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Special Focus on Control and Analysis for Stochastic Systems

Event-triggered fault detection for nonlinear discrete-time switched stochastic systems: a convex function method

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Dear editor,

To improve the safety and reliability of industrial processes, fault-detection problems in dynamic systems have attracted increasing research attention [1]. On the one hand, to enhance fault sensitivity, applying the H_- performance index to measure the minimum impact of the fault on the residual signal has been proposed. On the other hand, due to inevitable external disturbances, the fault-detection filter should restrain the impact of interference signals below a prescribed level while detecting faults. Besides, because the network bandwidth of communication is limited, the event-triggered fault-detection mechanism has been studied extensively to save network resources [2].

The main contributions of this study are as follows: (1) By considering the faults and external disturbances in the model, a mixed H_-/H_∞ fault-detection filtering is proposed to construct the residual signal. (2) An event-triggered scenario is recommended to reduce the burden of communication processes by determining whether measurements should be transmitted or not. (3) Based on a new difference operator associated with Lyapunov functions [3, 4], which only depends on the mathematical expectation of white noise $\{w_k\}$, a more practical result for the H_-/H_∞ fault detection filter for general nonlinear discrete switched stochastic systems is provided.

The notations are provided in Appendix A.

Problem description. In this study, we focus on the following discrete-time nonlinear switching stochastic system:

$$\begin{cases} x_{k+1}=d_{\sigma_k}(x_k,w_k)+g_{\sigma_k}(x_k,w_k)f_k+h_{\sigma_k}(x_k,w_k)v_k, \\ y_k=m_{\sigma_k}(x_k)+q_{\sigma_k}(x_k)f_k+z_{\sigma_k}(x_k)v_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^{n_y}$ is the measurement output, and $\{w_k\}_{k\in\mathcal{N}}$ is a sequence of independent n_w-dimensional random variables with an identical distribution that is defined on the complete filtered probability space $(\Omega,\mathcal{F},\{\mathcal{F}_k\}_{k\in\mathcal{N}},\mathcal{P})$, where $\mathcal{F}_k = \sigma(w_0,w_1,\ldots,w_{k-1})$, $\mathcal{F}_0=\{\phi,\Omega\}$. $f_k\in\mathbb{R}^{n_f}$ is the fault signal with $\{f_k\}_{k\in\mathcal{N}} \in l^2_\infty(\mathcal{N},\mathbb{R}^{n_f})$ to be detected and $v_k\in\mathbb{R}^{n_v}$ denotes the exogenous disturbance signal with $\{v_k\}_{k\in\mathcal{N}}\in l^2_\infty(\mathcal{N},\mathbb{R}^{n_v})$. The switching signal σ_k is a deterministic function of time, which takes its values in a finite set, $\sigma_k:\mathcal{N}\mapsto\mathcal{G}=\{1,2,\ldots,N\}$. Assume that the switching time series of switching signal σ_k is $\{\kappa_0,\kappa_1,\ldots\}$ with $\kappa_0=0$. Besides, when $\sigma_k=i$, let $d_{\sigma_k}(x_k,w_k)=d_i(x_k,w_k)$, and so are the other parameters in the system (1).

For system (1), we introduce the following fault detection filter:

$$\begin{cases} \hat{x}_{k+1}=\hat{d}_{\sigma_k}(\hat{x}_k)+\hat{n}_{\sigma_k}(\hat{x}_k)y_k,\ \hat{d}_{\sigma_k}(0)=0,\ \hat{n}_{\sigma_k}(0)=0, \\ r_k=\hat{l}_{\sigma_k}(\hat{x}_k)(y_k-m_{\sigma_k}(\hat{x}_k)),\ \hat{x}_0=0, \end{cases} \tag{2}$$

where $\hat{x}_k$ is the estimated value of x_k, r_k denotes the residual signal, $\hat{d}_i$, $\hat{n}_i$ and $\hat{l}_i$ are the filter functions that must be designed. Note that y_k is not directly sent to the filter at each instant k, but only at each triggering moment in $\{k_t\}_{t\in\mathcal{N}}$.

For producing $\{k_t\}_{t\in\mathcal{N}}$, design event generator functions $\phi_i(x,y):\mathbb{R}^+\times\mathbb{R}^+\mapsto\mathbb{R}$, $i\in\mathcal{G}$. Suppose $\phi_i(0,y)\leqslant 0$, $\phi_i(x,0)\geqslant 0$. Assume that $\phi_i(x,y)$ is continuously increasing for x and continuously decreasing for y. Then, $\{k_t\}_{t\in\mathcal{N}}$ can be computed iteratively using

$$\begin{cases} k_t=0,\quad t=0, \\ k_t=\min_k\{k>k_{t-1}|\phi_{\sigma_k}(\|y_k-y_{k_{t-1}}\|, \\ \qquad \|y_k\|)>0\},\quad t>1. \end{cases} \tag{3}$$

Setting $e^t_k=y_{k_t}-y_k$, let the extended variable $\tilde{x}_k=\left[\begin{smallmatrix} x_k \\ \hat{x}_k-x_k \end{smallmatrix}\right]$, and then the augmented system can be given as

$$\begin{cases} \tilde{x}_{k+1}=\tilde{d}_{\sigma_k}(\tilde{x}_k,w_k)+\tilde{g}_{\sigma_k}(\tilde{x}_k,w_k)f_k \\ \qquad +\tilde{h}_{\sigma_k}(\tilde{x}_k,w_k)v_k+\tilde{n}_{\sigma_k}(\tilde{x}_k)e^t_k, \\ r_k=\tilde{m}_{\sigma_k}(\tilde{x}_k)+\tilde{q}_i(\tilde{x}_k)f_k+\tilde{z}_i(\tilde{x}_k)v_k+\hat{l}_{\sigma_k}(\hat{x}_k)e^t_k. \end{cases} \tag{4}$$

In the following, we present the event-triggered fault detection problem:

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• Design a triggered communication condition (3) and a mixed H_-/H_∞ fault detection filter to construct an event-based residual signal, such that the residual signal has satisfactory performance.

• After obtaining the filter gain functions, we design a residual evaluation function $J_r(t)$ as

$$J_r(t) := \left(\left\{\frac{1}{t}\sum_{s=0}^{t} r_s' r_s\right\}\right)^{\frac{1}{2}}, \quad J_r(0) = 0, \tag{5}$$

and the predefined threshold $J_{\rm th}$ as

$$J_{\rm th} := \sup_{f(t)\equiv 0, v(t)\in l_\infty^2(\mathcal{N},\mathbb{R}^{n_v})} {\rm E}J_r(T), \tag{6}$$

where T is the evaluation window. $J_{\rm th}$ and $J_r(t)$ meet the following decision logic:

$$\begin{cases} J_r(t) > J_{\rm th} \Rightarrow \text{faults} \Rightarrow \text{alarm}, \\ J_r(t) \leqslant J_{\rm th} \Rightarrow \text{fault-free}. \end{cases} \tag{7}$$

Appendix B provides some preliminaries.

Main result. We first choose a set of convex Lyapunov functions and provide an event-triggered H_-/H_∞ fault detection filtering in the main theorem.

Assumption 1. $\varphi_i(x)$ and $\chi_i(y)$ are convex functions and $\varphi_i(x)$ is a $\mathcal{K}$-function. Moreover, $\phi_i(x,y) = \varphi_i(x) - \chi_i(y)$.

Theorem 1. Suppose Assumption 1 holds. If there exists a set of convex positive definite decrescent functions $\{V_i(\tilde{x}), i \in \mathcal{G}\}$ with $\mu_{i,1}\|\tilde{x}\|^2 \leqslant V_i(\tilde{x}) \leqslant \mu_{i,2}\|\tilde{x}\|^2$ and $V_i(\tilde{x}) \leqslant \mu_3 V_j(\tilde{x}), \forall i,j \in \mathcal{G}$, such that

$$\hbar_1(\tilde{x}) \leqslant 0, \tag{8}$$

$$\sup_{0\neq v\in\mathbb{R}^{n_v}} \left\{\frac{\hbar_2(\tilde{x},v)}{\|v\|^2}\right\} \leqslant \gamma^2, \tag{9}$$

$$\inf_{0\neq f\in\mathbb{R}^{n_f}} \left\{\frac{\hbar_3(\tilde{x},f)}{\|f\|^2}\right\} \geqslant \delta^2, \tag{10}$$

and switching dwell time T_a satisfies

$$T_a > \text{ceil}\left(\max_{i\in\mathcal{G}}\left(-\frac{\ln\mu_3}{\ln\lambda_i}\right)\right), \tag{11}$$

where

$$\begin{aligned}\hbar_1(\tilde{x}) =& {\rm E}\alpha_{i,1}V_i\left(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x},w_0)\right) + (1-\alpha_{i,1})\mu_{i,2} \\ &\cdot \left\|\frac{1}{1-\alpha_{i,1}}\tilde{n}_i(\tilde{x})\right\|^2 \left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{\alpha_{i,4}} + 2\|\tilde{m}_i(\tilde{x})\|^2\right.\right. \\ &\left.\left.\cdot \|m_i([I\ 0]\tilde{x})\|\right)\right]^2 - \lambda_i V_i(\tilde{x}) + 2\alpha_{i,4}\|\hat{l}_i(\hat{x})\|^2 \\ &\cdot \left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x})\|\right)\right]^2,\end{aligned}$$

$$\begin{aligned}\hbar_2(\tilde{x},v) =& {\rm E}\alpha_{i,2}V_i\left(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x},w_0)v\right) \\ &+ \alpha_{i,3}\mu_{i,2}\left\|\frac{\sqrt{2}(1-\alpha_{i,4})}{\alpha_{i,3}}\tilde{n}_i(\tilde{x})\right\|^2 \\ &\cdot\left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x})v\|\right)\right]^2 \\ &+ 2(1-\alpha_{i,4}) \\ &\cdot\|\hat{l}_i(\hat{x})\|^2\left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x})v\|\right)\right]^2 \\ &+ 2\|\tilde{z}_i(\tilde{x})v\|^2,\end{aligned}$$

$$\begin{aligned}\hbar_3(\tilde{x},f) =& -{\rm E}\alpha_{i,2}V_i\left(\frac{1}{\alpha_{i,2}}\tilde{g}_i(\tilde{x},w_0)f\right) - \alpha_{i,3}\mu_{i,2} \\ &\cdot\left\|\frac{\sqrt{2}(1-\alpha_{i,4})}{\alpha_{i,3}}\tilde{n}_i(\tilde{x})\right\|^2\left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{1-\alpha_{i,4}}\right.\right. \\ &\left.\left.\cdot\|q_i([I\ 0]\tilde{x})f\|\right)\right]^2 - 2(1-\alpha_{i,4})\|\hat{l}_i(\hat{x})\|^2 \\ &\cdot\left[\varphi_i^{-1}\circ\chi_i\left(\frac{1}{1-\alpha_{i,4}}\|q_i([I\ 0]\tilde{x})f\|\right)\right]^2 \\ &+ 2\|\tilde{q}_i(\tilde{x})f\|^2,\end{aligned}$$

$\mu_{i,1}$, $\mu_{i,2}$ μ_3 and $\alpha_{i,l} > 0$ are real numbers, $l \in \{1,2,3,4\}$, $\mu_3 \geqslant 1$, $\alpha_{i,1} + \alpha_{i,2} + \alpha_{i,3} = 1$, and $\alpha_{i,4} < \frac{\alpha_{i,3}}{\sqrt{2}(1-\alpha_{i,1})}$, then system (4) is internally stable. Moreover, system (4) satisfies the weighted H_∞ performance index γ for v and the weighted H_- performance index δ for f_k under the triggered communication scheme (3).

Appendix C provides the proof of Theorem 1. Appendix D provides a corollary about the linear case, and Appendix E shows a numerical example.

Conclusion. This study discussed a fault detection filter design for switching nonlinear stochastic systems using the event-triggered strategy. To simultaneously enhance the interference suppression ability and fault sensitivity of the fault-detection mechanism, a weighted mixed H_-/H_∞ fault detection filter was proposed. By choosing the convex Lyapunov function method and defining appropriate difference operators, we have researched the desired fault-detection filter and provided a sufficient condition about the existence in Theorem 1. The proposed results can be extended to many directions, such as systems with packed loss [5] and multiple nonlinear systems [6, 7].

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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