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Output feedback stabilization for power-integrator systems with unknown measurement sensitivity

Yu SHAO[1], Zong-Yao SUN[1*], Xue-Jun XIE[1] & Zhen-Guo LIU[2]

[1]*Institute of Automation, Qufu Normal University, Qufu* 273165*, China;*
[2]*Department of Automation, Shanxi University, Taiyuan* 030006*, China*

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Dear editor,

The technical limitations on sensors mean that some state variables will not be detected in any controlled system, so the actual values of an output function will always deviate somewhat from the nominal values. A practical example was given in [1], where the output of an infrared sensor that measures the distance between parts in a magnetic bearing suspension system suffers from measurement error [1]. Such systems require a novel approach for finding the effects of measurement error in global output feedback stabilization [2–5]. However, as pointed out by [1], unknown nonlinearities hinder the design and analysis of output-feedback control strategy, and as a result there are many unsolved problems that are worth studying from theoretical as well as practical perspectives.

On the other hand, control schemes that stabilize nonlinear systems with uncontrollable linearization around the origin have yielded interesting results by adding a power integrator method, like the one mentioned in [6].

This study considers the problem of global output feedback stabilization of a nonlinear system, which is described as

$$\begin{cases} \dot{x}_1(t) = x_2^p(t), \\ \dot{x}_2(t) = u^p(t), \\ y = \theta(t)x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2]^{\mathrm{T}} \in \mathbb{R}^2$, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the measured output, the continuous function $\theta(t)$ is unknown and bounded, and the initial condition is $x(0) = x_0$. $p \in \mathbb{R}^{<1}_{\rm odd} \triangleq \{\frac{m}{n} | m$ and n are positive odd integers, and $m < n\}$ is the order.

The following assumption is needed.

Assumption 1. For the continuous function $\theta(t)$, there exists a known positive parameter $\bar{\theta}$ that satisfies $|1-\theta(t)| \leqslant \bar{\theta} < \theta^* < 1$, where θ^* is the allowable sensitivity error of $\bar{\theta}$.

The function $\theta(t)$ represents the unknown measurement sensitivity, which describes the possible deviation between the state x_1 and the output y. An ideal case is $\theta(t) = 1$; that is $y = x_1$. However, in industrial applications, limitations on measurement techniques and tools mean that actual values of the output function could deviate from their nominal values to some extent inevitably. As a result, Assumption 1 is proposed to demonstrate such phenomenon. The condition $|1-\theta(t)| \leqslant \bar{\theta} < 1$ serves as the observable criteria of system (1). Information from $\theta(t)$ must be used in the design of the output feedback controller, so that a feasible range of $\theta(t)$ has to be determined in priority.

Lemmas 1–3 play a crucial role in the design and theoretical analysis of the controller.

Lemma 1 ([7]). For $r \in \mathbb{R}^{<1}_{\rm odd}$, the following inequality holds for any $0 < l < 1$ and t:

$$t^r + (1-t)^r + l^2 t^{1+r} \geqslant (2^r - 1) l^{1-r}.$$

Lemma 2 ([7]). Let c, d be positive real numbers and let $\gamma(x, y) > 0$ be a real-valued function. Then,

$$|x|^c |y|^d \leqslant \frac{c}{c+d} \gamma(x,y) |x|^{c+d} + \frac{d}{c+d} \gamma^{\frac{-c}{d}}(x,y) |y|^{c+d}.$$

Lemma 3 ([8]). For $p \in \mathbb{R}^{<1}_{\rm odd}$, the following inequalities hold for any $x, y \in \mathbb{R}$:

$$|x^p - y^p| \leqslant 2^{1-p} |x-y|^p, \quad |x+y|^p \leqslant |x^p| + |y^p|.$$

Main results. The main results of this study are summarized as follows.

Theorem 1. For system (1) with Assumption 1, if $p \in [p^*, 1)$ is odd, where p^* is a known positive constant, then there exists an output-feedback controller, and it guarantees that the closed-loop system is globally asymptotically stable.

Proof. The proof is composed of three parts.

* Corresponding author (email: sunzongyao@sohu.com)

Part I. Construction of the observer. Let

$$z_2 = x_2 - Lx_1, \quad e_2 = z_2 - \hat{z}_2; \tag{2}$$

then

$$\begin{cases} \dot{z}_2 = \dot{x}_2 - L\dot{x}_1 = u^p - Lx_2^p, \\ \dot{\hat{z}}_2 = u^p - L\hat{z}_2^p, \end{cases} \tag{3}$$

where $L \geqslant 1$ is a gain that will be specified later. From Eqs. (2) and (3), and $e_2 \triangleq z_2 - \hat{z}_2$, the following relation holds:

$$\begin{aligned} \dot{e}_2 &= \dot{z}_2 - \dot{\hat{z}}_2 = L(\hat{z}_2^p - x_2^p) \\ &= -L\big((e_2+\hat{z}_2)^p - \hat{z}_2^p\big) + L\big((e_2+\hat{z}_2)^p \\ &\quad - (e_2+\hat{z}_2+Lx_1)^p\big). \end{aligned} \tag{4}$$

Set $V_e = \frac{e_2^2}{2}$, and the time derivative of V_e along the trajectories of system (1) is

$$\begin{aligned} \dot{V}_e = &- Le_2^{p+1}\left(\left(1+\frac{\hat{z}_2}{e_2}\right)^p - \left(\frac{\hat{z}_2}{e_2}\right)^p\right) \\ &+ Le_2\left((e_2+\hat{z}_2)^p - (e_2+\hat{z}_2+Lx_1)^p\right). \end{aligned} \tag{5}$$

Lemma 1 implies that if we take $t = -\frac{\hat{z}_2}{e_2}$, $r = p$, $0 < l < 1$, then $1 - t = 1 + \frac{\hat{z}_2}{e_2}$. Thus

$$\begin{aligned} &- Le_2^{p+1}\left(\left(1+\frac{\hat{z}_2}{e_2}\right)^p - \left(\frac{\hat{z}_2}{e_2}\right)^p\right) \\ &\leqslant -L(2^p-1)l^{1-p}e_2^{p+1} + Ll^2\hat{z}_2^{p+1}, \end{aligned} \tag{6}$$

and from Lemma 3 it was observed that

$$\begin{aligned} &Le_2\left((e_2+\hat{z}_2)^p - (e_2+\hat{z}_2+Lx_1)^p\right) \\ &\leqslant L|e_2| \cdot 2^{1-p}|e_2+\hat{z}_2-e_2-\hat{z}_2-Lx_1|^p \\ &\leqslant e_2^{p+1} + c(L)x_1^{p+1}, \end{aligned} \tag{7}$$

where $c(L) = \frac{p}{p+1}(\frac{1}{1+p})^{\frac{1}{p}}(2^{1-p}L^{p+1})^{\frac{p+1}{p}}$. Substituting the estimates (6) and (7) into (5), we find that

$$\begin{aligned} \dot{V}_e \leqslant &- L(2^p-1)l^{1-p}e_2^{p+1} + e_2^{p+1} \\ &+ Ll^2\hat{z}_2^{p+1} + c(L)x_1^{p+1}. \end{aligned} \tag{8}$$

Part II. Construction of an output-feedback controller. Considering the system

$$\begin{cases} \dot{x}_1 = x_2^p = (z_2+Lx_1)^p = (\hat{z}_2+e_2+Lx_1)^p, \\ \dot{\hat{z}}_2 = u^p - L\hat{z}_2^p, \end{cases} \tag{9}$$

we introduce the following change of coordinates:

$$\xi_1 = x_1, \quad \xi_2 = \hat{z}_2 - \alpha_1, \tag{10}$$

where α_1 is the virtual control, which will be specified later. Then, there is

$$\dot{\xi}_1 = (\xi_2 + \alpha_1 + e_2 + Lx_1)^p. \tag{11}$$

Now, consider the scalar function $V_{11} = \frac{\xi_1^2}{2}$. From Lemmas 2 and 3, the time derivative of V_{11} along the trajectories of (10) is

$$\begin{aligned} \dot{V}_{11} &= \xi_1\left((\xi_2+\alpha_1+e_2+Lx_1)^p - \alpha_1^p\right) + \xi_1\alpha_1^p \\ &\leqslant e_2^{p+1}+\xi_2^{p+1}+(\bar{c}+c_1L^p)\xi_1^{p+1}+\xi_1\alpha_1^p, \end{aligned} \tag{12}$$

where $c_1 = 2^{1-p} > 1$, and $\bar{c} = \frac{2}{p+1}(\frac{p}{p+1})^p c_1^{p+1}$. The virtual control α_1 is chosen as

$$\alpha_1 = -(c_1L^p+\bar{c}+2L+c(L))^{\frac{1}{p}}\xi_1 \triangleq -g_1(L)\xi_1; \tag{13}$$

then, this inequality follows:

$$\dot{V}_{11} \leqslant -(2L+c(L))\xi_1^{p+1} + \xi_2^{p+1} + e_2^{p+1}. \tag{14}$$

Furthermore, the choice of $V_{12} = \frac{\xi_2^2}{2} + V_{11}$ yields

$$\begin{aligned} \dot{V}_{12} \leqslant \dot{V}_{11} &+ \xi_2u^p + L|\xi_2| \cdot |\xi_2 - g_1\xi_1|^p \\ &+ g_1|\xi_2| \cdot |\xi_2 - g_1\xi_1 + e_2 + L\xi_1|^p. \end{aligned} \tag{15}$$

Some terms need to be eliminated from the right hand side of (15). Firstly, Lemmas 2 and 3 mean that

$$\begin{aligned} &L|\xi_2| \cdot |\xi_2-g_1\xi_1|^p+g_1|\xi_2| \cdot |\xi_2-g_1\xi_1+e_2+L\xi_1|^p \\ &\leqslant e_2^{p+1} + \xi_1^{p+1} + c(g_1)\xi_2^{p+1}, \end{aligned} \tag{16}$$

where $c(g_1) = \frac{1}{p+1}(\frac{p}{p+1})^p(g_1^{p+1}+(g_1^{p+1}+Lg_1^p+L^pg_1)^{p+1})+L+g_1$. Substituting (16) in (15) leads to the following inequality:

$$\begin{aligned} \dot{V}_{12} \leqslant\ &\xi_2u^{*p} + c(g_1)\xi_2^{p+1} + 2e_2^{p+1} - L\xi_1^{p+1} \\ &+ (u^p - u^{*p})\xi_2 + L\xi_2^{p+1} - c(L)\xi_1^{p+1}. \end{aligned}$$

If $u^* = -(c(g_1)+2L)^{\frac{1}{p}}\xi_2 \triangleq -g_2\xi_2$, then, $\dot{V}_{12} \leqslant -L(\xi_1^{p+1} + \xi_2^{p+1}) + 2e_2^{p+1} + \xi_2(u^p - u^{*p}) - c(L)\xi_1^{p+1}$. Let $V_1 = V_e + V_{12}$; then the following inequality holds:

$$\begin{aligned} \dot{V}_1 \leqslant &- L(2^p-1)l^{1-p}e_2^{p+1} + Ll^2\hat{z}_2^{p+1} \\ &- L(\xi_1^{p+1}+\xi_2^{p+1}) + 3e_2^{p+1} + \xi_2(u^p - u^{*p}). \end{aligned}$$

The choice of $l = \frac{1}{\sqrt{2^{p+1}(1+g_1^{p+1})}}$ leads to

$$\begin{aligned} Ll^2\hat{z}_2^{p+1} &= Ll^2(\xi_2 - g_1\xi_1)^{p+1} \\ &\leqslant 2^pLl^2(\xi_2^{p+1} + g_1^{p+1}\xi_1^{p+1}) \\ &\leqslant \frac{L}{2}(\xi_2^{p+1} + \xi_1^{p+1}). \end{aligned}$$

Choose

$$\begin{cases} u^* = -g_2(L)\xi_2 = -g_2\hat{z}_2 - g_2g_1\xi_1, \\ u = -g_2\hat{z}_2 - g_2g_1\theta x_1. \end{cases} \tag{17}$$

It follows from Lemma 3 that

$$\begin{aligned} \xi_2(u^p - u^{*p}) &\leqslant 2^{1-p}|\xi_2| \cdot |g_2g_1|^p \cdot |1-\theta|^p \cdot |\xi_1|^p \\ &\leqslant 2^{1-p}|\xi_2| \cdot |g_2g_1|^p \cdot \bar{\theta}^p \cdot |\xi_1|^p. \end{aligned} \tag{18}$$

If $\bar{\theta} \leqslant 2^{\frac{p-1}{p}}|g_2g_1|^{-1}$ is selected to guarantee $2^{1-p} \cdot |g_2g_1|^p\bar{\theta}^p \leqslant 1$, then Eq. (18) can be rewritten as

$$\xi_2(u^p - u^{*p}) \leqslant |\xi_2| \cdot |\xi_1|^p \leqslant \xi_1^{p+1} + \tilde{c}\xi_2^{p+1}, \tag{19}$$

where $\tilde{c} = \frac{1}{p+1}(\frac{p}{p+1})^p$ is a constant, and $0 < \tilde{c} < 1$. Then,

$$\dot{V}_1 \leqslant -(L(2^p-1)l^{1-p}-3)e_2^{p+1} - \left(\frac{L}{2}-1\right)\xi_1^{p+1}$$

$$-\left(\frac{L}{2}-\tilde{c}\right)\xi_2^{p+1}. \tag{20}$$

Part III. Determination of L and p^*. The inequalities $L(2^p-1)l^{1-p}-3\geqslant 1$, $\frac{L}{2}-1\geqslant 1$, $\frac{L}{2}-\tilde{c}\geqslant 1$, $0<\tilde{c}<1$ guide the choice of L as

$$L\geqslant \max\left\{\frac{4}{(2^p-1)l^{1-p}},4\right\}. \tag{21}$$

Considering the selection of l, the inequality $L\geqslant\frac{4}{(2^p-1)l^{1-p}}$ is equivalent to

$$L\geqslant\frac{2^{\frac{5-p^2}{2}}}{2^p-1}\left(1+\left(c_1L^p+\bar{c}+2L+\frac{p}{p+1}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\cdot 2^{\frac{1-p^2}{p}}L^{\frac{(p+1)^2}{p}}\right)^{\frac{p+1}{p}}\right)^{\frac{1-p}{2}}. \tag{22}$$

Now, by choosing $p^*=0.7836$, it is easy to verify that $\frac{(p+1)^2\cdot(1-p^2)}{2p^2}<1$ for all $p\in[p^*,1)$. This in turn implies that the degree of the polynomial of L on the right hand side of (22) is less than 1, which ensures the existence of L. With this in mind, a delicate selection of L leads to

$$\dot{V}_1\leqslant -e_2^{p+1}-\xi_1^{p+1}-\xi_2^{p+1}. \tag{23}$$

Therefore, global output feedback stabilization can be guaranteed based on the stability theory.

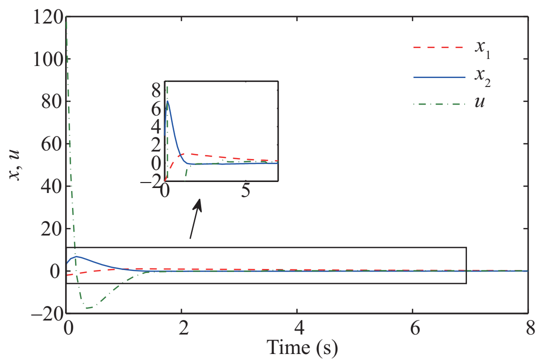

Figure 1 (Color online) Trajectories of x_1, x_2, and u.

Simulation. To confirm the validity of the control design, we consider the following nonlinear system:

$$\dot{x}_1=x_2^{\frac{9}{11}},\ \dot{x}_2=u^{\frac{9}{11}},\ y=\theta(t)x_1,$$

where $\theta(t)=1+0.25|\sin(10t)|$. Assumption 1 is satisfied with $|1-\theta(t)|\leqslant\bar{\theta}<\theta^*<1$. Following Theorem 1, the output feedback controller is constructed as follows: $u=-g_2g_1y-g_2\hat{z}_2$. For demonstration, initial values are chosen as $[x_1(0),x_2(0)]^{\rm T}=[-2,3]^{\rm T}$, and $L=4.9$. Figure 1 illustrates that the controller globally stabilizes the considered system.

Conclusion. This study solves the problem of global stabilization by constructing a novel continuous controller for a class of nonlinear systems with time-varying continuous output function. An extension of the above controller could be applied to develop an output feedback control scheme for higher-order nonlinear systems with uncertain output functions.

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