SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

September 2021, Vol. 64 192202:1–192202:14
https://doi.org/10.1007/s11432-020-3144-6

Finite-time command filtered adaptive control for nonlinear systems via immersion and invariance

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Received 1 June 2020/Revised 6 September 2020/Accepted 1 December 2020/Published online 23 August 2021

Abstract This paper investigates the problem of finite-time adaptive output tracking control for strict-feedback nonlinear systems with parametric uncertainties. Command signals and their derivatives are generated by a new command filter based on a second-order finite-time differentiator, which attenuates the chattering phenomenon. The parameter estimations are achieved by an immersion and invariance approach without requiring the certainty equivalence principle. The finite-time adaptive controller is constructed via a backstepping design method, a finite-time command filter, and a modified fractional-order error compensation mechanism. The proposed control strategy guarantees the finite-time boundedness of all signals in the closed-loop system, and the tracking error is driven into an arbitrarily small neighborhood of the origin in finite time. Finally, the new design technique is validated in a simulation example of the electromechanical system.

Keywords adaptive control, finite-time control, command-filtered backstepping, immersion and invariance

Citation Yu J P, Shi P, Chen X K, et al. Finite-time command filtered adaptive control for nonlinear systems via immersion and invariance. Sci China Inf Sci, 2021, 64(9): 192202, https://doi.org/10.1007/s11432-020-3144-6

1 Introduction

Over the past few decades, adaptive control of nonlinear systems with uncertainty parameterization has been widely investigated, and some crucial approaches for uncertain systems have been developed (see, e.g., [1–4] and references therein). The adaptive backstepping method has been proven to be especially effective for nonlinear systems with linear parametric uncertainty [5, 6]. Repeated differentiation of the virtual control signal, which leads to "explosion of complexity" in the backstepping design procedure, has been solved by dynamic surface control [7], which guarantees boundedness of the tracking error. Aided by dynamic surface control, later work [8] addressed the adaptive neural-network tracking control problem for nonlinear systems with arbitrary uncertainties. The authors of [9–11] applied dynamic surface control to nonlinear systems with time delay. However, as the aforementioned approaches ignore the effect of first-order filter error, they cannot guarantee better tracking performance.

As a powerful departure from the dynamic surface control technique, a novel command filter based on the backstepping method has been recently applied to nonlinear systems [12]. The command-filtered backstepping methodology has shown great potential in nonlinear systems with parameter uncertainties [13–16], and its tracking performance exceeds that of dynamic surface control [7–11]. However, these results require the so-called certain equivalence principle. Therefore, they may produce unacceptable dynamical behavior of the parameter estimation error, which degrades the transient performance of the closed-loop system. A new design tool for the stabilization and adaptive control of a class of nonlinear systems reliant upon immersion and invariance theory was recently proposed in [17] and was further developed in [18–20]. Unlike the classical adaptive control method, this method permits an extra term in the parameter estimation error to realize a modular control scheme with more flexible tunability

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than the usual scheme [5, 6]. Subsequently, the immersion and invariance-based approach has proven its effectiveness in many practical systems such as quadrotor unmanned aerial vehicles [21], air-breathing hypersonic vehicles [22], and vertical take-off and landing vehicles [23]. Nevertheless, these immersions and invariance-based adaptive control schemes only guarantee the asymptotic stability of nonlinear systems.

Finite-time stability is characterized by a faster convergence rate, higher tracking accuracy, and better disturbance rejection performance than asymptotic stability. Therefore, finite-time stabilization and tracking control problems have been widely studied for various nonlinear systems [24–28]. Many nonlinear systems with structure uncertainties have been solved by finite-time adaptive control schemes based on approximation approaches [29–32]. The authors of [33, 34] proposed distributed finite-time controllers for second- and higher-order nonlinear multi-agent systems, respectively. The authors of [35, 36] studied finite-time tracking control for strict-feedback nonlinear systems. They designed a sign function-based error compensation mechanism that eliminates the effect of filter errors. They also employed a Levant differentiator that accelerates the derivative approximation of the virtual control signal. Although the methods in [35, 36] achieve finite-time stability, they require an upper bound on the derivative of the virtual control signal. Furthermore, the sign function in the Levant differentiator may introduce the chattering phenomenon. To the authors' current knowledge, a finite-time command-filtered adaptive tracking control for nonlinear systems remains unsolved.

Motivated by the above discussion, we propose a finite-time command-filtered backstepping adaptive control scheme for strict-feedback nonlinear systems with parametric uncertainties. Our paper makes three main contributions to the existing literature.

(1) The finite-time command-filtered backstepping method [33–36] applies the Levant differentiator, which accelerates the convergence speed of the derivative approximation of the virtual control signal. In our approach, the Levant differentiator is replaced by a new finite-time convergent differentiator. The adopted finite-time command filter preserves the advantages of the Levant differentiator while attenuating the chattering phenomenon and relaxing the demand on the filter input signal.

(2) Different from command-filtered backstepping results under asymptotic stability in [13–16] and finite-time stability in [33–36], our scheme adopts an immersion and invariance approach for adaptive parameter estimation. Because it does not require the certainty equivalence principle, our control design expands the application scope of the command-filtered backstepping method.

(3) We replace the sign function-based error compensation mechanism in [34–36] with a modified fractional-order error compensation mechanism, which effectively eliminates the filter errors. We also design non-smooth fractional-power virtual control signals and an actual controller, together with parameter update laws. The whole closed-loop system operates with finite-time stability.

2 Problem formulation and preliminaries

Consider a single-input and single-output nonlinear system in parametric strict-feedback form:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + \psi_1^{\mathrm{T}}(x_1)\theta_1, \\ \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \psi_i^{\mathrm{T}}(\bar{x}_i)\theta_i, \quad i = 2, \ldots, n-1, \\ \dot{x}_n = f_n(x) + g_n(x)u + \psi_n^{\mathrm{T}}(x)\theta_n, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector with initial condition $x(0) = x_0$, $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the nonlinear system input signal and output signal, respectively. The $\theta_i \in \mathbb{R}^p$ is an unknown parameter vector. The functions $f_i : \mathbb{R}^i \mapsto \mathbb{R}$, $g_i : \mathbb{R}^i \mapsto \mathbb{R}$, and $\psi_i : \mathbb{R}^i \mapsto \mathbb{R}^p$ are known nonlinear functions.

The main objective of this paper is to propose a finite-time adaptive control scheme for a nonlinear system (1) such that the output tracking error is driven into an arbitrarily small neighborhood of the origin within finite-time as well as the finite-time boundedness of the closed-loop system signals is ensured.

Assumption 1. The desired signal y_d and its derivative $\dot{y}_d$ are all bounded and known.

Assumption 2. The desired signal y_d, the origin, and the initial condition x_0 are inside of open set $\Omega_d \subset \mathbb{R}^n$. For nonlinear system (1), g_i satisfies the inequality $\underline{\Xi}_i < |g_i| < \overline{\Xi}_i$, where $\underline{\Xi}_i$ and $\overline{\Xi}_i$ are known positive constants.

Lemma 1 ([36]). For given scalars $\lambda_1 > 0$, $\lambda_2 > 0$, $0 < \gamma < 1$, $0 < \eta < \infty$, if there exists a continuous positive definite function V with respect to a nonlinear system $\dot{x} = f(x)$ such that $\dot{V} \leqslant -\lambda_1 V - \lambda_2 V^{\gamma} + \eta$, then the solution of $\dot{x} = f(x)$ is practical finite-time stable, where the upper bound of the setting time T_r is $T_r \leqslant \max\{t_0 + \frac{1}{\theta_0 \lambda_1 (1-\iota)} \ln \frac{\theta_0 \lambda_1 V^{1-\iota}(t_0) + \lambda_2}{\lambda_2}, t_0 + \frac{1}{\lambda_1 (1-\iota)} \ln \frac{\lambda_1 V^{1-\iota}(t_0) + \theta_0 \lambda_2}{\theta_0 \lambda_2}\}$, and θ_0 satisfies $0 < \theta_0 < 1$.

Lemma 2 ([37]). Let ψ, φ be positive constants, and $\hbar(\mu, \nu)$ be a positive real valued function, and then

$$|\mu|^{\psi}|\nu|^{\varphi} \leqslant \frac{\psi \hbar(\mu, \nu) |\mu|^{\psi+\varphi}}{\psi + \varphi} + \frac{\varphi \hbar(\mu, \nu)^{-\frac{\psi}{\varphi}} |\nu|^{\psi+\varphi}}{\psi + \varphi}.$$

Lemma 3 ([24]). For any $\aleph_i \in \mathbb{R}$, $i = 1, 2, \ldots, n$, and $0 < \varpi \leqslant 1$, one has

$$\left(\sum_{i=1}^{n} |\aleph_i|\right)^{\varpi} \leqslant \sum_{i=1}^{n} |\aleph_i|^{\varpi} \leqslant n^{1-\varpi} \left(\sum_{i=1}^{n} |\aleph_i|\right)^{\varpi}.$$

Now, a second-order finite-time convergent differentiator [38] is introduced:

$$\begin{cases} \dot{\phi}_1 = \phi_2, \\ \dot{\phi}_2 = \dfrac{1}{\varepsilon^2}(-\mathrm{sat}_{\varepsilon_b}\{\mathrm{sgn}(\varphi_a(\phi_1, \phi_2))|\varphi_a(\phi_1, \phi_2)|^{\frac{\ell}{2-\ell}}\} - \mathrm{sat}_{\varepsilon_b}\{\mathrm{sgn}(\phi_2)|\varepsilon\phi_2|^{\ell}\}), \end{cases}$$

where ϕ_1, ϕ_2 are the system states of the second-order differentiator; $\varepsilon > 0$ is a perturbation parameter and $\ell \in (0, 1)$; $\varphi_a(\phi_1, \phi_2)$, $\mathrm{sat}_{\varepsilon_b}(\phi)$ are chosen as

$$\varphi_a(\phi_1, \phi_2) = \phi_1 - \alpha_r + \frac{1}{2-\ell}\mathrm{sgn}(\phi_2)|\varepsilon\phi_2|^{2-\ell},$$

$$\mathrm{sat}_{\varepsilon_b}(\phi) = \begin{cases} \phi, & |\phi| < \varepsilon_b, \\ \varepsilon_b \mathrm{sgn}(\phi), & |\phi| \geqslant \varepsilon_b, \end{cases}$$

where $\varepsilon_b > 0$ is a constant; α_r is the second-order differentiator input signal. By appropriately choosing ε, the following lemma is obtained.

Lemma 4 ([38]). Suppose that α_r is continuous and piecewise twice differentiable. For any $\varsigma > 0$, $\Omega > 0$, and $0 < \ell < 1$, the following relationship holds:

$$\phi_i - \alpha_r^{(i-1)} = \mathcal{O}(\varepsilon^{\rho\varsigma - i + 1}),$$

where $\rho\varsigma > 2$, $\rho = \ell/(2-\ell)$, for $t \geqslant \varepsilon\Omega(\Pi(\varepsilon)E(0))$, $i = 1$, and $t_k > t \geqslant t_{k-1} + \varepsilon\Omega(\Pi(\varepsilon)E_+(t_{k-1}))$, $k = 1, \ldots, j+1$, $i = 2$. $\Pi(\varepsilon)$, E, $E_+(t_{k-1})$ are given as $\Pi(\varepsilon) = \mathrm{diag}\{1, \varepsilon\}$, $E = [E_1, E_2]^{\mathrm{T}}$, $E_i = \phi_i - \alpha_r^{(i-1)}$, $E_+(t_{k-1}) = [E_1(t_{k-1}), E_2^+(t_{k-1})]^{\mathrm{T}}$, $E_2^+(t_{k-1}) = \phi_2 - \alpha_{r_+}(t_{k-1})$. $\mathcal{O}(\varepsilon^{\rho\varsigma - i + 1})$ stands for the approximation of ϕ_i and $\alpha_r^{(i-1)}(t)$.

3 Controller design and stability analysis

In this section, we propose a novel finite-time adaptive control scheme with help of the immersion and invariance approach, command filter technique, and backstepping recursive design method. The design procedure includes n steps, where the estimator $\hat{\theta}_i$ and virtual control signal α_i are firstly proposed at each step. Then, the stability of the whole closed-loop system is analyzed in the end.

3.1 Controller design

Let the error variable for each step be

$$z_1 = x_1 - y_d, \quad z_i = x_i - x_{i,c}, \quad i = 2, \ldots, n, \tag{2}$$

where $x_{i,c}$ is the output of finite-time command filter with virtual control signal α_{i-1} as the input signal, and the finite-time command filter is given as

$$\begin{cases} \dot{\phi}_{i,1} = \phi_{i,2}, \\ \dot{\phi}_{i,2} = \dfrac{1}{\varepsilon_i^2}\left(-\mathrm{sat}\left\{\left(\phi_{i,1} - \alpha_{i-1} + \dfrac{3}{5}(\varepsilon_i\phi_{i,2})^{\frac{5}{3}}\right)^{\frac{1}{5}}\right\} - \mathrm{sat}\{(\varepsilon_i\phi_{i,2})^{\frac{1}{3}}\}\right), \end{cases} \tag{3}$$

where $\varepsilon_i > 0$ is a constant, and $x_{i,c}(t) = \phi_{i,1}(t)$, $\dot{x}_{i,c}(t) = \phi_{i,2}(t)$.

Remark 1. In contrast to the asymptotic convergence differentiator [12–16], the second-order finite-time differentiator (3) makes the output of the command filter faster approximate the derivative of the virtual control signal. Unlike the finite-time differentiator with a sign function [33–36], the chattering phenomenon can be further attenuated in our results. Thus, both the finite time convergence and chattering phenomenon are simultaneously settled.

Based on the immersion and invariance methodology, define the estimator errors

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i + \beta_i(x_i), \quad i = 1, \ldots, n, \tag{4}$$

where $\beta_i(x_i)$ is smooth function yet to be given later, and $\hat{\theta}_i$ is an estimation of unknown parameter θ_i.

In order to remove the effect of filter error, a modified fractional-order compensation mechanism is constructed as

$$\dot{\xi}_1 = -k_1\xi_1 + g_1(x_{2,c} - \alpha_1) + g_1\xi_2 - h_1\xi_1^{\gamma}, \tag{5}$$

$$\dot{\xi}_i = -k_i\xi_i + g_i(x_{(i+1),c} - \alpha_i) - g_{i-1}\xi_{i-1} + g_i\xi_{i+1} - h_i\xi_i^{\gamma}, \quad i = 2, \ldots, n-1, \tag{6}$$

$$\dot{\xi}_n = -k_n\xi_n - g_{n-1}\xi_{n-1} - h_n\xi_n^{\gamma}, \tag{7}$$

where $k_i > 0$, $h_i > 0$ are design parameters; $\frac{1}{2} < \gamma = \frac{\gamma_1}{\gamma_2} < 1$, γ_1, γ_2 are positive odd integers, and the initial condition of ξ_i is $\xi_i(0) = 0$.

Remark 2. Note that the certain equivalence principle in our proposed method is not invoked for the control design procedure. The introduction of the function $\beta_i(\cdot)$ makes the parameter estimation not only be an integral action but enhance the flexibility of the estimator design.

Remark 3. The finite-time command filter errors may be still larger due to the increase of order n, so it is a difficult task for better tracking performance. Here, a modified fractional-order error compensation mechanism is designed to skillfully avoid the effect of filter errors, and it has finite-time convergence; meanwhile, the chattering problem in [34–36] caused by the sign function-based error compensation mechanism is weakened.

Before the detailed recursive design, the compensated tracking error ν_i is introduced as

$$\nu_i = z_i - \xi_i, \quad i = 1, \ldots, n. \tag{8}$$

Step 1. In light of (1), (2), and (8), one has

$$\dot{\nu}_1 = \dot{x}_1 - \dot{y}_d - \dot{\xi}_1 = f_1 + g_1(z_2 + x_{2,c} - \alpha_1) + g_1\alpha_1 + \psi_1^{\mathrm{T}}\big(\hat{\theta}_1 + \beta_1 - \tilde{\theta}_1\big) - \dot{y}_d - \dot{\xi}_1. \tag{9}$$

Substituting (5) into (9) yields

$$\dot{\nu}_1 = f_1 + g_1\nu_2 + g_1\alpha_1 + \psi_1^{\mathrm{T}}\big(\hat{\theta}_1 + \beta_1 - \tilde{\theta}_1\big) - \dot{y}_d + k_1\xi_1 + h_1\xi_1^{\gamma}. \tag{10}$$

The virtual control signal α_1 is designed as

$$\alpha_1 = \frac{1}{g_1}\big(- f_1 - \psi_1^{\mathrm{T}}\big(\hat{\theta}_1 + \beta_1\big) + \dot{y}_d - k_1 z_1 - s_1\nu_1^{\gamma}\big) \tag{11}$$

with $s_1 > 0$ being a design parameter. From (10) and (11), one has

$$\dot{\nu}_1 = -k_1\nu_1 - s_1\nu_1^{\gamma} + h_1\xi_1^{\gamma} - \psi_1^{\mathrm{T}}\tilde{\theta}_1 + g_1\nu_2. \tag{12}$$

By using the definition of $\tilde{\theta}_1$, we obtain that

$$\dot{\tilde{\theta}}_1 = \dot{\hat{\theta}}_1 + \frac{\partial\beta_1}{\partial x_1}\dot{x}_1 = \dot{\hat{\theta}}_1 + \frac{\partial\beta_1}{\partial x_1}\big(f_1 + g_1x_2 + \psi_1^{\mathrm{T}}\big(\hat{\theta}_1 + \beta_1 - \tilde{\theta}_1\big)\big). \tag{13}$$

Select the following parameter update law $\hat{\theta}_1$ to cancel the known dynamic:

$$\dot{\hat{\theta}}_1 = -\frac{\partial\beta_1}{\partial x_1}\big(f_1 + g_1x_2 + \psi_1^{\mathrm{T}}\big(\hat{\theta}_1 + \beta_1\big)\big) - \Gamma_1\sigma_1\big(\hat{\theta}_1 + \beta_1\big), \tag{14}$$

where $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ is a constant matrix, and $\sigma_1 > 0$ is a design parameter; $\beta_1 = \Gamma_1 \int_0^{x_1} \psi_1(\chi)\mathrm{d}\chi$.

Thus, one has the error dynamic

$$\dot{\tilde{\theta}}_1 = -\Gamma_1\big(\psi_1\psi_1^{\mathrm{T}}\tilde{\theta}_1 + \sigma_1\big(\hat{\theta}_1 + \beta_1\big)\big). \tag{15}$$

Step i ($i = 2, \ldots, n-1$). Similar to the above procedure, the virtual control signal α_i and parameter update law $\hat{\theta}_i$ are given. According to (2) and (8), one has

$$\dot{\nu}_i = \dot{x}_i - \dot{x}_{i,c} - \dot{\xi}_i = f_i + g_i(z_{i+1} + (x_{(i+1),c} - \alpha_i)) + g_i\alpha_i + \psi_i^{\mathrm{T}}\big(\hat{\theta}_i + \beta_i - \tilde{\theta}_i\big) - \dot{x}_{i,c} - \dot{\xi}_i. \tag{16}$$

By combining (6) and (16), it follows that

$$\dot{\nu}_i = f_i + g_i\nu_{i+1} + g_i\alpha_i + \psi_i^{\mathrm{T}}\big(\hat{\theta}_i + \beta_i - \tilde{\theta}_i\big) - \dot{x}_{i,c} + k_i\xi_i + g_{i-1}\xi_{i-1} + h_i\xi_i^{\gamma}. \tag{17}$$

Construct virtual control signal α_i as

$$\alpha_i = \frac{1}{g_i}\big(-f_i - \psi_i^{\mathrm{T}}\big(\hat{\theta}_i + \beta_i\big) - g_{i-1}z_{i-1} + \dot{x}_{i,c} - k_i z_i - s_i\nu_i^{\gamma}\big), \tag{18}$$

where $s_i > 0$ is a design parameter. Furthermore, the following equation holds:

$$\dot{\nu}_i = -k_i\nu_i - s_i\nu_i^{\gamma} + h_i\xi_i^{\gamma} - \psi_i^{\mathrm{T}}\tilde{\theta}_i - g_{i-1}\nu_{i-1} + g_i\nu_{i+1}. \tag{19}$$

Using the definition of $\tilde{\theta}_i$ yields

$$\dot{\tilde{\theta}}_i = \dot{\hat{\theta}}_i + \frac{\partial\beta_i}{\partial x_i}\dot{x}_i = \dot{\hat{\theta}}_i + \frac{\partial\beta_i}{\partial x_i}\big(f_i + g_i x_{i+1} + \psi_i^{\mathrm{T}}\big(\hat{\theta}_i + \beta_i - \tilde{\theta}_i\big)\big). \tag{20}$$

Choose the update law $\hat{\theta}_i$ as

$$\dot{\hat{\theta}}_i = -\frac{\partial\beta_i}{\partial x_i}\big(f_i + g_i x_{i+1} + \psi_i^{\mathrm{T}}\big(\hat{\theta}_i + \beta_i\big)\big) - \Gamma_i\sigma_i\big(\hat{\theta}_i + \beta_i\big) \tag{21}$$

with any constant matrix $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$, where σ_i is a positive design parameter; $\beta_i = \Gamma_i \int_0^{x_i} \psi_i(\bar{x}_{i-1}, \chi)\mathrm{d}\chi$.

By substituting (21) into (20), the dynamics of the estimation error is rewritten as

$$\dot{\tilde{\theta}}_i = -\Gamma_i\big(\psi_i\psi_i^{\mathrm{T}}\tilde{\theta}_i + \sigma_i\big(\hat{\theta}_i + \beta_i\big)\big). \tag{22}$$

Step n. The actual controller u will be designed in this step. On the basis of (1), (2), and (8), the following equation is obtained:

$$\dot{\nu}_n = \dot{x}_n - \dot{x}_{n,c} - \dot{\xi}_n = f_n + g_n u + \psi_n^{\mathrm{T}}\big(\hat{\theta}_n + \beta_n - \tilde{\theta}_n\big) - \dot{x}_{n,c} - \dot{\xi}_n. \tag{23}$$

Choose the actual control input u as

$$u = \frac{1}{g_n}\big(-f_n - \psi_n^{\mathrm{T}}\big(\hat{\theta}_n + \beta_n\big) - g_{n-1}z_{n-1} + \dot{x}_{i,c} - k_n z_n - s_n\nu_n^{\gamma}\big), \tag{24}$$

where s_n is a positive design parameter. In light of (23) and (24), one has

$$\dot{\nu}_n = -k_n\nu_n - s_n\nu_n^{\gamma} + h_n\xi_n^{\gamma} - \psi_n^{\mathrm{T}}\tilde{\theta}_n - g_{n-1}\nu_{n-1}. \tag{25}$$

From the definition of $\tilde{\theta}_n$, one can obtain the following equation:

$$\dot{\tilde{\theta}}_n = \dot{\hat{\theta}}_n + \frac{\partial\beta_n}{\partial x_n}\dot{x}_n = \dot{\hat{\theta}}_n + \frac{\partial\beta_n}{\partial x_n}\big(f_n + g_n u + \psi_n^{\mathrm{T}}\big(\hat{\theta}_n + \beta_n - \tilde{\theta}_n\big)\big). \tag{26}$$

From (26), define the parameter update law $\hat{\theta}_n$ as

$$\dot{\hat{\theta}}_n = -\frac{\partial\beta_n}{\partial x_n}\big(f_n + g_n u + \psi_n^{\mathrm{T}}\big(\hat{\theta}_n + \beta_n\big)\big) - \Gamma_n\sigma_n\big(\hat{\theta}_n + \beta_n\big), \tag{27}$$

where $\Gamma_n > 0$, $\sigma_n > 0$, and $\beta_n = \Gamma_n \int_0^{x_n} \psi_n(\bar{x}_{n-1}, \chi)\mathrm{d}\chi$.

Thus, substituting (27) into (26) yields

$$\dot{\tilde{\theta}}_n = -\Gamma_n\big(\psi_n\psi_n^{\mathrm{T}}\tilde{\theta}_n + \sigma_n\big(\hat{\theta}_n + \beta_n\big)\big). \tag{28}$$

3.2 Stability analysis

Theorem 1. Consider the closed-loop nonlinear system composed by (1), the actual controller (24), the virtual control signals (11), (18), and the parameter update laws (14), (21), (27). Under the Assumptions 1 and 2, finite-time boundedness for all signals in the closed-loop system is guaranteed for any bounded initial conditions, while the output tracking error can converge to a sufficiently small neighborhood of origin in finite time by tuning the design parameters.

Proof. The derivatives of compensated tracking error ν_i, compensation signal ξ_i, and estimate error $\tilde{\theta}_i$ are rewritten as

$$\begin{cases} \dot{\nu}_1 = -k_1\nu_1 - s_1\nu_1^{\gamma} + h_1\xi_1^{\gamma} - \psi_1^{\mathrm{T}}\tilde{\theta}_1 + g_1\nu_2, \\ \dot{\nu}_i = -k_i\nu_i - s_i\nu_i^{\gamma} + h_i\xi_i^{\gamma} - \psi_i^{\mathrm{T}}\tilde{\theta}_i - g_{i-1}\nu_{i-1} + g_i\nu_{i+1}, \\ \dot{\nu}_n = -k_n\nu_n - s_n\nu_n^{\gamma} + h_n\xi_n^{\gamma} - \psi_n^{\mathrm{T}}\tilde{\theta}_n - g_{n-1}\nu_{n-1}, \\ \dot{\xi}_1 = -k_1\xi_1 + g_1(x_{2,c} - \alpha_1) + g_1\xi_2 - h_1\xi_1^{\gamma}, \\ \dot{\xi}_i = -k_i\xi_i + g_i(x_{(i+1),c} - \alpha_i) - g_{i-1}\xi_{i-1} + g_i\xi_{i+1} - h_i\xi_i^{\gamma}, \\ \dot{\xi}_n = -k_n\xi_n - g_{n-1}\xi_{n-1} - h_n\xi_n^{\gamma}, \\ \dot{\tilde{\theta}}_1 = -\Gamma_1\big(\psi_1\psi_1^{\mathrm{T}}\tilde{\theta}_1 + \sigma_1\big(\hat{\theta}_1 + \beta_1\big)\big), \\ \dot{\tilde{\theta}}_i = -\Gamma_i\big(\psi_i\psi_i^{\mathrm{T}}\tilde{\theta}_i + \sigma_i\big(\hat{\theta}_i + \beta_i\big)\big), \\ \dot{\tilde{\theta}}_n = -\Gamma_n\big(\psi_n\psi_n^{\mathrm{T}}\tilde{\theta}_n + \sigma_n\big(\hat{\theta}_n + \beta_n\big)\big). \end{cases} \tag{29}$$

The Lyapunov function $V(\nu_1,\ldots,\nu_n,\xi_1,\ldots,\xi_n,\tilde{\theta}_1,\ldots,\tilde{\theta}_n)$ is constructed as

$$V = \frac{1}{2}\sum_{i=1}^{n}\nu_i^2 + \frac{1}{2}\sum_{i=1}^{n}\xi_i^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i. \tag{30}$$

By taking the derivative of V, one has

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{n}\nu_i\dot{\nu}_i + \sum_{i=1}^{n}\xi_i\dot{\xi}_i + \sum_{i=1}^{n}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\tilde{\theta}}_i \\ =& \nu_1\big(-k_1\nu_1 - s_1\nu_1^{\gamma} + h_1\xi_1^{\gamma} - \psi_1^{\mathrm{T}}\tilde{\theta}_1 + g_1\nu_2\big) \\ &+ \sum_{i=2}^{n-1}\nu_i\big(-k_i\nu_i - s_i\nu_i^{\gamma} + h_i\xi_i^{\gamma} - \psi_i^{\mathrm{T}}\tilde{\theta}_i - g_{i-1}\nu_{i-1} + g_i\nu_{i+1}\big) \\ &+ \nu_n\big(-k_n\nu_n - s_n\nu_n^{\gamma} + h_n\xi_n^{\gamma} - \psi_n^{\mathrm{T}}\tilde{\theta}_n - g_{n-1}\nu_{n-1}\big) \\ &+ \xi_1\big(-k_1\xi_1 + g_1(x_{2,c} - \alpha_1) + g_1\xi_2 - h_1\xi_1^{\gamma}\big) \\ &+ \sum_{i=2}^{n-1}\xi_i\big(-k_i\xi_i + g_i(x_{(i+1),c} - \alpha_i) - g_{i-1}\xi_{i-1} + g_i\xi_{i+1} - h_i\xi_i^{\gamma}\big) \\ &+ \xi_n\big(-k_n\xi_n - g_{n-1}\xi_{n-1} - h_n\xi_n^{\gamma}\big) - \sum_{i=1}^{n}\tilde{\theta}_i^{\mathrm{T}}\big(\psi_i\psi_i^{\mathrm{T}}\tilde{\theta}_i + \sigma_i\big(\hat{\theta}_i + \beta_i\big)\big). \end{aligned} \tag{31}$$

From Lemma 2, the following inequality holds:

$$\nu_i\xi_i^{\gamma} \leqslant |\nu_i||\xi_i|^{\gamma} \leqslant \frac{1}{1+\gamma}|\nu_i|^{1+\gamma} + \frac{\gamma}{1+\gamma}|\xi_i|^{1+\gamma}. \tag{32}$$

By substituting (32) into (31), one yields

$$\begin{aligned} \dot{V} \leqslant& -\sum_{i=1}^{n}k_i\nu_i^2 - \sum_{i=1}^{n}\left(s_i - \frac{h_i}{1+\gamma}\right)\nu_i^{1+\gamma} - \sum_{i=1}^{n}k_i\xi_i^2 - \sum_{i=1}^{n}\frac{h_i}{1+\gamma}\xi_i^{1+\gamma} \\ &+ \sum_{i=1}^{n-1}|g_i||\xi_i||x_{(i+1),c} - \alpha_i| - \sum_{i=1}^{n}\nu_i\psi_i^{\mathrm{T}}\tilde{\theta}_i - \sum_{i=1}^{n}\tilde{\theta}_i^{\mathrm{T}}\big(\psi_i\psi_i^{\mathrm{T}}\tilde{\theta}_i + \sigma_i\big(\hat{\theta}_i + \beta_i\big)\big). \end{aligned} \tag{33}$$

From Lemma 4, $|x_{(i+1),c}-\alpha_i| = \mathcal{O}_i(\varepsilon_i^{\rho\varsigma})$ is guaranteed in finite time. Base on Assumption 2, then

$$\begin{aligned}\dot V \leqslant & -\sum_{i=1}^n k_i\nu_i^2 - \sum_{i=1}^n \left(s_i - \frac{h_i}{1+\gamma}\right)\nu_i^{1+\gamma} - \sum_{i=1}^n \left(k_i - \frac{1}{2}\right)\xi_i^2 - \sum_{i=1}^n \frac{h_i}{1+\gamma}\xi_i^{1+\gamma} \\ & + \sum_{i=1}^n \frac{\Xi_i^2}{2}\mathcal{O}_i(\varepsilon_i^{2\rho\varsigma}) - \sum_{i=1}^n \nu_i\psi_i^{\mathrm{T}}\tilde\theta_i - \sum_{i=1}^n \tilde\theta_i^{\mathrm{T}}\big(\psi_i\psi_i^{\mathrm{T}}\tilde\theta_i + \sigma_i\big(\hat\theta_i+\beta_i\big)\big). \end{aligned} \tag{34}$$

In light of Lemma 2, one has

$$-\nu_i\psi_i^{\mathrm{T}}(\bar x_i)\tilde\theta_i \leqslant \frac{1}{4}\nu_i^2 + \tilde\theta_i^{\mathrm{T}}\psi_i(\bar x_i)\psi_i^{\mathrm{T}}(\bar x_i)\tilde\theta_i, \tag{35}$$

$$-\tilde\theta_i^{\mathrm{T}}\big(\hat\theta_i+\beta_i\big) \leqslant -\frac{1}{2}\tilde\theta_i^{\mathrm{T}}\tilde\theta_i + \frac{1}{2}\theta_i^{\mathrm{T}}\theta_i. \tag{36}$$

Substituting (35) and (36) into (34) yields

$$\begin{aligned}\dot V \leqslant & -\sum_{i=1}^n \left(k_i-\frac{1}{4}\right)\nu_i^2 - \sum_{i=1}^n \left(s_i - \frac{h_i}{1+\gamma}\right)\nu_i^{1+\gamma} - \sum_{i=1}^n \left(k_i-\frac{1}{2}\right)\xi_i^2 \\ & - \sum_{i=1}^n \frac{h_i}{1+\gamma}\xi_i^{1+\gamma} - \sum_{i=1}^n \frac{\sigma_i}{2}\tilde\theta_i^{\mathrm{T}}\tilde\theta_i + \sum_{i=1}^n \frac{\sigma_i}{2}\theta_i^{\mathrm{T}}\theta_i + \sum_{i=1}^n \frac{\Xi_i^2}{2}\mathcal{O}_i(\varepsilon_i^{2\rho\varsigma}) \\ = & -\sum_{i=1}^n \left(k_i-\frac{1}{4}\right)\nu_i^2 - \sum_{i=1}^n \left(s_i - \frac{h_i}{1+\gamma}\right)\nu_i^{1+\gamma} - \sum_{i=1}^n \left(k_i-\frac{1}{2}\right)\xi_i^2 \\ & - \sum_{i=1}^n \frac{h_i}{1+\gamma}\xi_i^{1+\gamma} - \sum_{i=1}^n \frac{\sigma_i}{2}\tilde\theta_i^{\mathrm{T}}\tilde\theta_i - \sum_{i=1}^n \sigma_i\left(\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2}\right)^{\frac{1+\gamma}{2}} \\ & + \sum_{i=1}^n \sigma_i\left(\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2}\right)^{\frac{1+\gamma}{2}} + \sum_{i=1}^n \frac{\sigma_i}{2}\theta_i^{\mathrm{T}}\theta_i + \sum_{i=1}^n \frac{\Xi_i^2}{2}\mathcal{O}_i(\varepsilon_i^{2\rho\varsigma}). \end{aligned} \tag{37}$$

By applying Lemma 2 to the term $(\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2})^{\frac{1+\gamma}{2}}$, there exists a constant $0<\kappa<1$ such that

$$\left(\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2}\right)^{\frac{1+\gamma}{2}} \leqslant \kappa\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2} + \frac{(1-\gamma)}{2}\left(\frac{\kappa^{-1}(1+\gamma)}{2}\right)^{\frac{1+\gamma}{1-\gamma}}. \tag{38}$$

Substituting (38) into (37) and applying Lemma 3 yield

$$\begin{aligned}\dot V \leqslant & -\sum_{i=1}^n \left(k_i-\frac{1}{4}\right)\nu_i^2 - \sum_{i=1}^n \left(s_i - \frac{h_i}{1+\gamma}\right)(\nu_i^2)^{\frac{1+\gamma}{2}} - \sum_{i=1}^n \left(k_i-\frac{1}{2}\right)\xi_i^2 \\ & - \sum_{i=1}^n \frac{h_i}{1+\gamma}(\xi_i^2)^{\frac{1+\gamma}{2}} - \sum_{i=1}^n (1-\kappa)\sigma_i\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2} - \sum_{i=1}^n \sigma_i\left(\frac{\tilde\theta_i^{\mathrm{T}}\tilde\theta_i}{2}\right)^{\frac{1+\gamma}{2}} \\ & + \sum_{i=1}^n \frac{\sigma_i}{2}\theta_i^{\mathrm{T}}\theta_i + \sum_{i=1}^n \frac{\Xi_i^2}{2}\mathcal{O}_i(\varepsilon_i^{2\rho\varsigma}) + \sum_{i=1}^n \sigma_i\frac{(1-\gamma)}{2}\left(\frac{\kappa^{-1}(1+\gamma)}{2}\right)^{\frac{1+\gamma}{1-\gamma}} \\ \leqslant & -\lambda_1 V - \lambda_2 V^{\frac{1+\gamma}{2}} + \eta, \end{aligned} \tag{39}$$

where

$$\lambda_1 = \min\left\{2\left(k_i-\frac{1}{4}\right), 2\left(k_i-\frac{1}{2}\right), \frac{(1-\kappa)\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\right\},$$

$$\lambda_2 = \min\left\{2^{\frac{1+\gamma}{2}}\left(s_i-\frac{h_i}{1+\gamma}\right), 2^{\frac{1+\gamma}{2}}\frac{h_i}{1+\gamma}, \sigma_i\left(\frac{1}{\lambda_{\max}(\Gamma_i^{-1})}\right)^{\frac{1+\gamma}{2}}\right\},$$

$$\eta=\sum_{i=1}^{n}\frac{\Xi_i^2}{2}\mathcal{O}_i(\varepsilon_i^{2\rho\varsigma})+\sum_{i=1}^{n}\sigma_i\frac{(1-\gamma)}{2}\left(\frac{\kappa^{-1}(1+\gamma)}{2}\right)^{\frac{1+\gamma}{1-\gamma}}+\sum_{i=1}^{n}\frac{\sigma_i}{2}\theta_i^{\rm T}\theta_i.$$

Thus, Eq. (39) is equivalent to the following inequality:

$$\dot{V}\leqslant-\Theta\lambda_1V-(1-\Theta)\lambda_1V-\lambda_2V^{\frac{1+\gamma}{2}}+\eta, \tag{40}$$

or

$$\dot{V}\leqslant-\lambda_1V-\Theta\lambda_2V^{\frac{1+\gamma}{2}}-(1-\Theta)\lambda_2V^{\frac{1+\gamma}{2}}+\eta, \tag{41}$$

where $0<\Theta<1$. According to (40), if $V>\eta/((1-\Theta)\lambda_1)$, then $\dot{V}\leqslant-\Theta\lambda_1V-\lambda_2V^{\frac{1+\gamma}{2}}$. In light of Lemma 1, ν_i, ξ_i, and $\tilde{\theta}_i$ will converge into the following region:

$$\left(\nu_i,\xi_i,\tilde{\theta}_i\right)\in\left\{V\leqslant\frac{\eta}{(1-\Theta)\lambda_1}\right\} \tag{42}$$

in finite time $T_1\leqslant(2/(\Theta\lambda_1(1-\gamma)))\ln((\Theta\lambda_1V^{(1-\gamma)/2}(0)+\lambda_2)/\lambda_2)$. Based on (41), $\dot{V}\leqslant-\lambda_1V-\Theta\lambda_2V^{\frac{1+\gamma}{2}}$ can be obtained if $V^{\frac{1+\gamma}{2}}>\eta/((1-\Theta)\lambda_2)$. Similarly, ν_i, ξ_i, and $\tilde{\theta}_i$ are driven into the following region:

$$\left(\nu_i,\xi_i,\tilde{\theta}_i\right)\in\left\{V\leqslant\left(\frac{\eta}{(1-\Theta)\lambda_2}\right)^{\frac{2}{1+\gamma}}\right\} \tag{43}$$

in finite time $T_2\leqslant(2/(\lambda_1(1-\gamma)))\ln((\lambda_1V^{(1-\gamma)/2}(0)+\Theta\lambda_2)/\Theta\lambda_2)$. Therefore, the finite-time boundedness of all signals ν_i, ξ_i, and $\tilde{\theta}_i$ in the closed-loop systems is guaranteed.

Then, ν_1 and ξ_1 will converge into the region

$$|\nu_1|\leqslant\min\left\{\sqrt{\frac{2\eta}{(1-\Theta)\lambda_1}},\sqrt{2\left(\frac{\eta}{(1-\Theta)\lambda_2}\right)^{\frac{2}{1+\gamma}}}\right\}, \tag{44}$$

$$|\xi_1|\leqslant\min\left\{\sqrt{\frac{2\eta}{(1-\Theta)\lambda_1}},\sqrt{2\left(\frac{\eta}{(1-\Theta)\lambda_2}\right)^{\frac{2}{1+\gamma}}}\right\} \tag{45}$$

in finite time $T=\max\{(2/(\Theta\lambda_1(1-\gamma)))\ln((\Theta\lambda_1V^{\frac{(1-\gamma)}{2}}(0)+\lambda_2)/\lambda_2),(2/(\lambda_1(1-\gamma)))\ln((\lambda_1V^{\frac{(1-\gamma)}{2}}(0)+\Theta\lambda_2)/\Theta\lambda_2)\}$. For $t\geqslant T$, it follows that

$$|z_1|\leqslant|\nu_1|+|\xi_1|\leqslant\min\left\{2\sqrt{\frac{2\eta}{(1-\Theta)\lambda_1}},2\sqrt{2\left(\frac{\eta}{(1-\Theta)\lambda_2}\right)^{\frac{2}{1+\gamma}}}\right\}.$$

Furthermore, by adjusting the design parameters, one can make the output tracking error z_1 arbitrarily small in finite time.

Remark 4. Compared with the command-filtered backstepping asymptotic convergence results [12–16], the fractional-power virtual control signals and controller are designed, which makes the closed-loop system with a faster convergence speed. In contrast to these studies in [33–36], an immersion and invariance methodology is used to address the parameter estimation, which removes the requirement of the certainty equivalence principle. Thus, our method further improves the control quality and relax some limitations.

4 Simulation example

The dynamics equation of the electromechanical system in [39] is shown as

$$\begin{cases}M\ddot{q}+B\dot{q}+N\sin(q)=I,\\ L\dot{I}+K_B\dot{q}+RI=V,\end{cases}$$

where $M = J/K_\tau + mL_0^2/3K_\tau + M_0L_0^2/K_\tau + 2M_0R_0^2/5K_\tau$, $B = B_0/K_\tau$, $N = mL_0G/2K_\tau + M_0L_0G/K_\tau$. $J = 1.625\times10^{-3}$ Kg·m^2, $K_\tau = 0.9$ N·m/A, $m = 0.506$ Kg, $L_0 = 0.305$ m, $M_0 = 0.434$ Kg, $R_0 = 0.023$ m, $B_0 = 1.625\times10^{-2}$ N·m·s/rad, $G = 9.8$ m/s^2, $L = 0.025$ H, $K_B = 0.9$ N·m/A, $R = 5\ \Omega$. Define $x_1 = q$, $x_2 = \dot{q}$, $x_3 = I$, and $u = V$; then

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \dfrac{1}{M}x_3 + [\sin(x_1)\ x_2][\theta_{21}\ \theta_{22}]^{\mathrm{T}}, \\ \dot{x}_3 = \dfrac{1}{L}u + [x_2\ x_3][\theta_{31}\ \theta_{32}]^{\mathrm{T}}, \\ y = x_1, \end{cases} \tag{46}$$

where $\theta_2 = [\theta_{21}\ \theta_{22}]^{\mathrm{T}}$, $\theta_{21} = -N/M$, $\theta_{22} = -B/M$, $\theta_3 = [\theta_{31}\ \theta_{32}]^{\mathrm{T}}$, $\theta_{31} = -K_B/L$, and $\theta_{32} = -R/L$.

By following the recursive design in Subsection 3.1, the detailed controller design procedure for the electromechanical system (46) is given as follows.

Define the following tracking errors and compensated tracking errors:

$$z_1 = x_1 - y_d, \qquad \nu_1 = z_1 - \xi_1, \tag{47}$$

$$z_2 = x_2 - x_{2,c}, \qquad \nu_2 = z_2 - \xi_2, \tag{48}$$

$$z_3 = x_3 - x_{3,c}, \qquad \nu_3 = z_3 - \xi_3, \tag{49}$$

where ξ_i, $i = 1, 2, 3$ is the compensation signal yet to be given later; $x_{2,c}$ and $x_{3,c}$ are the output of finite-time command filter with virtual control signal α_1 and α_2 as the input signal, respectively; the finite-time command filter is introduced as

$$\begin{cases} \dot{\phi}_{i,1} = \phi_{i,2}, \\ \dot{\phi}_{i,2} = \dfrac{1}{\varepsilon_i^2}\left(-\operatorname{sat}\left\{ \left(\phi_{i,1} - \alpha_{(i-1)} + \dfrac{3}{5}(\varepsilon_i\phi_{i,2})^{\frac{5}{3}} \right)^{\frac{1}{5}} \right\} - \operatorname{sat}\left\{ (\varepsilon_i\phi_{i,2})^{\frac{1}{3}} \right\} \right), \end{cases} \tag{50}$$

where $\varepsilon_i > 0$ is a constant, and $x_{i,c}(t) = \phi_{i,1}(t)$, $\dot{x}_{i,c}(t) = \phi_{i,2}(t)$, $i = 2, 3$.

Step 1. In light of (46)–(48), one has

$$\dot{\nu}_1 = \dot{x}_1 - \dot{y}_d - \dot{\xi}_1 = z_2 + (x_{2,c} - \alpha_1) + \alpha_1 - \dot{y}_d - \dot{\xi}_1. \tag{51}$$

In order to handle the effect of filter error $(x_{2,c} - \alpha_1)$, define the following compensation signal ξ_1:

$$\dot{\xi}_1 = -k_1\xi_1 + (x_{2,c} - \alpha_1) + \xi_2 - h_1\xi_1^{\gamma}, \quad \xi_1(0) = 0, \tag{52}$$

where k_1 and h_1 are positive constants. Substituting (52) into (51) yields

$$\dot{\nu}_1 = \dot{x}_1 - \dot{y}_d - \dot{\xi}_1 = \nu_2 + \alpha_1 - \dot{y}_d + k_1\xi_1 + h_1\xi_1^{\gamma}. \tag{53}$$

The virtual control signal α_1 is designed as

$$\alpha_1 = -k_1z_1 - s_1\nu_1^{\gamma} + \dot{y}_d \tag{54}$$

with $s_1 > 0$ being a design parameter. From (53) and (54), one has

$$\dot{\nu}_1 = -k_1\nu_1 - s_1\nu_1^{\gamma} + h_1\xi_1^{\gamma} + \nu_2. \tag{55}$$

Step 2. With help of (46), (48), and (49), one has

$$\dot{\nu}_2 = \frac{1}{M}\left(z_3 + (x_{3,c} - \alpha_2)\right) + \frac{1}{M}\alpha_2 + [\sin(x_1)\ x_2]\big(\hat{\theta}_2 + \beta_2 - \tilde{\theta}_2\big) - \dot{x}_{2,c} - \dot{\xi}_2. \tag{56}$$

To deal with the effect of filter error $(x_{3,c} - \alpha_2)$, the following compensation signal ξ_2 is defined:

$$\dot{\xi}_2 = -k_2\xi_2 + \frac{1}{M}\left(x_{3,c} - \alpha_2\right) + \frac{1}{M}\xi_3 - \xi_1 - h_2\xi_2^{\gamma}, \quad \xi_2(0) = 0, \tag{57}$$

where $k_2 > 0$ and $h_2 > 0$ are design parameters. By combining (56) and (57), it follows

$$\dot{\nu}_2 = \frac{1}{M}\nu_3 + \frac{1}{M}\alpha_2 + [\sin(x_1)\ x_2]\big(\hat{\theta}_2 + \beta_2 - \tilde{\theta}_2\big) - \dot{x}_{2,c} + k_2\xi_2 + \xi_1 + h_2\xi_2^{\gamma}. \tag{58}$$

Construct virtual control signal α_2 as

$$\alpha_2 = M\big(-k_2z_2 - s_2\nu_2^{\gamma} + \dot{x}_{2,c} - [\sin(x_1)\ x_2]\big(\hat{\theta}_2 + \beta_2\big) - z_1\big), \tag{59}$$

where $s_2 > 0$ is a design parameter. Furthermore, the following equation holds:

$$\dot{\nu}_2 = -k_2\nu_2 - s_2\nu_2^{\gamma} + h_2\xi_2^{\gamma} - [\sin(x_1)\ x_2]\tilde{\theta}_2 - \nu_1 + \frac{1}{M}\nu_3. \tag{60}$$

Choose the update law $\hat{\theta}_2$ as

$$\dot{\hat{\theta}}_2 = -\frac{\partial\beta_2}{\partial x_2}\left(\frac{1}{M}x_3 + [\sin(x_1)\ x_2]\big(\hat{\theta}_2 + \beta_2\big)\right) - \Gamma_2\sigma_2\big(\hat{\theta}_2 + \beta_2\big) \tag{61}$$

with a constant matrix $\Gamma_2 = \Gamma_2^{\mathrm{T}} > 0$, where σ_2 is a positive design parameter and $\beta_2 = \Gamma_2\int_0^{x_2}[\sin(x_1)\ \ \chi]^{\mathrm{T}}\mathrm{d}\chi$. The dynamics of the estimation error is rewritten as

$$\dot{\tilde{\theta}}_2 = -\Gamma_2\big([\sin(x_1)\ x_2]^{\mathrm{T}}[\sin(x_1)\ x_2]\tilde{\theta}_2 + \sigma_2\big(\hat{\theta}_2 + \beta_2\big)\big). \tag{62}$$

Step 3. On the basis of (46) and (49), the following equation is obtained:

$$\dot{\nu}_3 = \frac{1}{L}u + [x_2\ x_3]\big(\hat{\theta}_3 + \beta_3 - \tilde{\theta}_3\big) - \dot{x}_{3,c} - \dot{\xi}_3. \tag{63}$$

Define the compensation signal ξ_3 as

$$\dot{\xi}_3 = -k_3\xi_3 - \frac{1}{M}\xi_2 - h_3\xi_3^{\gamma}, \quad \xi_3(0) = 0, \tag{64}$$

where k_3 and h_3 are positive constants. Choose the actual control input u as

$$u = L\left(-k_3z_3 - s_3\nu_3^{\gamma} + \dot{x}_{3,c} - [x_2\ x_3]\big(\hat{\theta}_3 + \beta_3\big) - \frac{1}{M}z_2\right), \tag{65}$$

where s_3 is a positive design parameter. In light of (63)–(65), one has

$$\dot{\nu}_3 = -k_3\nu_3 - s_3\nu_3^{\gamma} + h_3\xi_3^{\gamma} - [x_2\ x_3]\tilde{\theta}_3 - \frac{1}{M}\nu_2. \tag{66}$$

Define the parameter update law $\hat{\theta}_3$ as

$$\dot{\hat{\theta}}_3 = -\frac{\partial\beta_3}{\partial x_3}\left(\frac{1}{L}u + [x_2\ x_3]\big(\hat{\theta}_3 + \beta_3\big)\right) - \Gamma_3\sigma_3\big(\hat{\theta}_3 + \beta_3\big), \tag{67}$$

where $\Gamma_3 > 0$, $\sigma_3 > 0$, and $\beta_3 = \Gamma_3\int_0^{x_3}[x_1x_2\ \ \chi]^{\mathrm{T}}\mathrm{d}\chi$. Thus, one has

$$\dot{\tilde{\theta}}_3 = -\Gamma_3\big([x_2\ x_3]^{\mathrm{T}}[x_2\ x_3]\tilde{\theta}_3 + \sigma_3\big(\hat{\theta}_3 + \beta_3\big)\big). \tag{68}$$

In order to show the advantages of the finite-time command-filtered backstepping control (FTCFB) method, the comparison is derived with command-filtered backstepping (CFB) in [13]. In the simulation, the desired reference signal $y_d(t)$ is chosen as $y_d(t) = 0.5(\sin(t) + \sin(0.5t))$. The initial condition of the nonlinear system is $[x_1(0), x_2(0), x_3(0)]^{\mathrm{T}} = [0.5, 0.5, 0.5]^{\mathrm{T}}$. The designed parameters are defined as the following three cases.

Case 1. $k_1 = k_2 = k_3 = 20$, $h_1 = h_2 = h_3 = 1$, $s_1 = s_2 = s_3 = 30$, $\sigma_2 = 0.01$, $\sigma_3 = 0.01$, $\Gamma_2 = \mathrm{diag}\{2, 2\}$, $\Gamma_3 = \mathrm{diag}\{2, 2\}$, $\gamma = 3/5$.

Case 2. $k_1 = k_2 = k_3 = 20$, $\Gamma_1 = \mathrm{diag}\{2, 2\}$, $\Gamma_2 = \mathrm{diag}\{2, 2\}$.

Case 3. $k_1 = k_2 = k_3 = 40$, $\Gamma_1 = \mathrm{diag}\{2, 2\}$, $\Gamma_2 = \mathrm{diag}\{2, 2\}$.

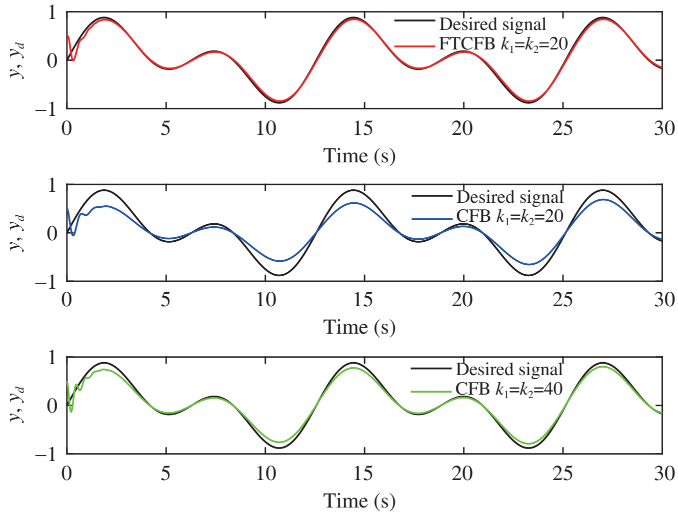

Figure 1 (Color online) The desired signal y_d and system output y of case 1 (red), case 2 (blue), and case 3 (green).

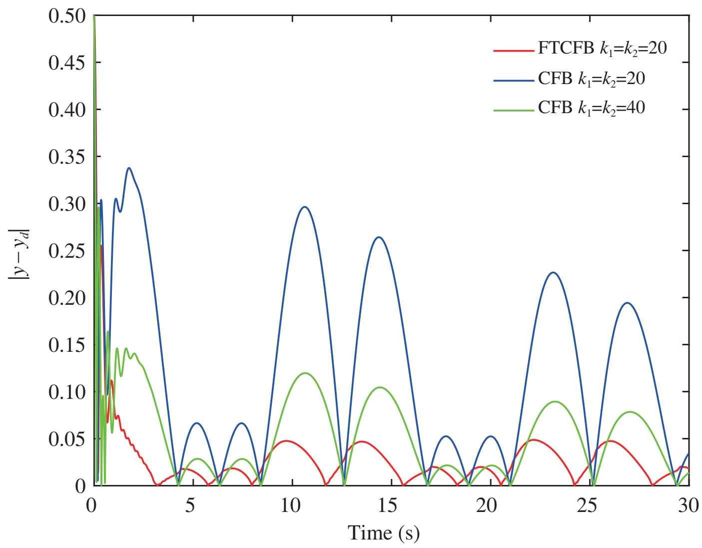

Figure 2 (Color online) The tracking error $|y - y_d|$ of case 1 (red), case 2 (blue), and case 3 (green).

Figures 1–5 provide the simulation results. The trajectories of the desired signal y_d and system output y are shown in Figure 1. The tracking error is given in Figure 2. Figures 3 and 4 give the trajectories of the parameter update laws $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$. Figure 5 illustrates the response of input signal u.

Define the performance index of the tracking error as $\sum_{k=1}^{N} |y(k)-y_d(k)|$, and $\sum_{k=1}^{N} |u(k)|$ is the control input index. The sampling period is 0.01 s, and calculate the tracking error and control input indexes from 0 to 30 s, and the number of sampling data N is 3001. The comparison results among cases 1–3 are shown in Table 1.

Remark 5. Compared with the traditional command-filtered backstepping approach in [13], whether similar or different design parameters are chosen, it is obvious from Figures 1, 2, 5, and Table 1 that the faster convergence rate and the better tacking performance are simultaneously achieved in our proposed method. However, it also brings some disadvantages, such as larger control energy and more design parameters, which will be settled in our future research.

5 Conclusion

This paper proposed a novel finite-time adaptive controller that solves the output tracking problem for strict-feedback nonlinear systems with parametric uncertainties. By integrating an immersion and in-

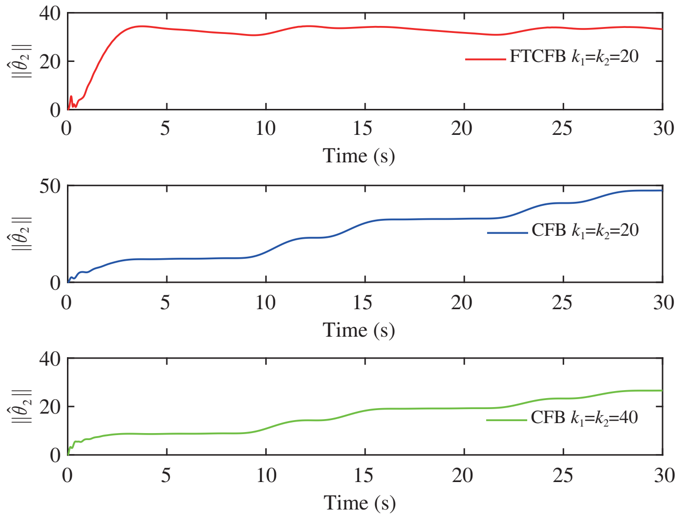

Figure 3 (Color online) The parameter update law $\|\hat{\theta}_2\|$ of case 1 (red), case 2 (blue), and case 3 (green).

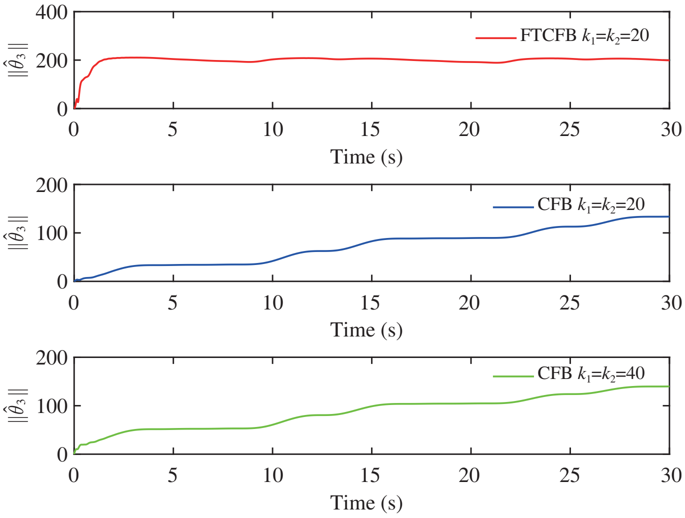

Figure 4 (Color online) The parameter update law $\|\hat{\theta}_3\|$ of case 1 (red), case 2 (blue), and case 3 (green).

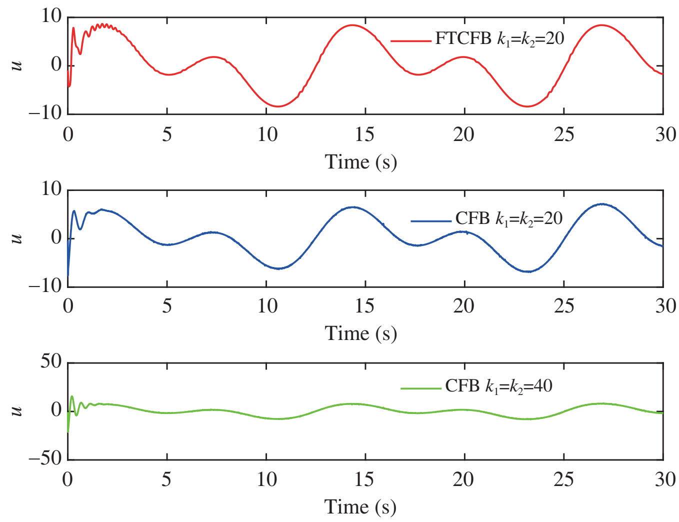

Figure 5 (Color online) The control input signal u of case 1 (red), case 2 (blue), and case 3 (green).

Table 1 Performance comparison for cases 1–3

Performance index	Case 1	Case 2	Case 3
$\sum_{k=1}^{N} \lvert u(k)\rvert$	12317	9358.3	11698
$\sum_{k=1}^{N} \lvert y(k) - y_d(k)\rvert$	85.32	382.39	163.59

variance methodology and a fractional power error compensation mechanism into the command-filtered backstepping design procedure, the proposed scheme preserves a finite-time trait, and avoids the "explosion of complexity" problem, and removes the requirement of the certainty equivalence principle in existing frameworks. The developed controller guarantees that all signals in the closed-loop system have finite-time stability, and the output tracking error is regulated to a small neighborhood around zero in finite time. Based on the present results, we will develop an observer-based finite-time command-filtered adaptive control and a fixed-time command-filtered adaptive control for nonlinear systems in future work.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61973179, U1813201, 61973131, 61703059), Japan Society for the Promotion of Science (Grant No. C-18K04212), Taishan Scholar Special Project Fund (Grant No. TSQN20161026), and Natural Science Foundation of Jiangsu Province (Grant No. BK20170291).

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