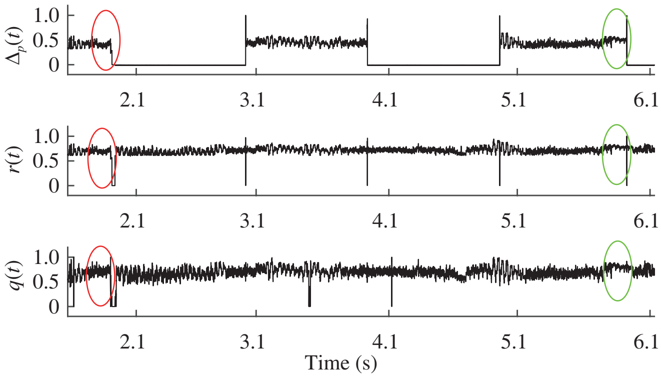

Figure 1 (Color online) Normalized time series of the three variables. The red circles contain the data samples used for model training and the green circles contain the data samples used for performance verification.

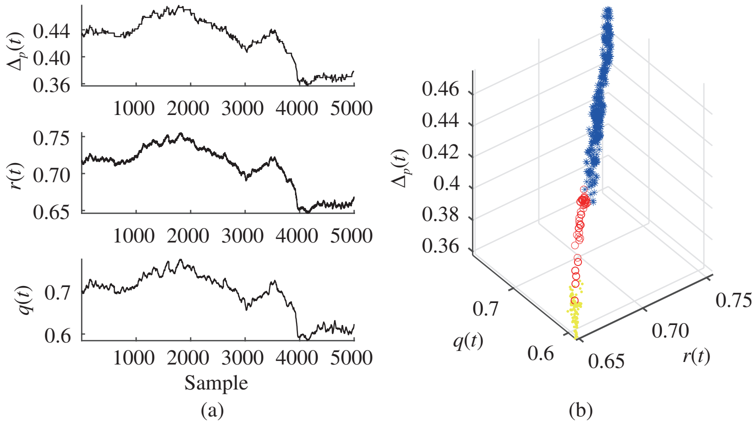

Figure 2 (Color online) (a) Normalized time series of the historical normal data; (b) the 3D plot of the data.

5 Application case study

The developed Bayesian BRB method is applied to the abnormal condition detection of a condensate pump used in a real coal-fired power plant in China. A dataset consisting of 11836800 data points in the time period between January 15, 2015 and June 1, 2015 is collected. The dataset involves three process variables, namely, the difference in pressure between the input and output flows $\Delta_p(t)$, the rotational speed of the pump $r(t)$ and the flow rate $q(t)$. The pressure difference $\Delta_p(t)$ is closely related to the rotational speed $r(t)$ and the flow rate $q(t)$. Hence, $\Delta_p(t)$ is used as model output while $r(t)$ and $q(t)$ as model inputs. Figure 1 presents the normalized time series of the three variables.

During the time period, three faults occurred at 10:40, January 24, 23:35, March 25 and 13:00, May 25 respectively. The faults caused the pressure difference dropping to zero, indicating that the pump stopped working. The other two variables, however, are not zero since the backup pump started working and the two pumps used the same transducer. As is analyzed in [20], the dataset exhibits clear time-varying characteristics. During the model construction procedures, the work in [20] used extensive prior knowledge about the relationship between the output variable $\Delta_p(t)$ and the input variables $r(t)$ and $q(t)$. Such prior knowledge, however, is generally not available in many practical cases. In this section, the Bayesian online BRB model is applied to make full use of its capability in handling nonlinearity and time-varying characteristics. For comparison, principal component analysis and traditional univariate alarm method [31] are considered.

5.1 Model training for alarm system design

For alarm design, the data samples corresponding to the first fault are used. The data samples are divided into a historical normal dataset and a historical abnormal dataset, each consisting of 5000 samples.

The time series of the historical normal dataset are shown in Figure 2. The dataset was divided into three clusters, following the method used in [20]. From Figure 2, it can be clearly seen that there are

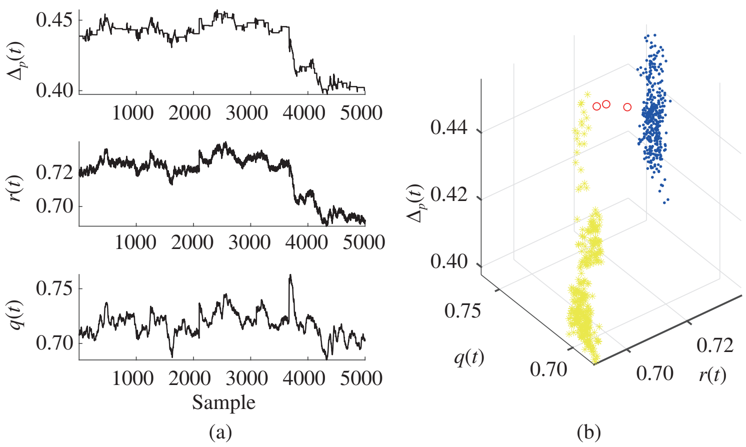

Figure 3 (Color online) (a) Normalized time series of the historical abnormal data; (b) the 3D plot of the data.

Table 1 Initial referential values for the input and output variables

Variable	$q(t)$			$r(t)$			$\Delta_p(t)$		
	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	D_1	D_2	D_3
Referential value	0.55	0.70	0.80	0.65	0.75	0.85	0.35	0.45	0.55

Table 2 Initial belief rule base

Number	Rule	Rule weight	Belief degree		
			$\beta_{1,k}$	$\beta_{2,k}$	$\beta_{3,k}$
1	$A_{1,1} \wedge A_{2,1}$	1	0.1639	0.8361	0
2	$A_{1,1} \wedge A_{2,2}$	1	0.1609	0.8391	0
3	$A_{1,1} \wedge A_{2,3}$	1	0	0.6719	0.3281
4	$A_{1,2} \wedge A_{2,1}$	1	0.1700	0.8300	0
5	$A_{1,2} \wedge A_{2,2}$	1	0	0.6719	0.3281
6	$A_{1,2} \wedge A_{2,3}$	1	0	0.6719	0.3281
7	$A_{1,3} \wedge A_{2,1}$	1	0	0.1857	0.8143
8	$A_{1,3} \wedge A_{2,2}$	1	0	0.1857	0.8143
9	$A_{1,3} \wedge A_{2,3}$	1	0	0.1857	0.8143

three clusters in the dataset, with 3750 blue points, 1043 yellow points and 207 red points.

In contrast, Figure 3 shows the historical abnormal dataset. For the historical abnormal dataset, an anomaly happens at around the 3670th sample. This time, however, there are only 3 red points, which clearly indicates the occurrence of an abnormal condition. In addition, it can be seen that the flow rate $q(t)$ suddenly increase, while the other variables decrease. This is in sharp contrast to Figure 2, where all the three variables share the same trend.

Based on the historical datasets, the BRB model can now be constructed using the procedures proposed in Section 3. In order to build a BRB model, the initial referential values of the input and output variables should be obtained, as shown in Table 1. The initial referential values in Table 1 can be determined using expert knowledge or observation from data. In this application, they are determined by investigating the distribution of the input and output variables to ensure that the intervals defined by these referential values equally cover the whole dataset. In addition, the initial attribute weights are set as $\delta_1 = 1, \delta_2 = 1$. And a total of 9 initial belief rules are built, as shown in Table 2. In fact, determination of the number of belief rules is problem-specific. In general, if a process involves more variables, more rules should be constructed to get a satisfactory tracking accuracy. Obviously, more rules will result in more parameters to be estimated and hence heavier computation load. In the case of condensate pump, it is found that adopting 9 rules can better balance the computation burden and tracking accuracy via a number of tests on the training dataset. In Table 2, the initial belief degrees are determined by roughly estimating the percentage of output values falling into the subspace spanned by the input referential values in each rule. Using the trial and error approach, the transition probability of each parameter and the observation

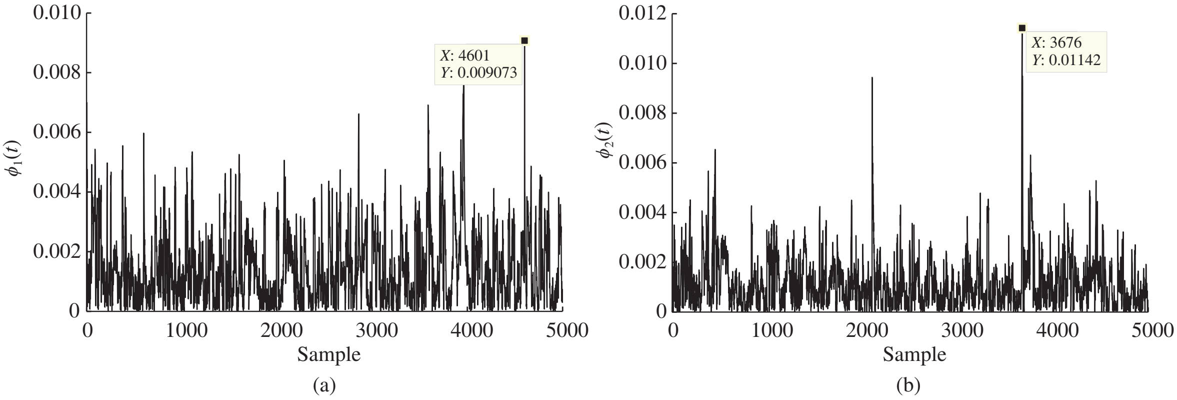

Figure 4 Absolute tracking error for (a) the historical normal dataset and (b) the historical abnormal dataset.

probability are initialized to be as follows:

$$\begin{aligned}
&A^k_{m_k}(t) \sim N(A^k_{m_k}(t-1), 0.0001),\\
&D_j(t) \sim N(D_j(t-1), 0.00001),\\
&\theta_k(t) \sim N(\theta_k(t-1), 0.0001),\\
&\delta_i(t) \sim N(\delta_i(t-1), 0.0001),\\
&\beta_{jk}(t) \sim N(\beta_{jk}(t-1), 0.0001),\\
&p(y(t)|\mathbf{\Theta}(t)) \sim N(y(t), 0.0001).
\end{aligned} \tag{24}$$

With all the parameters initialized, a normal sample set with 500 samples are used to train the initial model. Although the BRB method involves many parameters, the number of samples used to train the initial model will be relatively small if a good initial model with appropriate parameters determined. This is true in this case as only 500 samples are sufficient to generate a good initial model. Based on the initial model, the sequential Monte Carlo sampling is then used to update the model parameters online. For SMC, the number of sampling is set as $N_s = 200$ and the resampling threshold is set as $N_{\rm thr} = 20$. Increasing N_s results in higher tracking accuracy, however, at the cost of heavier computation burden. Through multiple trials, it is found that increasing N_s to more than 200 does not show significant improvement on tracking accuracy.

For the online updating algorithm, once a fault is detected, the Bayesian BRB parameters stop updating and the tracking errors continuously exceed the control limit. If the system gets back to normal, the tracking errors will not violate the control limit and the Bayesian BRB parameters begin to update again. The tracking errors for the historical normal and abnormal datasets are shown in Figure 4.

It can be seen that the maximum of $\phi_2(t)$ was observed at the 3676th sample, which is in accordance with the previous knowledge about the occurrence of the fault. According to (23), the alarm threshold is set as $\phi_{tp} = 0.01025$.

5.2 Performance verification

In order to verify the performance of the proposed method, a test normal dataset and a test abnormal dataset corresponding to the green circles in Figure 1 are considered. In the test normal dataset, a total of 5000 data points are collected during the time from 9:32:10 to 10:23:20 on May 17, 2015. On the other hand, another 5000 data points during 11:29:40 to 12:53:00 on May 25, 2015 are used as the test abnormal dataset, during which a fault occurred. The time series of both datasets are shown in Figures 5 and 6.

Comparing Figures 5 and 6, we can see that when the fault occurred, the trend of flow rate $q(t)$ differs from those of $\Delta_p(t)$ and $r(t)$. This is in accordance with the analysis in Subsection 5.1. More specifically, around the 3045th data point of the abnormal dataset, all the three variables increase. Around the 3200th sample, $q(t)$ reaches its peak and begins to decrease. This can be explained. The fault caused an emergency shutdown of the pump at around 13:00:00 and the backup pump is triggered. Using the same parameter settings as in Subsection 5.1, the Bayesian BRB inference generates the absolute tracking

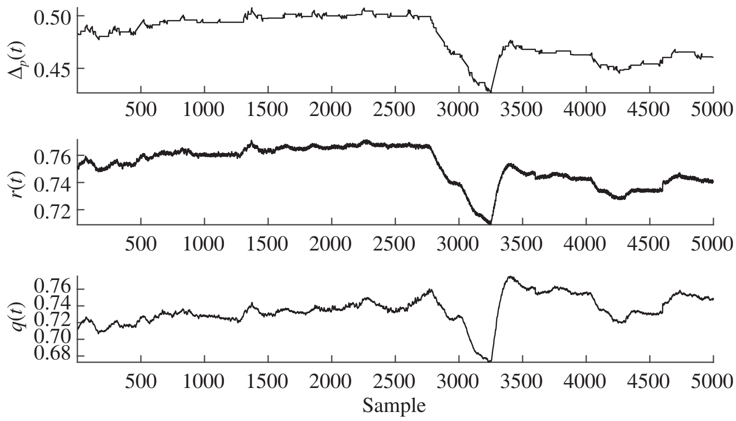

Figure 5 Normalized time series of the test normal data.

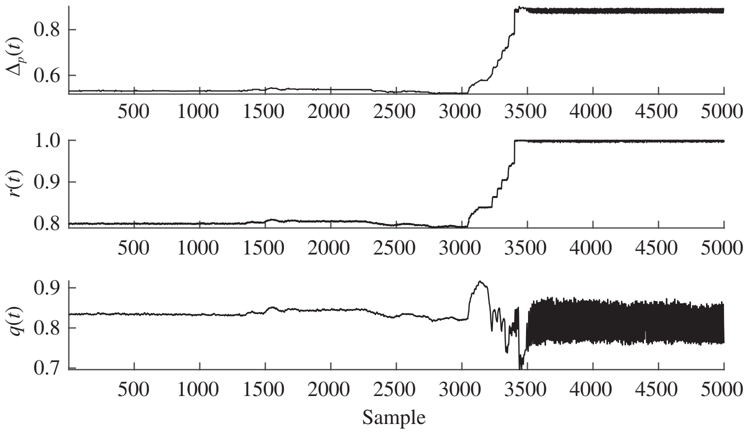

Figure 6 Normalized time series of the test abnormal data.

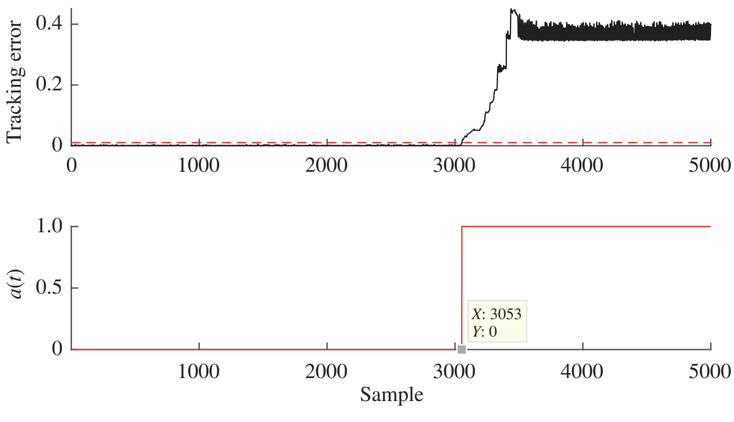

Figure 7 (Color online) Values of $\phi(t)$ and the alarm sequence for the test abnormal dataset.

errors and the alarm variable $a(t)$ for both the test normal dataset and test abnormal dataset, as shown in Figure 7.

From Figure 7 it can be seen that satisfactory alarm results have been obtained. No alarm was triggered before the 3045th sample as all the tracking errors are below the threshold. In contrast, the proposed method triggered alarm for the test abnormal dataset at the 3053rd data point, just a few data points after the fault occurred.

For comparison, the monitoring statistics and alarm sequence for the test abnormal dataset using PCA and the univariate method proposed by [31] are presented in Figures 8 and 9. For PCA, the number of principal components (PCs) retained is set as 2 because 2 PCs are sufficient of retaining more than 90% variance.

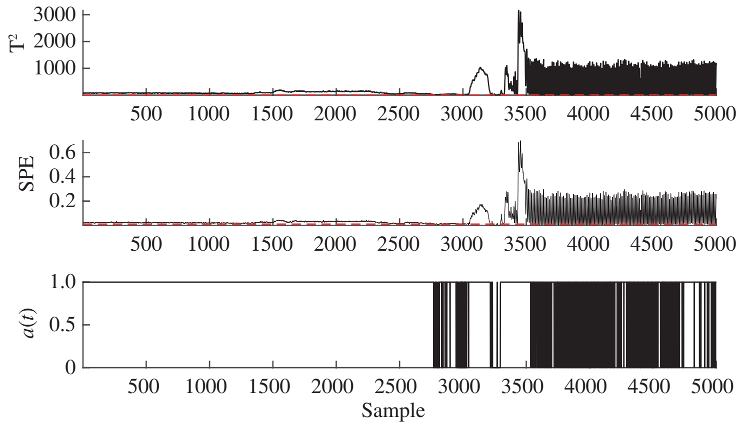

Figure 8 (Color online) Monitoring statistics and alarm sequence for the test abnormal dataset using PCA.

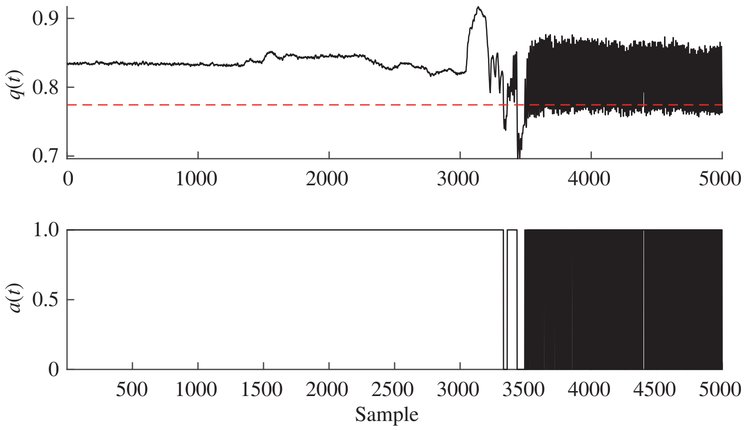

Figure 9 (Color online) Monitoring results and alarm sequence for the test abnormal dataset using the univariate method.

From Figures 8 and 9 it can be seen that neither PCA nor the univariate method can capture the time-varying characteristics of the system. As a result, a significant number of false alarms are observed. This clearly indicates the advantages of the proposed method in accommodating time-varying and nonlinear systems.

In addition, to test the efficiency of the Bayesian BRB method, the computation time is recorded. The test is performed on a personal computer with the CPU of Intel® Core™ i5-1035G1@1.00 GHz 1.19 GHz and RAM of 16 G. The computation time for 5000 normal data samples is 126.09 s and the average time for each sample is 0.025 s. For the test data samples, the computation time for the first 3053 samples is 75.73 s and the average time for each sample is 0.0248 s. After the 3053rd sample, a fault was detected and the model stopped updating. It can be seen that the online tracking algorithm is fast and efficient.

6 Conclusion

This study proposed a Bayesian BRB inference multivariate alarm system for time-varying nonlinear processes. Compared with conventional methods, the BRB method does not require the prior knowledge of the input-output relationships and is able to handle general nonlinear processes. By introducing an SMC-based inference method, the tracking error of the BRB model can be obtained online, which is further used to update the multivariate alarm system. Application to the alarm design of a condensate pump indicates that, compared to conventional methods, the proposed Bayesian BRB method can effectively accommodate the nonlinear and time-varying characteristics of the process and significantly reduce the amount of false and nuisance alarms.

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