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Global adaptive stabilization for planar nonlinear systems with unknown input powers

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Dear editor,

Despite the development of extensive strategies [1, 2], adaptive control still requires thorough investigations to improve its capability and cover more uncertain systems. This study aims to demonstrate the powerful ability of adaptive control in compensating multiple serious uncertainties, particularly those in input powers, by investigating global stabilization for the following planar nonlinear system with unknown input powers:

$$\begin{cases} \dot{x}_1 = g_1(x)[x_2]^{p_1(t)} + f_1(x), \\ \dot{x}_2 = g_2(x)[u]^{p_2(t)} + f_2(x), \end{cases} \tag{1}$$

where $[\cdot]^a = \mathrm{sign}(\cdot)|\cdot|^a$ for $a > 0$; $x = [x_1, x_2]^{\mathrm{T}} \in \mathbb{R}^2$ is the system state with the initial condition $x(t_0) = x_0$; $u \in \mathbb{R}$ is the control input; $p_1(t)$ and $p_2(t)$ are unknown smooth functions, referred to as the input powers of the system; $g_i : \mathbb{R}^2 \to \mathbb{R}$, $i = 1, 2$ and $f_i : \mathbb{R}^2 \to \mathbb{R}$, $i = 1, 2$ are unknown continuous functions, referred to as the control coefficients and nonlinearities of the system, respectively.

The following assumptions are made for the input powers, control coefficients, and nonlinearities of system (1); these show that the system undergoes multiple serious uncertainties, particularly in input powers $p_i(t)$.

Assumption 1. Unknown input powers $p_1(t)$ and $p_2(t)$ satisfy

$$1 \leqslant p_i(t) \leqslant \bar{p}, \quad i = 1, 2,$$

for $\forall t \geqslant t_0$, where $\bar{p}$ is an unknown positive constant.

Assumption 2. The signs of $g_1(x)$ and $g_2(x)$ are known, and the following holds for $\forall x \in \mathbb{R}^2$:

$$\begin{cases} a_1\underline{g}_1(x_1) \leqslant |g_1(x)| \leqslant b_1\bar{g}_1(x), \\ a_2\underline{g}_2(x) \leqslant |g_2(x)| \leqslant b_2\bar{g}_2(x), \end{cases}$$

where a_i and b_i are unknown positive constants, and $\underline{g}_i(\cdot)$ and $\bar{g}_i(\cdot)$ are known positive smooth functions.

Assumption 3. There exists an unknown positive constant θ such that for $\forall x \in \mathbb{R}^2$,

$$\begin{cases} |f_1(x)| \leqslant \theta h_1(x_1)|x_1|^{\bar{p}}, \\ |f_2(x)| \leqslant \theta h_2(x)\big(|x_1|^{\bar{p}} + |x_2|^{\bar{p}}\big), \end{cases}$$

where $h_i(\cdot)$ are known positive smooth functions.

Assumption 1 shows that the unknown input powers have an unknown constant upper bound; this is essentially different from other closely related studies [3–5], where the input powers are unknown but can be dominated by known constants. Assumptions 2 and 3 show that serious uncertainties also exist in the control coefficients and nonlinearities of system (1); however, related studies [3–5] exclude uncertainties in control coefficients, although the systems considered in these studies are n-dimensional ones.

Remarkably, the unknown upper bound of the input power renders the compensation strategy adopted in [5] (i.e., introducing terms of lower and higher powers with respect to the input powers) inapplicable. In this study, a powerful adaptive controller is designed, which has the ability to compensate the serious uncertainties in the system, particularly those in the input power. Notably, the key to designing the controller lies in an appropriate design of the adaptive dynamic and the choice of design functions of dynamic and system states, that can ensure the existence of solutions and global stability of the closed-loop systems. It should also be pointed out that, although similar problems can be solved by the switching adaptive strategy [6, 7], the designed controller is discontinuous and is more likely to cause chattering behavior in practical implementation.

Adaptive controller design. Before proceeding to the control design, we assume, without loss of generality, that the signs of $g_1(x)$ and $g_2(x)$ are positive, and $h_1(x_1) \geqslant 1$ and $h_2(x) \geqslant 1$. More importantly, we introduce design functions $\lambda_i(y, z) \geqslant 1$, $i = 1, 2$, $\forall y \in \mathbb{R}$, $\forall z \geqslant 1$, satisfying

$$\begin{cases} \lim\limits_{z\to+\infty} \inf\limits_{y\in\mathbb{R}} \lambda_1(y, z) = +\infty, \\ \lim\limits_{|y|\to+\infty} \inf\limits_{z\geqslant1} \lambda_1(y, z) = +\infty, \end{cases} \tag{2}$$

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and

$$\begin{cases} \lim\limits_{z\to+\infty}\inf\limits_{y\in\mathbb{R}}\lambda_2(y,z)=+\infty, \\ \lim\limits_{z\to+\infty}\inf\limits_{y\in\mathbb{R}\setminus\{0\}}\dfrac{\lambda_2(y,z)}{|y|^z}=+\infty, \\ \lim\limits_{|y|\to+\infty}\inf\limits_{z\geqslant 1}\dfrac{\lambda_2(y,z)}{|y|^c}=+\infty,\ \forall c\geqslant 1. \end{cases} \quad (3)$$

Remarkably, $\lambda_1(\cdot)$ and $\lambda_2(\cdot)$ can be constructed explicitly, and an example is provided in Appendix A.

The following adaptive controller is designed:

$$\begin{cases} u=-\beta_2(x,K)[\xi_2]^{\overline{\frac{1}{\lambda_1(x_1^2+\xi_2^2,K)}}}, \\ \xi_2=x_2-\alpha_1(x_1,K)=:x_2+\beta_1(x_1,K)x_1, \end{cases} \quad (4)$$

where K is generated by the following dynamic:

$$\dot{K}=|x_1|^K+|\xi_2|^K,\quad K(t_0)=1, \quad (5)$$

and

$$\begin{cases} \beta_1(\cdot)=\left(\underline{g}_1^{-1}(x_1)+\underline{g}_1^{-\frac{1}{\lambda_1(x_1,K)}}(x_1)\right) \\ \qquad\cdot\lambda_2(x_1,K)h_1(x_1), \\ \beta_2(\cdot)=\left(\underline{g}_2^{-1}(x)+\underline{g}_2^{-\frac{1}{\lambda_1(x_1^2+\xi_2^2,K)}}(x)\right) \\ \qquad\cdot\lambda_2\big(x_1^2+\xi_2^2,K\big)H\big(x,K,\lambda_1\big(x_1^2+\xi_2^2,K\big)\big), \end{cases} \quad (6)$$

with $H(\cdot)\geqslant 1$ being a known smooth function to be determined later and strictly increasing on its third argument. Remarkably, controller (4) designed in this study is essentially different from those in the related studies. On one hand, from (5), we see that the dynamic gain $K(t)$ appears in the powers of the system state; this provides the controller with the powerful ability to compensate unknown input powers. On the other hand, the design functions on system states and dynamic gain are introduced, which are quite critical to ensure the existence of solutions and global stability of the closed-loop systems, as can be seen in the later stability analysis.

Main results. Let $V(x,K)=\frac{1}{2}x_1^2+\frac{1}{2}\xi_2^2$. Then, based on controller (4), we obtain the following proposition, which gives the appropriate estimation of the time derivative of $V(\cdot)$ and whose proof is given in Appendix B.

Proposition 1. There exist $H(\cdot)$ as in (6) and an unknown positive constant Θ, such that under controller (4) and along the trajectories of system (1), the following holds:

$$\begin{aligned} \dot{V}\leqslant& -a_1\left(\underline{g}_1^{1-p_1(t)}(x_1)+\underline{g}_1^{1-\frac{p_1(t)}{\lambda_1(x_1,K)}}(x_1)\right)\lambda_2(x_1,K) \\ &\cdot h_1(x_1)|x_1|^{1+p_1(t)}+\big(\theta h_1(x_1)+2\big)|x_1|^{1+\bar{p}} \\ &+2|x_1|^{1+p_1(t)}+|x_1|^{1+K}-a_2\Bigg(\underline{g}_2^{1-p_2(t)}(x) \\ &+\underline{g}_2^{1-\frac{p_2(t)}{\lambda_1(x_1^2+\xi_2^2,K)}}(x)\Bigg)\lambda_2\big(x_1^2+\xi_2^2,K\big) \\ &\cdot H\big(x,K,\lambda_1\big(x_1^2+\xi_2^2,K\big)\big)|\xi_2|^{1+\frac{p_2(t)}{\lambda_1(x_1^2+\xi_2^2,K)}} \\ &+\Theta H(x,K,\bar{p})\big(|\xi_2|^{1+p_1(t)}+|\xi_2|^{1+K}\big). \end{aligned} \quad (7)$$

The main results of this study are as follows.

Theorem 1. Consider system (1) under Assumptions 1–3. The designed adaptive controller (4) guarantees that all the signals of the closed-loop system are bounded on $[t_0,+\infty)$, and the original system state $x(t)$ and the control input $u(t)$ ultimately converge to zero.

Proof. From system (1) and controller (4), we see that the vector field of the closed-loop system is continuous in (t,x,K). Then, from Theorem 1.1 (Page 14) and Theorem 2.1 (Page 17) in [8], the closed-loop system has at least one solution (not necessarily unique) starting from the initial condition $(t_0,x_0,K(t_0))$. Let $[t_0,T_f)$ be the maximal existence interval on which all solutions exist. The following proof is divided into three parts.

Part I. $K(t)$ is bounded on $[t_0,T_f)$. Suppose by contradiction that $\lim_{t\to T_f}K(t)=+\infty$. Then, from (2)–(7), there always exists t_1 such that for $\forall t\in[t_1,T_f)$,

$$\dot{V}(t)\leqslant-|x_1(t)|^{K(t)}-|\xi_2(t)|^{K(t)}, \quad (8)$$

where $V(t)$ simply denotes $V(x(t),K(t))$. The derivation process of (8) is provided in Appendix C.

Note that (8) directly implies that for $\forall t\in[t_1,T_f)$,

$$\begin{aligned} &\int_{t_1}^{t}\big(|x_1(s)|^{K(s)}+|\xi_2(s)|^{K(s)}\big)\mathrm{d}s \\ &\leqslant V(t_1)-V(t)<+\infty. \end{aligned} \quad (9)$$

On the other hand, by (5), we have for $\forall t\in[t_1,T_f)$,

$$K(t)-K(t_1)=\int_{t_1}^{t}\big(|x_1(s)|^{K(s)}+|\xi_2(s)|^{K(s)}\big)\mathrm{d}s,$$

which together with $\lim_{t\to T_f}K(t)=+\infty$ directly contradicts (9). Thus, we prove that $K(t)$ is bounded on the maximal existence interval $[t_0,T_f)$.

Part II. $T_f=+\infty$, and the system state $x(t)$ is bounded on $[t_0,+\infty)$. Actually, it suffices to prove that the system state $x(t)$ is bounded on $[t_0,T_f)$ with $T_f\leqslant+\infty$. Suppose by contradiction that $\lim_{t\to T_f}\|x(t)\|=+\infty$ with $T_f\leqslant+\infty$. Then, from (2)–(7) and the boundedness of $K(t)$ on $[t_0,T_f)$, there exists t_2 such that for $\forall t\in[t_2,T_f)$,

$$\dot{V}(t)\leqslant 0. \quad (10)$$

The detailed derivation process of (10) is provided in Appendix D.

From (10), it follows that for $\forall t\in[t_2,T_f)$,

$$0\leqslant V(t)\leqslant V(t_2)<+\infty,$$

which together with (4) and the boundedness of $K(t)$ on $[t_0,T_f)$, directly contradicts the fact that $\lim_{t\to T_f}\|x(t)\|=+\infty$. Thus, we prove that $x(t)$ is bounded on $[t_0,T_f)$ with $T_f\leqslant+\infty$.

Part III. The system signals $(x(t),u(t))$ ultimately converge to zero.

From (5) and the boundedness of $K(t)$ on $[t_0,+\infty)$, it follows that, for $\forall t\in[t_0,+\infty)$,

$$\int_{t_0}^{t}\big(|x_1(s)|^{K(s)}+|\xi_2(s)|^{K(s)}\big)\mathrm{d}s<+\infty.$$

Moreover, by the boundedness of $x(t)$ on $[t_0,+\infty)$, we obtain that $\xi_2(t)$ and $u(t)$ are also bounded on $[t_0,+\infty)$. This, together with (1), (4), and the boundedness of $K(t)$, implies that $\dot{x}_1(t)$ and $\dot{\xi}_2(t)$ are bounded on $[t_0,+\infty)$. Thus, by using Barbălat lemma, we obtain that $\lim_{t\to+\infty}x_1(t)=0$ and $\lim_{t\to+\infty}\xi_2(t)=0$, while $\lim_{t\to+\infty}x_2(t)=0$ and $\lim_{t\to+\infty}u(t)=0$ directly follow from (4).

Concluding remarks. In this study, a novel adaptive controller has been designed to achieve global adaptive stabilization for a class of planar nonlinear systems with serious uncertainties in input powers. Remarkably, the proposed adaptive control strategy is rather suggestive, and may be used to solve other control problems. In addition, the considered system has some certain practical significance. When the nonlinearities and input powers of system (1) are suitably degenerated, system (1) can be used to describe single-link robot arm systems. Moreover, future work can focus on how to achieve global stabilization for more general n-dimensional nonlinear systems with unknown input powers. Such systems can be used to describe underactuated, weakly coupled, unstable mechanical systems.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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