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• **Supplementary File** •

Event-triggered fault detection for nonlinear discrete-time switched stochastic systems: convex function method

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Appendix A Notations

S': the transpose of the matrix S or vector S; I_n: $n \times n$ identity matrix; $S > 0$ ($S < 0$): the matrix S is a positive definite (negative definite) symmetric matrix; $\mathcal{R}^n$: the n-dimensional real vector space with 2-norm $\|x\| = \sqrt{\sum_{k=0}^{n} x_k^2}$ for $x = [x_1\ x_2\ \cdots\ x_n]'$; $\mathcal{R}^{n\times m}$: the $n \times m$ real matrix space; $\mathcal{R}^+$: a set containing all positive real numbers and zero. $\mathcal{N} := \{0, 1, 2, \cdots, \}$; E: the mathematical expectation operator; $l^2_\infty(\mathcal{N}, \mathcal{R}^{nv})$: the space of all non-anticipative stochastic processes $\{v_k \in \mathcal{R}^{nv}\}_{k\in\mathcal{N}}$ with the norm

$$\|v\|_{l^2_\infty} := \left\{E\left[\sum_{k=0}^{\infty}\|v_k\|^2\right]\right\}^{1/2} < \infty;$$

If $l(x) > 0$ for any $x \neq 0$, and $l(0) = 0$, then function $l(x)$ is called a positive function; $\mathcal{K}$: the family of all continuous positive increasing functions $\mu(\cdot)$; $ceil(x)$: the smallest integer greater than or equal to x.

Appendix B Preliminaries

For the considered discrete-time nonlinear switching stochastic system (1), an event-triggered fault detection filter is adopted as

$$\begin{cases}\hat{x}_{k+1} = \hat{d}_{\sigma_k}(\hat{x}_k) + \hat{n}_{\sigma_k}(\hat{x}_k)y_{k_t}, \hat{d}_{\sigma_k}(0) = 0, \hat{n}_{\sigma_k}(0) = 0,\\ r_k = \hat{l}_{\sigma_k}(\hat{x}_k)(y_{k_t} - m_{\sigma_k}(\hat{x}_k)), \hat{x}_0 = 0,\end{cases} \tag{B1}$$

where $k \in [k_t, k_{t+1})$. Besides, the parameters in the augmented system (4) is given as

$$\tilde{d}_i(\tilde{x}_k, w_k) = \begin{bmatrix} d_i(x_k)\\ \hat{d}_i(\hat{x}_k) - d_i(x_k, w_k) + \hat{n}_i(\hat{x}_k)m_i(x_k)\end{bmatrix},$$

$$\tilde{g}_i(\tilde{x}_k, w_k) = \begin{bmatrix} g_i(x_k, w_k)\\ \hat{n}_i(\hat{x}_k)q_i(x_k) - g_i(x_k, w_k)\end{bmatrix},\ \tilde{h}_i(\tilde{x}_k, w_k) = \begin{bmatrix} h_i(x_k, w_k)\\ \hat{n}_i(\hat{x}_k)z_i(x_k) - h_i(x_k, w_k)\end{bmatrix},$$

$$\tilde{n}_i(\tilde{x}_k) = \begin{bmatrix} 0\\ \hat{n}_i(\hat{x}_k)\end{bmatrix},\ \tilde{m}_i(\tilde{x}_k) = \hat{l}_i(\hat{x}_k)m_i(x_k) - \hat{l}_i(\hat{x}_k)m_i(\hat{x}_k),$$

$$\tilde{q}_i(\tilde{x}_k) = \hat{l}_i(\hat{x}_k)q_i(x_k),\ \tilde{z}_i(\tilde{x}_k) = \hat{l}_i(\hat{x}_k)z_i(x_k).$$

Definition 1. For a switching signal $\{\sigma_k\}$, if there exist $\tau_0 > 0$, $\tau_a > 0$ such that

$$N_\sigma(\tau_1, \tau_2) \leqslant \tau_0 + \frac{\tau_2 - \tau_1}{\tau_a},$$

in which $\tau_2 \geqslant \tau_1 \geqslant 0$ are any positive numbers, $N_\sigma(\tau_1, \tau_2)$ stands for the switching numbers over an interval (τ_1, τ_2), then $\{\sigma_k\}$ has an average dwell time τ_a

Definition 2. System $x_{k+1} = f(x_k, w_k)$ is called exponentially stable in mean square (ESMS), if there exist $\rho \geqslant 1$, $\varrho \in (0, 1)$ such that

$$E\|x_k\|^2 \leqslant \rho E\|x_{k_0}\|^2 \varrho^{(k-k_0)}$$

holds for any $0 \leqslant k_0 \leqslant k < +\infty$.

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We need to measure the residual signal r_k and then calculate it in real time to judge if the fault is occurring under external interferences or not. So, the comprehensive ability to suppress external interferences and reflect fault information is important for r_k. For reasonably analyzing the capacity of fault detection filters, our purpose is to design a mixed H_-/H_∞ fault detection filter for discrete-time nonlinear switching stochastic system (1). We next present the definition of mixed weighted H_-/H_∞ fault detection filter.

Definition 3. A fault detection filter (B1) is called the event-triggered mixed H_-/H_∞ fault detection filter under triggered communication scheme (3), if for positive numbers $\gamma > 0$ and $\delta > 0$ given in advance, the following three requirements are met simultaneously:

- System (4) is internally stable, which means that system

$$\begin{cases} \tilde{x}_{k+1} = \tilde{d}_{\sigma_k}(\tilde{x}_k, w_k) + \tilde{n}_{\sigma_k}(\tilde{x}) e_k^t, \\ \tilde{x}_0 = \begin{bmatrix} x_0 \\ 0 \end{bmatrix} \in \mathcal{R}^{n\eta} \end{cases}$$

is ESMS.

- System (4) is externally stable, which means that when $f_k \equiv 0$ and $\tilde{x}_0 = 0$ in system (4) , for positive real numbers $\gamma > 0$, $0 < \lambda < 1$, and any nonzero $v_k \in l^2_\infty(\mathcal{N}, \mathcal{R}^{n_v})$, the weighted l_2-gain from v_k to r_k of the following system

$$\begin{cases} \tilde{x}_{k+1} = \tilde{d}_{\sigma_k}(\tilde{x}_k, w_k) + \tilde{h}_{\sigma_k}(\tilde{x}_k, w_k) v_k + \tilde{n}_{\sigma_k}(\tilde{x}_k) e_k^t, \\ r_k = \tilde{m}_{\sigma_k}(\tilde{x}_k) + \tilde{z}_i(\tilde{x}_k) v_k + \hat{l}_{\sigma_k}(\hat{x}_k) e_k^t \end{cases}$$

is no larger than γ, i.e., the weighted H_∞ index

$$\|\mathcal{L}_{v,r}\|_\infty^2 := \sup_{f_k \equiv 0, \tilde{x}_0, v_k \not\equiv 0, v_k \in l^2_\infty(\mathcal{N}, \mathcal{R}^{n_v})} \frac{\|\lambda^k r_k\|^2_{l^2_\infty}}{\|v_k\|^2_{l^2_\infty}} \leqslant \gamma^2,$$

which means that $\gamma > 0$ is the disturbance attenuation level.

- The residual signal is sensitive enough to the fault, that is, when $v_k \equiv 0$ and $\tilde{x}_0 = 0$ in system (4), for positive real numbers $\delta > 0$, $0 < \lambda < 1$ and any nonzero $f_k \in l^2_\infty(\mathcal{N}, \mathcal{R}^{n_f})$, the weighted l_2-gain from f_k to r_k of the following system

$$\begin{cases} \tilde{x}_{k+1} = \tilde{d}_{\sigma_k}(\tilde{x}_k, w_k) + \tilde{g}_{\sigma_k}(\tilde{x}_k, w_k) f_k + \tilde{n}_{\sigma_k}(\tilde{x}_k) e_k^t, \\ r_k = \tilde{m}_{\sigma_k}(\tilde{x}_k) + \tilde{q}_i(\tilde{x}_k) f_k + \hat{l}_{\sigma_k}(\hat{x}_k) e_k^t \end{cases}$$

is no less than δ, i.e., the weighted H_- index

$$\|\mathcal{L}_{f,r}\|_-^2 = \inf_{v_k \equiv 0, \tilde{x}_0 = 0, f_k \not\equiv 0, f_k \in l^2_\infty(\mathcal{N}, \mathcal{R}^{n_f})} \frac{\|\lambda^k r_k\|^2_{l^2_\infty}}{\|f_k\|^2_{l^2_\infty}} \geqslant \delta^2,$$

which means that $\delta > 0$ are is the fault sensitivity level.

According to the description of the event-triggered fault detection problems, The event-triggered H_-/H_∞ fault detection scheme is given in Figure B1.

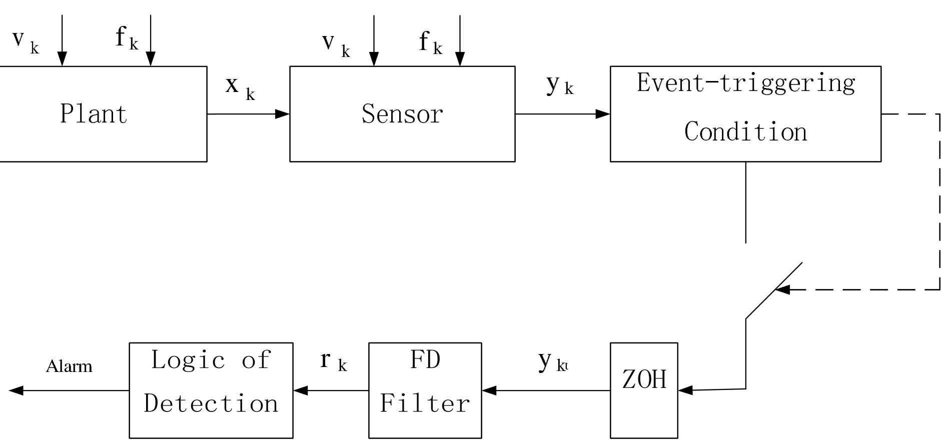

Figure B1 The diagram of the proposed event-triggered H_-/H_∞ FD scheme.

Definition 4. [1] We consider a continuous function $W(x)$ defined on $\mathcal{R}^n$.

(i) $W(x)$ is called a positive definite function (resp. a decrescent function) if $W(0) = 0$ (resp. $W(x) \geqslant 0$), and there exists $\mu \in \mathcal{K}$, such that

$$W(x) \geqslant \mu(\|x\|), (resp. W(x) \leqslant \mu(\|x\|)), \quad \forall x \in \mathcal{R}^n.$$

(iii) $W(x)$ is said to be radially unbounded if

$$\liminf_{\|x\| \to \infty} W(x) = \infty. \tag{B2}$$

Lemma 1. [2] If $\eta \in \mathcal{R}^d$ is a measurable random variable about σ-field $\mathcal{G}$, and $\zeta \in \mathcal{R}^n$ is independent of $\mathcal{G}$, then

$$E[g(\zeta, \eta)|\mathcal{G}] = E[g(\zeta, x)]_{x=\eta}, \quad a.s.$$

holds for any bounded or nonnegative function $g : \mathcal{R}^n \times \mathcal{R}^d \mapsto \mathcal{R}$.

Appendix C The Proof of Theorem 1

The proof is divided naturally into three parts. The first step is to define some appropriate difference operators. Choose a convex switch Lyapunov function $V_{\sigma_k}(\tilde{x})$, $V_{\sigma_k}: \mathcal{R}^{n_{\tilde{x}}} \mapsto \mathcal{R}^+$. For any $k \in [\kappa_j, \kappa_{j+1})$, assume that $\min_l\{k - k_l > 0 | l \in \mathcal{N}\} = t$ and $\sigma_k = i \in \mathcal{G}$. Since $\{w_k\}_{k\in\mathcal{N}}$ is independently identically distributed and x_k, $\hat{x}_k$, v_k, f_k and e_k^t are $\mathcal{F}_k$-measurable, according to Lemma 1, we have

$$\begin{aligned}E[V_i(\tilde{x}_{k+1})|\mathcal{F}_k] =& E[V_i(\tilde{d}_i(\tilde{x}_k, w_k) + \tilde{g}_i(\tilde{x}_k, w_k)f_k + \tilde{h}_i(\tilde{x}_k, w_k)v_k + \tilde{n}_i(\tilde{x}_k)e_k^t)|\mathcal{F}_k]\\ =& E[V_i(\tilde{d}_i(\tilde{x}, w_k) + \tilde{g}_i(\tilde{x}, w_k)f + \tilde{h}_i(\tilde{x}, w_k)v + \tilde{n}_i(\tilde{x})e)]_{\tilde{x}=\tilde{x}_k, v=v_k, f=f_k, e=e_k^t}\\ =& E[V_i(\tilde{d}_i(\tilde{x}, w_0) + \tilde{g}_i(\tilde{x}, w_0)v + \tilde{h}_i(\tilde{x}, w_0)v + \tilde{n}_i(\tilde{x})e)]_{\tilde{x}=\tilde{x}_k, v=v_k, f=f_k, e=e_k^t}.\end{aligned}$$

We define

$$\begin{aligned}&\Delta_{\lambda_i} V_i(\tilde{x}_k) := E[V_i(\tilde{d}_i(\tilde{x}_k, w_k) + \tilde{n}_i(\tilde{x})e_k^t)|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k),\\ &\Delta_{f,\lambda_i} V_i(\tilde{x}_k) := E[V_i(\tilde{d}_i(\tilde{x}_k, w_k) + \tilde{g}_i(\tilde{x}_k, w_k)f_k + \tilde{n}_i(\tilde{x}_k)e_k^t)|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k)\end{aligned}$$

and

$$\Delta_{v,\lambda_i} V_i(\tilde{x}_k) := E[V_i(\tilde{d}_i(\tilde{x}_k, w_k) + \tilde{h}_i(\tilde{x}_k, w_k)v_k + \tilde{n}_i(\tilde{x}_k)e_k^t)|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k)$$

as the difference operators of the function V_i with $\lambda_i \in (0, 1)$.

The second step is to prove internal stability and external stability in Definition 3. When $v_k \equiv 0$ and $f_k \equiv 0$, due to the convexness of V_i, for any nonnegative scalars $\alpha_{i,1}$ and $\alpha_{i,2}$ with $\alpha_{i,1} + \alpha_{i,2} = 1$, we can get that

$$\begin{aligned}\Delta_{\lambda_i} V_i(\tilde{x}_k) \leqslant& E[\alpha_{i,1} V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k, w_k)) + \alpha_{i,2} V_i(\frac{1}{\alpha_{i,2}}\tilde{n}_i(\tilde{x}_k)e_k^t)|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k)\\ \leqslant& E[\alpha_{i,1} V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k, w_0)) + \alpha_{i,2}\mu_{i,2}\|\frac{1}{\alpha_{i,2}}\tilde{n}_i(\tilde{x}_k)\|^2\|e_k^t\|^2|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k). \qquad \text{(C1)}\end{aligned}$$

Based on Assumption 1, φ_i is a $\mathcal{K}$-function. Thus the inverse function φ_i^{-1} exists which is also a $\mathcal{K}$-function. Considering the event triggering condition (3), it follows that

$$\begin{aligned}&\varphi_i(\|e_k^t\|) - \chi_i(\|y_k\|) \leqslant 0\\ \Rightarrow& \|e_k^t\|^2 \leqslant (\varphi_i^{-1} \circ \chi_i(\|y_k\|))^2, k \in [k_t, k_{t+1}], \forall t \in \mathcal{N}. \qquad \text{(C2)}\end{aligned}$$

Substituting (C2) into (C1), we can get that

$$\begin{aligned}&\Delta_{\lambda_i} V_i(\tilde{x}_k)\\ \leqslant& E[\alpha_{i,1} V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k, w_0)) + \alpha_{i,2}\mu_{i,2}\|\frac{1}{\alpha_{i,2}}\tilde{n}_i(\tilde{x}_k)\|^2(\varphi_i^{-1} \circ \chi_i(\|m_i([I\ 0]\tilde{x}_k)\|))^2|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k)\\ =& E[\alpha_{i,1} V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}, w_0)) + \alpha_{i,2}\mu_{i,2}\|\frac{1}{\alpha_{i,2}}\tilde{n}_i(\tilde{x})\|^2(\varphi_i^{-1} \circ \chi_i(\|m_i([I\ 0]\tilde{x})\|))^2 - \lambda_i V_i(\tilde{x})]_{\tilde{x}=\tilde{x}_k}.\end{aligned}$$

From condition (8), we have

$$E[V_i(\tilde{x}_{k+1})|\mathcal{F}_k] - \lambda_i V_i(\tilde{x}_k) \leqslant 0. \qquad \text{(C3)}$$

Taking a mathematical expectation in (C3), it follows that

$$E[V_i(\tilde{x}_{k+1})] \leqslant \lambda_i E V_i(\tilde{x}_k). \qquad \text{(C4)}$$

For (C4), taking the summation on both sides of it from $k = \kappa_j$ to $k < \kappa_{j+1}$, there is

$$E V_i(\tilde{x}_k) \leqslant \lambda_i^{k-\kappa_j} E V_i(\tilde{x}_{\kappa_j}).$$

In view of $V_i(\tilde{x}) \leqslant \mu_3 V_j(\tilde{x})$, for any $i, j \in \mathcal{G}$, we obtain

$$V_{\sigma_{\kappa_{j+1}}}(\tilde{x}_{\kappa_{j+1}}) \leqslant \mu_3 V_i(\tilde{x}_{\kappa_{j+1}}).$$

Therefore, considering Definition 1, for any $k \geqslant k_0$,

$$\begin{aligned}E V_{\sigma_k}(\tilde{x}_k) \leqslant \mu_3^{N_0}\bar{\lambda}^{k-\kappa_0} E V_{\sigma_{\kappa_0}}(\tilde{x}_{\kappa_0}) \ \Rightarrow&\ \mu_{\sigma_k,1} E\|\tilde{x}_k\|^2 \leqslant \mu_3^{N_0}\bar{\lambda}^{k-\kappa_0}\mu_{\sigma_{\kappa_0},2} E\|\tilde{x}_{\kappa_0}\|^2\\ \Rightarrow&\ E\|\tilde{x}_k\|^2 \leqslant \mu_3^{N_0}\bar{\mu}\bar{\lambda}^{k-\kappa_0} E\|\tilde{x}_{\kappa_0}\|^2,\end{aligned}$$

in which

$$\bar{\mu} = \max_{\forall i,j\in\mathcal{G}}(\frac{\mu_{i,2}}{\mu_{j,1}}), \quad \bar{\lambda} = \max_{\forall i\in\mathcal{G}}(\mu_3^{\frac{1}{Ta}}\lambda_i).$$

By condition (11), $\mu_3 > 1$ and $\lambda_i < 1$, we can conclude that system (4) is ESMS by Definition 2. For the rest of the proof, we prove that $\|\mathcal{L}_{v,r}\| \leqslant \gamma$. For any nonnegative numbers $\alpha_{i,1}$, $\alpha_{i,2}$ and $\alpha_{i,3}$ with $\alpha_{i,1} + \alpha_{i,2} + \alpha_{i,3} = 1$, it follows that

$$\Delta_{v_k,\lambda} V_i(\tilde{x}_k)$$

$$\leqslant E\left[V_i\alpha_{i,1}(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k,w_0))+\alpha_{i,2}V_i(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x}_k,w_0)v_k)+\alpha_{i,3}V_i(\frac{1}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)e_k^t)|\mathcal{F}_k\right]-\lambda_iV_i(\tilde{x}_k).$$

Then, based on Young's inequality, it leads to

$$\begin{aligned}
&\Delta_{v_k,\lambda}V_i(\tilde{x}_k)\\
\leqslant&E\bigg[\alpha_{i,1}V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k,w_0))+\alpha_{i,2}V_i(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x}_k,w_0)v_k)+\alpha_{i,3}\mu_{i,2}\|\frac{1}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2\|e_k^t\|^2-\lambda_iV_i(\tilde{x}_k)\\
&+2\|\tilde{m}_i(\tilde{x}_k)\|^2+2\|\hat{l}_i(\hat{x}_k)e_k^t\|^2+2\|\tilde{z}_i(\tilde{x}_k)v_k\|^2-\gamma^2\|v_k\|^2-\|r_k\|^2+\gamma^2\|v_k\|^2\Big|\mathcal{F}_k\bigg]\\
\leqslant&E\bigg[\alpha_{i,1}V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k,w_0))+\alpha_{i,2}V_i(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x}_k,w_0)v_k)-\lambda_iV_i(\tilde{x}_k)\\
&+\alpha_{i,3}\mu_{i,2}\|\frac{1}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\|m_i([I\ 0]\tilde{x}_k)\|+\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2+2\|\tilde{m}_i(\tilde{x}_k)\|^2\\
&+2\|\hat{l}_i(\hat{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\|m_i([I\ 0]\tilde{x}_k)\|+\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2+2\|\tilde{z}_i(\tilde{x}_k)v_k\|^2-\gamma^2\|v_k\|^2\\
&-\|r_k\|^2+\gamma^2\|v_k\|^2\Big|\mathcal{F}_k\bigg].
\end{aligned}\tag{C5}$$

In view of Assumption 1, for $0<\alpha_{i,4}<\frac{\alpha_{i,3}}{\sqrt{2}(1-\alpha_{i,1})}<1$, we can get that

$$\begin{aligned}
&\alpha_{i,3}\mu_{i,2}\|\frac{1}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\|m_i([I\ 0]\tilde{x}_k)\|+\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2\\
\leqslant&\alpha_{i,3}\mu_{i,2}\|\frac{\sqrt{2}\alpha_{i,4}}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x}_k)\|)]^2\\
&+\alpha_{i,3}\mu_{i,2}\|\frac{\sqrt{2}(1-\alpha_{i,4})}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2.
\end{aligned}\tag{C6}$$

By Lemma 1, submitting (C6) into (C5) yields that

$$\begin{aligned}
&\Delta_{v_k,\lambda}V_i(\tilde{x}_k)\\
\leqslant&E\bigg[\alpha_{i,1}V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x}_k,w_0))+\alpha_{i,2}V_i(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x}_k,w_0)v_k)\\
&+\alpha_{i,3}\mu_{i,2}\|\frac{\sqrt{2}\alpha_{i,4}}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x}_k)\|)]^2\\
&+\alpha_{i,3}\mu_{i,2}\|\frac{\sqrt{2}(1-\alpha_{i,4})}{\alpha_{i,3}}\tilde{n}_i(\tilde{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2-\lambda_iV_i(\tilde{x}_k)+2\|\tilde{m}_i(\tilde{x}_k)\|^2\\
&+2\alpha_{i,4}\|\hat{l}_i(\hat{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x}_k)\|)]^2\\
&2(1-\alpha_{i,4})\|\hat{l}_i(\hat{x}_k)\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x}_k)v_k\|)]^2\\
&+2\|\tilde{z}_i(\tilde{x}_k)v_k\|^2-\gamma^2\|v_k\|^2-\|r_k\|^2+\gamma^2\|v_k\|^2\Big|\mathcal{F}_k\bigg]\\
\leqslant&E\bigg[\alpha_{i,1}V_i(\frac{1}{\alpha_{i,1}}\tilde{d}_i(\tilde{x},w_0))+\alpha_{i,2}V_i(\frac{1}{\alpha_{i,2}}\tilde{h}_i(\tilde{x},w_0)v)\\
&+(1-\alpha_{i,1})\mu_{i,2}\|\frac{1}{1-\alpha_{i,1}}\tilde{n}_i(\tilde{x})\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x})\|)]^2\\
&+\alpha_{i,3}\mu_{i,2}\|\frac{\sqrt{2}(1-\alpha_{i,4})}{\alpha_{i,3}}\tilde{n}_i(\tilde{x})\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x})v\|)]^2-\lambda_iV_i(\tilde{x})+2\|\tilde{m}_i(\tilde{x})\|^2\\
&+2\alpha_{i,4}\|\hat{l}_i(\hat{x})\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{\alpha_{i,4}}\|m_i([I\ 0]\tilde{x})\|)]^2+2(1-\alpha_{i,4})\|\hat{l}_i(\hat{x})\|^2[\varphi_i^{-1}\circ\chi_i(\frac{1}{1-\alpha_{i,4}}\|z_i([I\ 0]\tilde{x})v\|)]^2\\
&+2\|\hat{z}_i(\hat{x})v\|^2-\gamma^2\|v\|^2\bigg]_{\tilde{x}=\tilde{x}_k,v=v_k,}-\|r_k\|^2+\gamma^2\|v_k\|^2.
\end{aligned}$$

Because conditions (8) and (10) hold, it follows that

$$E[V_i(\tilde{x}_{k+1})|\mathcal{F}_k]-\lambda_iV_i(\tilde{x}_k)\leqslant-\|r_k\|^2+\gamma^2\|v_k\|^2.$$

So,

$$EV_i(\tilde{x}_{k+1})-\lambda_iEV_i(\tilde{x}_k)\leqslant-E\|r_k\|^2+\gamma^2E\|v_k\|^2.\tag{C7}$$

When $\tilde{x}_0=0$, setting $\lambda=\max_{i\in\mathcal{G}}\lambda_i$, for any $T\geqslant0$, we have

$$E\sum_{k=0}^{T}\lambda^{T-k}\mu_3^{N_0+\frac{T-k}{T_a}}\gamma^2\|v_k\|^2-E\sum_{k=0}^{T}\lambda^{T-k}\mu_3^{N_0+\frac{T-k}{T_a}}\|r_k\|^2\geqslant0$$

$$\Rightarrow E\sum_{k=0}^{T}\lambda^{T-k}\gamma^2\|v_k\|^2 - E\sum_{k=0}^{T}\lambda^{T-k}\mu_3^{-\frac{k}{T_a}}\|r_k\|^2 \geqslant 0. \tag{C8}$$

Considering the condition about switching dwell time given in (11), (C8) leads to

$$E\sum_{k=0}^{T}\lambda^{T-k}\gamma^2\|v_k\|^2 - E\sum_{k=0}^{T}\lambda^{T-k}\mu_3^{\frac{k\ln\lambda}{\ln\mu_3}}\|r_k\|^2 \geqslant 0.$$

Letting $T\to\infty$, we can get

$$E\sum_{k=0}^{\infty}\gamma^2\|v_k\|^2 - E\sum_{k=0}^{\infty}\lambda^{k}\|r_k\|^2 \geqslant 0.$$

The second part of the proof ends. The remaining task is to prove the weighted H_- system performance $\|\mathcal{L}_{f,r}\|_-^2 \geqslant \delta^2$. Considering $-\Delta_{f,\lambda_i}V_i(\tilde{x})$, similar to the proof process of the second part, we can obtain $E\sum_{k=0}^{\infty}\delta^2\|f_k\|^2 - E\sum_{k=0}^{\infty}\lambda^k\|r_k\|^2 \leqslant 0$ from (8) and (10).

Appendix D The Linear Case

Specially, consider the linear case with following form

$$\begin{cases} x_{k+1} = A_{\sigma_k}x_k + B_{\sigma_k}v_k + F_{\sigma_k}f_k + (C_{\sigma_k}x_k + D_{\sigma_k}v_k + G_{\sigma_k}f_k)w_k, \\ y_k = M_{\sigma_k}x_k + Q_{\sigma_k}v_k + Z_{\sigma_k}f_k, \end{cases} \tag{D1}$$

where A_i, B_i, F_i, C_i, D_i, G_i, M_i, Q_i and Z_i are given system parameter matrices, $i\in\mathcal{G}$. $\{w_k\in\mathcal{R}\}_{k\in\mathcal{N}}$ stands for the independent white noise processes with $E[w_k]=0$, $E[w_k^2]=0$, $E[w_iw_j]=1$ for $i\neq j$. In order to monitor the operation status of system (D1), we introduce the following fault detection filter

$$\begin{cases} \hat{x}_{k+1} = \hat{A}_{\sigma_k}\hat{x}_k + \hat{L}_{\sigma_k}(y_{k_t} - M_{\sigma_k}\hat{x}_k), \\ r_k = \hat{M}_{\sigma_k}(y_{k_t} - M_{\sigma_k}\hat{x}_k), \end{cases} \tag{D2}$$

where $\hat{A}_i$, $\hat{L}_i$ and $\hat{M}_i$ are the observer gain matrices to be determined. The triggered rule is in the form of

$$\begin{cases} k_t = 0, \quad t=0, \\ k_t = \min_k\{k > k_{t-1}, \|y_k - y_{k_{t-1}}\| - \theta_{\sigma_k}\|y_k\| > 0\}, t>1, t\in\mathcal{N}, \end{cases} \tag{D3}$$

where θ_i is the event-triggering parameter needed to be designed. Therefore, the augmented system for $\tilde{x} = \begin{bmatrix} x \\ x-\hat{x} \end{bmatrix}$ can be written as

$$\begin{cases} \tilde{x}_{k+1} = \tilde{A}_{\sigma_k}\tilde{x}_k + \tilde{B}_{\sigma_k}v_k + \tilde{F}_{\sigma_k}f_k + \tilde{L}_{\sigma_k}e_k^t + (\tilde{C}_{\sigma_k}\tilde{x}_k + \tilde{D}_{\sigma_k}v_k + \tilde{G}_{\sigma_k}f_k)w_k, \\ r_k = \tilde{M}_{\sigma_k}\tilde{x}_k + \tilde{N}_{\sigma_k}v_k + \tilde{H}_{\sigma_k}f_k - \hat{M}_{\sigma_k}e_k^t, \end{cases} \tag{D4}$$

where

$$\tilde{A}_{\sigma_k} = \begin{bmatrix} A_{\sigma_k} & 0 \\ A_{\sigma_k} - \hat{A}_{\sigma_k} & \hat{A}_{\sigma_k} - \hat{L}_{\sigma_k}M_{\sigma_k} \end{bmatrix},\ \tilde{B}_{\sigma_k} = \begin{bmatrix} B_{\sigma_k} \\ B_{\sigma_k} - \hat{L}_{\sigma_k}N_{\sigma_k} \end{bmatrix},\ \tilde{F}_{\sigma_k} = \begin{bmatrix} F_{\sigma_k} \\ F_{\sigma_k} - \hat{L}_{\sigma_k}H_{\sigma_k} \end{bmatrix},$$
$$\tilde{C}_{\sigma_k} = \begin{bmatrix} C_{\sigma_k} & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{D}_{\sigma_k} = \begin{bmatrix} D_{\sigma_k} \\ 0 \end{bmatrix},\ \tilde{G}_{\sigma_k} = \begin{bmatrix} G_{\sigma_k} \\ 0 \end{bmatrix},\ \tilde{L}_{\sigma_k} = \begin{bmatrix} 0 \\ \hat{L}_{\sigma_k} \end{bmatrix},$$
$$\tilde{M}_{\sigma_k} = \begin{bmatrix} \hat{M}_{\sigma_k}M_{\sigma_k} & -\hat{M}_{\sigma_k}M_{\sigma_k} \end{bmatrix},\ \tilde{N}_{\sigma_k} = \hat{M}_{\sigma_k}N_{\sigma_k},\ \tilde{H}_{\sigma_k} = \hat{M}_{\sigma_k}H_{\sigma_k}.$$

According to Theorem 1, we can directly obtain the following result about system (D4).

Corollary 1. For the given $\lambda>0$ and $\delta>0$ and any $i,j\in\mathcal{G}$, if there exist positive definite matrices P_i with $\mu_{i,1}I_{n_{\tilde{x}}} \leqslant P_i \leqslant \mu_{i,2}I_{n_{\tilde{x}}}$, $P_i\leqslant\mu_3P_j$, matrices $\hat{A}_i$, $\hat{L}_i$ and $\hat{M}_i$, as well as real numbers $\alpha_{i,1}$, $\alpha_{i,2}$, $\alpha_{i,3}$, $\alpha_{i,4}$ and θ_i, such that

$$\begin{aligned}
&\frac{1}{\alpha_{i,1}}(\tilde{A}_i'P_i\tilde{A}_i + \tilde{C}_i'P_i\tilde{C}_i) - \lambda_iP_i + 2\tilde{M}_i'\tilde{M}_i + \left(\frac{\mu_{i,2}}{1-\alpha_{i,1}}\|\tilde{L}_i\|^2 + 2\|\hat{M}_i\|^2\right)\frac{\theta_i}{\alpha_{i,4}^2}[I\ 0]'M_i'M_i[I\ 0] < 0, \\
&\frac{1}{\alpha_{i,2}}(\tilde{B}_i'P_i\tilde{B}_i + \tilde{D}_i'P_i\tilde{D}_i) + 2\tilde{N}_i'\tilde{N}_i + \left[2(1-\alpha_{i,4})\|\hat{M}_i\|^2 + \frac{2\mu_{i,2}(1-\alpha_{i,4})^2}{\alpha_{i,3}}\|\tilde{L}_i\|^2\right]\frac{\theta_i}{(1-\alpha_{i,4})^2}Q_i'Q_i < \gamma^2I, \\
&-\frac{1}{\alpha_{i,2}}(\tilde{F}_i'P_i\tilde{F}_i + \tilde{G}_i'P_i\tilde{G}_i) + \tilde{H}_i'\tilde{H}_i - \left[2(1-\alpha_{i,4})\|\hat{M}_i\|^2 + \frac{2\mu_{i,2}(1-\alpha_{i,4})^2}{\alpha_{i,3}}\|\tilde{L}_i\|^2\right]\frac{\theta_i}{(1-\alpha_{i,4})^2}Z_i'Z_i > \delta^2I,
\end{aligned}$$

where $\alpha_{i,1}+\alpha_{i,2}+\alpha_{i,3}=1$ and $\alpha_{i,4} < \frac{\alpha_{i,3}}{\sqrt{2}(1-\alpha_{i,1})}$, $T_a > ceil(\max_{\forall i\in\mathcal{G}}(-\frac{\ln u_3}{\ln\lambda_i}))$. Then a mixed H_-/H_∞ fault detection filter (D2) for system (D1) and event-triggered communication scheme (D3) are designed.

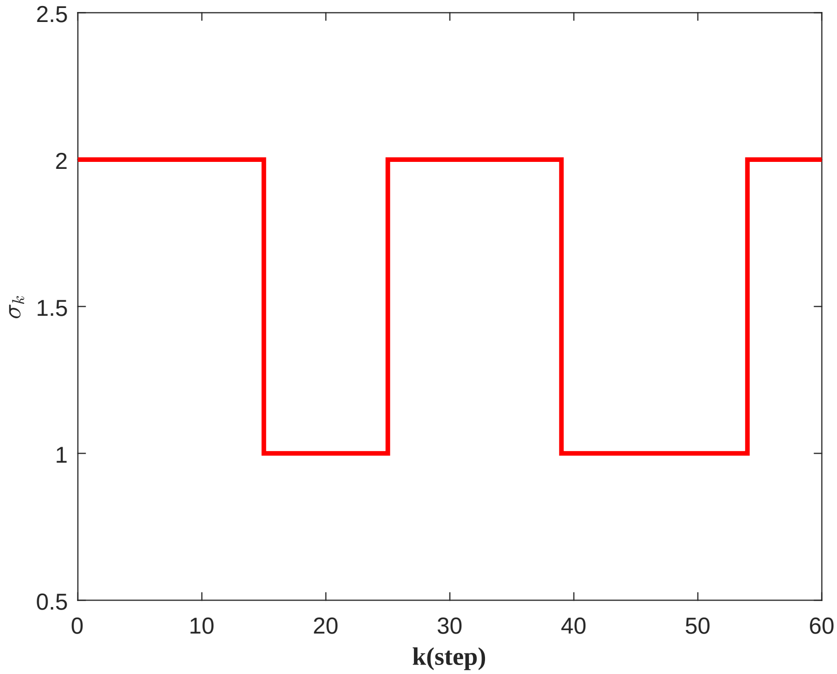

Figure E1 The switching signal σ_k.

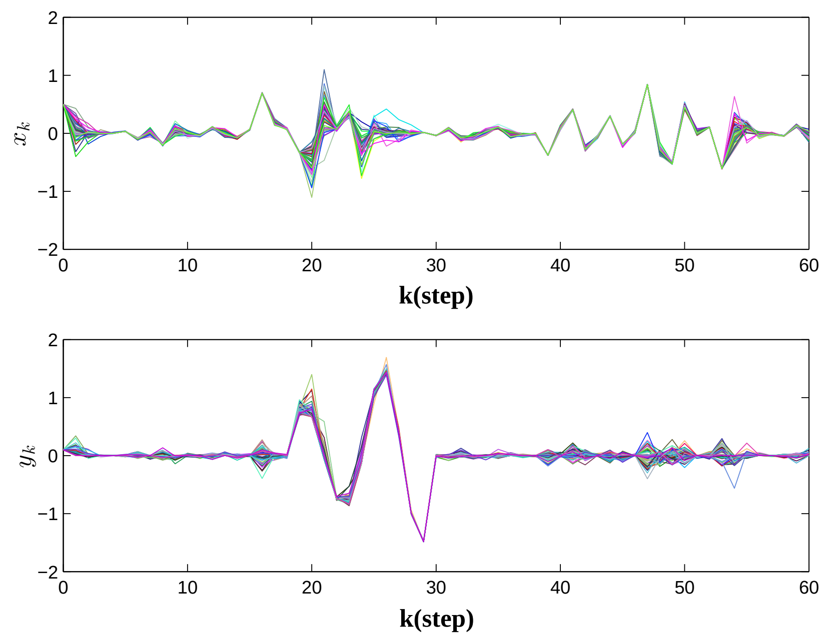

Figure E2 The state x_k and output y_k of system (E1).

Appendix E Numerical Example

Consider 1-D nonlinear system (1) as the following

$$\begin{cases} x_{k+1} = a_{\sigma_k} w_k x_k + b_{\sigma_k} v_k + c_{\sigma_k} sin(x_k) w_k f_k, \\ y_k = d_{\sigma_k} x_k + q_{\sigma_k} f_k, \end{cases} \tag{E1}$$

where $i \in \{1, 2\}$ and $x_0 = 2$, $\{w_k\}_{k \in \mathcal{N}}$ is as in (D1). Meanwhile, the function about the event-triggered condition is chosen as $\phi_i(x, y) = \varphi_i x^2 - \chi_i y^2$, where φ_i and χ_i are positive real number. For detecting the fault f_k in (E1), we construct the following

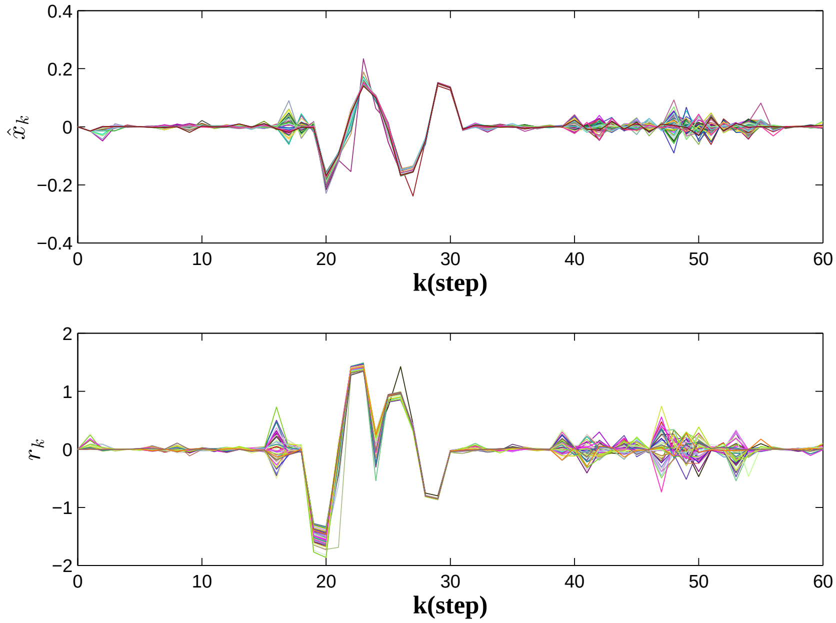

Figure E3 The state $\hat{x}_k$ and output r_k of system (E1).

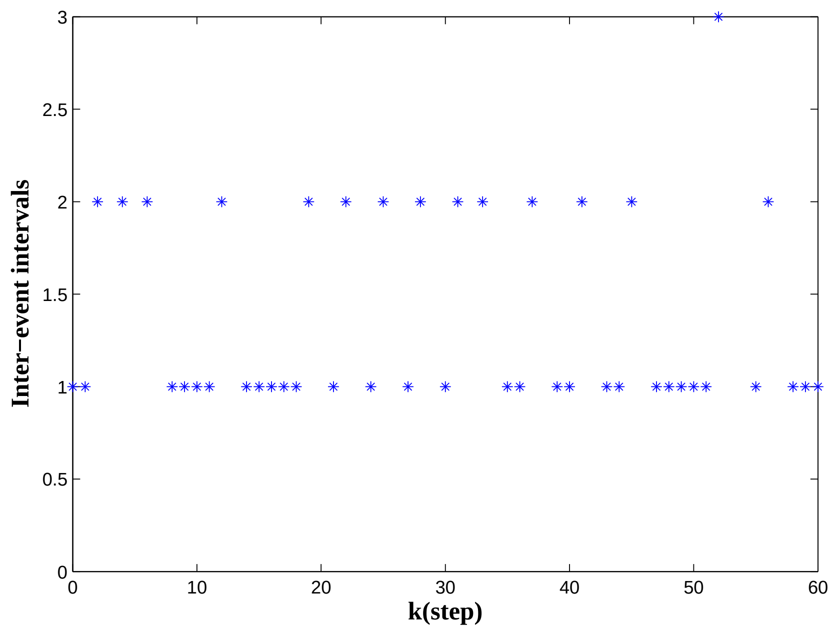

Figure E4 Triggering time and interval in a test.

fault detection filter

$$\begin{cases} \hat{x}_{k+1} = \hat{d}_{\sigma_k}\hat{x}_k + \hat{n}_{\sigma_k} y_{k_t}, \\ r_k = \hat{l}_{\sigma_k}(y_{k_t} - d_{\sigma_k}\hat{x}_k). \end{cases} \tag{E2}$$

Take

$$\mathbb{V}_i = \{V_i : V_i(\tilde{x}) = (P_i + R_i)x^2 + R_i\hat{x}^2 - 2R_i x\hat{x}, P_i, R_i \in \mathcal{R},\ P_i, R_i > 0\}$$

and

$$\mu_{i,2} = max\{P_i, R_i\}.$$

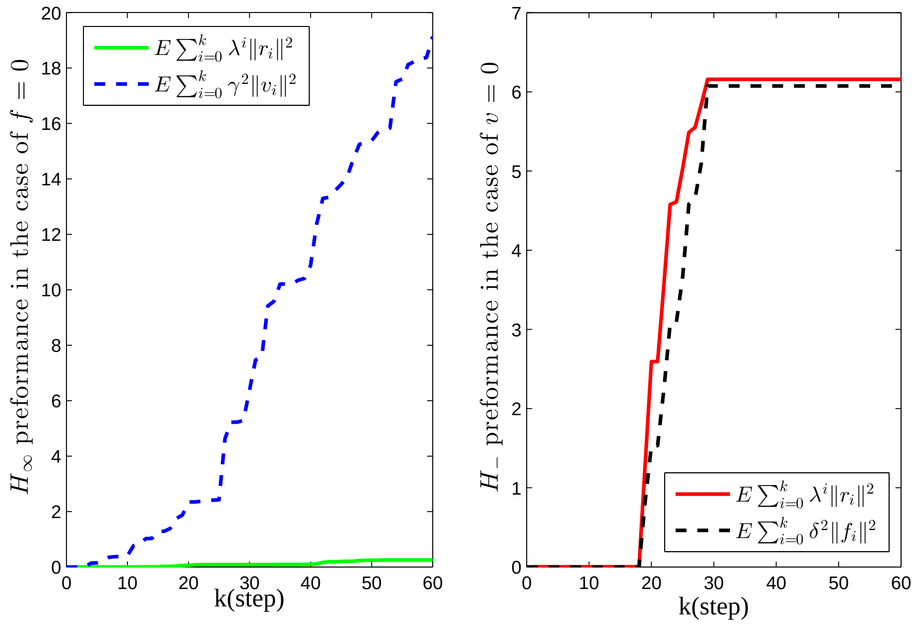

Figure E5 H_- performance and H_∞ performance for system (E1) and fault detection filter (E4).

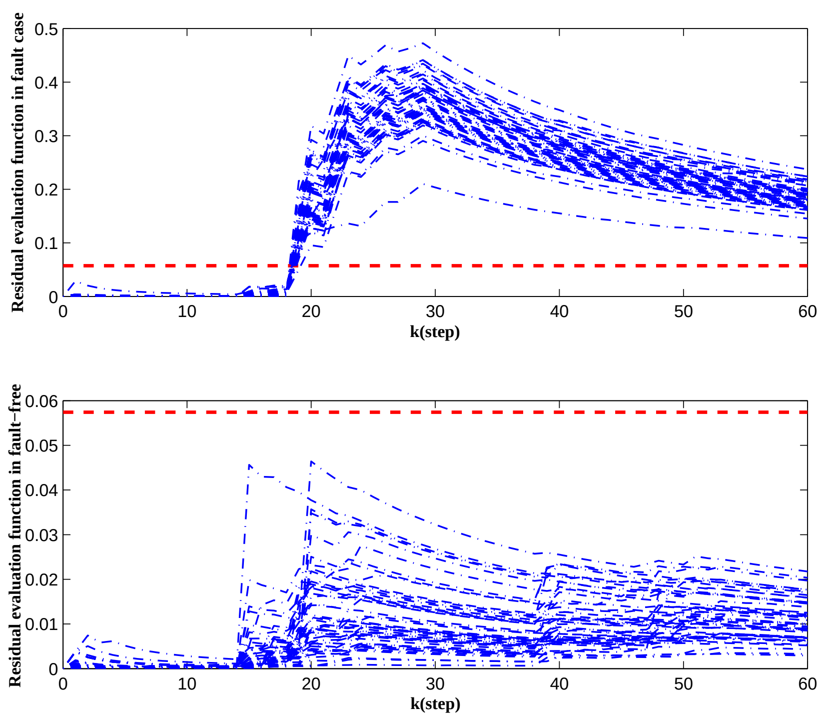

Figure E6 Residual evaluation and threshold with $J_{th}=0.0575$

Then through Theorem 1, the parameters in (E2) need to satisfy the following inequalities

$$\begin{cases}\begin{bmatrix}\frac{1}{\alpha_{i,1}}P_i a_i^2-\lambda_i P_i+\frac{\chi_i}{(1-\alpha_{i,1})\alpha_{i,4}^2\varphi_i^2}\mu_{i,2}\hat{n}_i^2 d_i^2+2\hat{l}_i^2 d_i^2+2\frac{\chi_i}{\alpha_{i,4}\varphi_i^2}\hat{l}_i^2 d_i^2 & R_i\hat{n}_i^2 d_i\hat{d}_i-4\hat{l}_i^2 d_i^2\\ * & \frac{1}{\alpha_{i,1}}R_i\hat{d}_i-\lambda_i R_i+2\hat{l}_i^2 d_i^2\end{bmatrix}\leqslant 0,\\ \frac{1}{\alpha_{i,2}}P_i b_i^2+\frac{1}{\alpha_{i,2}}R_i\leqslant\gamma^2,\\ -[\frac{1}{\alpha_{i,2}}P_i c_i^2+R_i(q_i\hat{n}_i-c_i)^2]-\mu_{i,2}\frac{2\chi_i}{\varphi_i^2\alpha_{i,3}}\hat{n}_i^2 q_i^2-2\hat{l}^2\frac{\chi_i}{\varphi_i^2(1-\alpha_{i,4})}q_i^2+2\hat{l}^2 q_i^2\geqslant\delta^2,\end{cases}\tag{E3}$$

and switching dwell time T_a meets the condition (11). The model parameters are assumed to be $a_1 = -0.0541$, $a_2 = 0.3277$, $b_1 = -1.7896$, $b_2 = 0.2106$, $c_1 = -0.6645$, $c_2 = 0.4750$, $d_1 = 0.2030$, $d_2 = 0.3039$, $q_1 = 0.8588$, $q_2 = 1.4963$. If we set $\gamma = 2$ and $\delta = 1$, then by solving (E3) and (11), there are $\alpha_{1,1} = 0.0239$, $\alpha_{1,2} = 0.5067$, $\alpha_{1,3} = 0.4694$, $\alpha_{1,4} = 0.2701$, $\alpha_{2,1} = 0.2647$, $\alpha_{2,2} = 0.2828$, $\alpha_{2,3} = 0.4525$, $\alpha_{2,4} = 0.1802$, $P_1 = 0.9261$, $P_2 = 0.5503$, $R_1 = 0.7451$, $R_2 = 0.5363$, $\lambda_1 = 0.9726$, $\lambda_2 = 0.9186$, $\mu_3 = 1.2429$. The average dwell time $T_a = 8$. Meanwhile, the fault detection filter can be designed as

$$\begin{cases} \hat{x}_{k+1} = -0.4408\hat{x}_k - 0.2302y_{k_t}, \\ r_k = -1.8484(y_{k_t} - 0.2030\hat{x}_k), \end{cases} \quad \text{when } \sigma_k = 1,$$
$$\begin{cases} \hat{x}_{k+1} = -0.0752\hat{x}_k - 0.1469y_{k_t}, \\ r_k = 0.8217(y_{k_t} - 0.3039\hat{x}_k), \end{cases} \quad \text{when } \sigma_k = 2. \tag{E4}$$

and the event-triggering conditions are obtained by

$$\phi_1(\|y_k - y_{k_{t-1}}\|, \|y_k\|) = 0.5197\|y_k - y_{k_{t-1}}\|^2 - 0.2245\|y_k\|^2$$

and

$$\phi_2(\|y_k - y_{k_{t-1}}\|, \|y_k\|) = 0.8217\|y_k - y_{k_{t-1}}\|^2 - 0.1373\|y_k\|^2.$$

Besides, in order to exhibit the effectiveness of the obtained design strategy, choose the external disturbance signal $v_k = 0.9^k$, the fault signal

$$f_k = \begin{cases} 1, & k \in \{10, 11, \cdots, 20\}, \\ 0, & \text{otherwise}. \end{cases}$$

Figure E1 displays the switching signal σ_k. The trajectories of systems (E1) and (E2) in fifty simulation results are shown in Figures E2 and E3, respectively. It should be pointed out that the results of each experiment are drawn by the curve of random color. The amount of measured data transmitted is decreasing efficiently. In Figure E4, we draw the event-triggered transmission intervals in one experiment. It is shown that the energy and network bandwidth has been saved significantly. Based on (6)-(7), we can conclude that the designed detection threshold $J_{th} = 0.0575$ with evaluation window $T = 10$. In the fault case, the fault can be detected (see Figure E6). When fault $f_k = 0$ and only external disturbance v occurs, the decision logic will not alarm. Besides, H_∞ performance and H_- performance are shown in Figure E5. In general, the desired fault detection objective can be achieved by using our obtained design strategy.

References

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