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• **Supplementary File** •

An interactive multiobjective evolutionary algorithm based on decomposition and compression

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Appendix A Test problems used in the experiments

The formulations and brief introductions of the 20 test problems used in our experiments are as follows.

P1: Water resources planning

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = e^{0.01x_1} x_1^{0.02} x_2^2 \\ & f_2(\mathbf{x}) = 0.5x_2^2 \\ & f_3(\mathbf{x}) = -e^{0.005x_1} x_1^{0.001} x_2^2 \\ \text{subject to} \quad & 0.01 \leqslant x_1 \leqslant 1.3 \\ & 0.01 \leqslant x_2 \leqslant 10. \end{aligned}$$

This is a simplified water resources planning problem [1–3]. A multipurpose dam was to be constructed at a certain site. The cost of construction (f_1) and the water loss (f_2) are to be minimized, and the total storage capacity ($-f_3$) of the reservoir is to be maximized. The decision variables x_1 and x_2 represent the total man-hours devoted to building the dam and the mean radius of the lake impounded (in miles), respectively. In [1, 2], the decision variables have no upper bounds and their lower bounds are 0. In [3], their lower bounds are modified to be 0.01. In this paper, we follow the modification in [3].

P2: Water quality management

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = -4.07 - 2.27x_1 \\ & f_2(\mathbf{x}) = -2.60 - 0.03x_1 - 0.02x_2 - \frac{0.01}{1.39 - x_1^2} - \frac{0.30}{1.39 - x_2^2} \\ & f_3(\mathbf{x}) = -8.21 + \frac{0.71}{1.09 - x_1^2} \\ & f_4(\mathbf{x}) = -0.96 + \frac{0.96}{1.09 - x_2^2} \\ \text{subject to} \quad & 0.3 \leqslant x_1, x_2 \leqslant 1.0. \end{aligned}$$

This problem deals with the pollution of the (hypothetical) Fast Water Valley on a stretch of 100 river miles [4]. Four objectives: the dissolved oxygen (DO) level at Fortuna (f_1), the DO level at the state line (f_1), the percent return on investment at the Fresh Fishery (f_3), and the addition to the tax rate for Fortuna (f_4) are considered. The decision variables represent the proportionate amount of biochemical oxygen demanding material removed from water discharge at the Fresh Fishery and at Fortuna, respectively.

In [5], the nonsmooth function $f_5(\mathbf{x}) = \max[|x_1 - 0.65|, |x_2 - 0.65|]$ which is to be minimized is added into P2. The resulted five-objective problem is used as P3 in our paper.

P4: Location problem

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = \varphi(x_1, x_2) \\ & f_2(\mathbf{x}) = \varphi(x_1 - 1.2, x_2 - 1.5) \\ & f_3(\mathbf{x}) = \varphi(x_1 + 0.3, x_2 - 3.0) \\ & f_4(\mathbf{x}) = \varphi(x_1 - 1.0, x_2 + 0.5) \\ & f_5(\mathbf{x}) = \varphi(x_1 - 0.5, x_2 - 1.7) \\ \text{subject to} \quad & -4.9 \leqslant x_1 \leqslant 3.2 \\ & -3.5 \leqslant x_1 \leqslant 6.0, \end{aligned}$$

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where

$$\varphi(x_1,x_2) = -3(1-x_1)^2 e^{-x_1^2-(x_2+1)^2} + 10\left(\frac{1}{4}x_1 - x_1^3 - x_2^5\right) e^{-x_1^2-x_2^2} - \frac{1}{3}e^{-(x_1+1)^2-x_2^2}.$$

The goal of this problem is to locate a pollution monitoring station [3,6]. The decision variables represent the coordinates of the station. The five objectives describe five experts' different forecasts for the information loss.

P5: ZDT1

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = x_1 \\ & f_2(\mathbf{x}) = g \cdot (1 - \sqrt{f_1/g}) \\ \text{subject to} \quad & 0 \leqslant x_i \leqslant 1, i \in \{1,\ldots,n\}, \end{aligned}$$

where $g(x_2,\ldots,x_n) = 1 + 9 \cdot \Sigma_{i=2}^n x_i/(n-1)$, $n = 30$ is the number of decision variables. This problem is presented in [7] and is often used to test the performance of multiobjective evolutionary algorithms.

P8: Seven hard functions

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1-x_1)^2 \\ & f_2(\mathbf{x}) = \max[x_1^2 + (x_2-1)^2 + x_2 - 1, -x_1^2 - (x_2-1)^2 + x_2 + 1] \\ & f_3(\mathbf{x}) = \max[x_1^2 + x_2^4, (2-x_1)^2 + (2-x_2)^2, 2e^{-x_1+x_2}] \\ & f_4(\mathbf{x}) = \max[5x_1 + x_2, -5x_1 + x_2, x_1^2 + x_2^2 + 4x_2] \\ & f_5(\mathbf{x}) = \max[x_1^2 + x_2^2, x_1^2 + x_2^2 + 10(-4x_1 - x_2 + 4), x_1^2 + x_2^2 - 10(x_1 + 2x_2 - 6)] \\ & f_6(\mathbf{x}) = \max[-x_1 - x_2, -x_1 - x_2 + (x_1^2 + x_2^2 - 1)] \\ & f_7(\mathbf{x}) = -x_1 + 20\max[x_1^2 + x_2^2 - 1, 0] \\ \text{subject to} \quad & -100 \leqslant x_1, x_2 \leqslant 100. \end{aligned}$$

Miettinen and Mäkelä have collected seven standard test functions (Rosenbrock, Crescent, CB2, Dem, QL, LQ and Mifflin) from nondifferentiable optimization to form the above seven-objective optimization problem [3,8]. In our paper, except for using this problem, we also split it into a two-objective problem (P6) by minimizing the first two objectives and a five-objective problem (P7) by considering the last five objectives.

P9-P10: DTLZ1

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = \tfrac{1}{2}x_1x_2\cdots x_{k-1}(1+g(\mathbf{x}_k)) \\ & f_2(\mathbf{x}) = \tfrac{1}{2}x_1x_2\cdots(1-x_{k-1})(1+g(\mathbf{x}_k)) \\ & \vdots \\ & f_k(\mathbf{x}) = \tfrac{1}{2}(1-x_1)(1+g(\mathbf{x}_k)) \\ \text{subject to} \quad & 0 \leqslant x_i \leqslant 1, i \in \{1,\ldots,n\}, \end{aligned}$$

where

$$g(\mathbf{x}_k) = 100\left[|\mathbf{x}_k| + \sum_{x_i \in \mathbf{x}_k}((x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)))\right].$$

$|\mathbf{x}_k| = 5$ is suggested and the total number of decision variables is $n = k + |\mathbf{x}_k| - 1$. This is a scalable problem given in [9]. Its Pareto front is a linear hyperplane with the form $\sum_{i=1}^k f_i = 0.5$. In our paper, we set P9 with $k = 3, n = 7$ and P10 with $k = 4, n = 8$, respectively.

P11-P12: DTLZ2

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = (1+g(\mathbf{x}_k))\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_{k-2}\pi/2)\cos(x_{k-1}\pi/2) \\ & f_2(\mathbf{x}) = (1+g(\mathbf{x}_k))\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_{k-2}\pi/2)\sin(x_{k-1}\pi/2) \\ & f_3(\mathbf{x}) = (1+g(\mathbf{x}_k))\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\sin(x_{k-2}\pi/2) \\ & \vdots \\ & f_{k-1}(\mathbf{x}) = (1+g(\mathbf{x}_k))\cos(x_1\pi/2)\sin(x_2\pi/2) \\ & f_k(\mathbf{x}) = (1+g(\mathbf{x}_k))\sin(x_1\pi/2) \\ \text{subject to} \quad & 0 \leqslant x_i \leqslant 1, i \in \{1,\ldots,n\} \end{aligned}$$

where

$$g(\mathbf{x}_k) = \sum_{x_i \in \mathbf{x}_k}(x_i - 0.5)^2.$$

This is also a scalable problem introduced in [9]. The total number of variables is $n = k + |\mathbf{x}_k| - 1$ where $|\mathbf{x}_k| = 10$ is recommended. In our paper, P11 and P12 correspond to $k = 3, n = 12$ and $k = 4, n = 13$, respectively.

P13: UF1

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = x_1 \qquad + \frac{2}{|J_1|}\sum\nolimits_{j\in J_1}[x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2 \\ & f_2(\mathbf{x}) = 1 - \sqrt{x_1} + \frac{2}{|J_2|}\sum\nolimits_{j\in J_2}[x_j - \sin(6\pi x_1 + \frac{j\pi}{n})]^2 \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \leqslant j \leqslant n\}$ and $J_2 = \{j|j \text{ is even and } 2 \leqslant j \leqslant n\}$ [10]. The search space of decision vectors is $[0,1] \times [-1,1]^{n-1}$. The Pareto front of UF1 is convex.

P14: UF7

$$\begin{aligned} \text{minimize} \quad & f_1(\mathbf{x}) = \sqrt[5]{x_1} \qquad + \frac{2}{|J_1|} \sum\nolimits_{j \in J_1} y_j^2 \\ & f_2(\mathbf{x}) = 1 - \sqrt[5]{x_1} + \frac{2}{|J_2|} \sum\nolimits_{j \in J_2} y_j^2 \end{aligned}$$

where

$$J_1 = \{j | j \text{ is odd and } 2 \leqslant j \leqslant n\},$$
$$J_2 = \{j | j \text{ is even and } 2 \leqslant j \leqslant n\},$$
$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n$$

The search space of decision vectors is $[0, 1] \times [-1, 1]^{n-1}$ [10]. The Pareto front of UF7 is linear.

P15-P20 are two-objective WFG2-WFG4 and three-objective WFG2-WFG4 [11], respectively. The formulations of WFG problems are complex. Readers can refer to [11] for details. WFG2's Pareto front is convex and disconnected. The Pareto fronts of WFG3 and WFG4 are linear and concave, respectively.

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