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• **Supplementary File** •

Ergodic Rate Analysis for Full-duplex NOMA Networks with Energy Harvesting

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Appendix A Proof of Proposition 1

In this section, the downlink ergodic data rate for the FD-SWIPT based transmission model is given by

$$R^{DL} = \sum_{i \in \{1,2\}} R_i^{DL}. \tag{A1}$$

where R_i^{DL} represents the downlink ergodic data rate of U_i.

By applying [1, Eq.4.337.2], we can express R_1^{DL} as

$$\begin{aligned} R_1^{DL} &= \frac{1}{\ln 2}\int_0^{+\infty} [\ln\left(1+\left(\varrho_1\lambda_1+\rho_{1,1}\right)x\right) \\ &\quad -\ln\left(1+\varrho_1\lambda_1 x\right)] e^{-x} dx \\ &= \frac{1}{\ln 2}\left[\exp\left(\frac{1}{\varrho_1\lambda_1+\rho_{1,1}}\right)E_1\left(\frac{1}{\varrho_1\lambda_1+\rho_{1,1}}\right)\right. \\ &\quad \left. -\exp\left(\frac{1}{\varrho_1\lambda_1}\right)E_1\left(\frac{1}{\varrho_1\lambda_1}\right)\right]. \end{aligned} \tag{A2}$$

Likewise, R_2^{DL} is given by

$$\begin{aligned} R_2^{DL} &= \frac{1}{\ln 2}\left[\exp\left(\frac{1}{\rho_{1,2}+\varrho_2\lambda_2+\rho_{2,2}}\right)\right. \\ &\quad \times E_1\left(\frac{1}{\rho_{1,2}+\varrho_2\lambda_2+\rho_{2,2}}\right) \\ &\quad \left. -\exp\left(\frac{1}{\rho_{1,2}+\varrho_2\lambda_2}\right)E_1\left(\frac{1}{\rho_{1,2}+\varrho_2\lambda_2}\right)\right], \end{aligned} \tag{A3}$$

where $E_1(x) = \int_x^{+\infty} \frac{e^{-t}}{t} dt = -\mathtt{Ei}(-x)$, in which $\mathtt{Ei}(\cdot)$ denotes the exponential integral function [1, Eq.8.211.1].

Furthermore, by substituting (A2) and (A3) into (A1), we can complete the proof of Proposition 1.

Appendix B Proof of Proposition 2

In this section, the uplink ergodic data rate for the FD-SWIPT based transmission model is given by

$$R^{UP} = \sum_{i \in \{1,2\}} R_i^{UP}. \tag{B1}$$

where R_i^{UP} represents the uplink ergodic data rate of U_i.

The cdf of $\gamma_{S,x_1'}^{UP}$ can be calculated as

$$\begin{aligned} F_{\gamma_{S,x_1'}^{UP}}(x) &= \Pr\left(\frac{\vartheta_1|h_{S,1}|^4}{\vartheta_2|h_{S,2}|^4+1} < x\right) \\ &= \int_0^{+\infty} \Pr\{|h_{S,1}|^2 < \sqrt{x\left(\vartheta_2 y^2+1\right)/\vartheta_1}\} f_{|h_{S,2}|^2}(y)dy \\ &= 1-\underbrace{\int_0^{+\infty} e^{-\sqrt{x\left(\vartheta_2 y^2+1\right)/\vartheta_1}-y} dy}_{\Xi}. \end{aligned} \tag{B2}$$

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By applying Gaussian cheybshev quadrature, Ξ can be expressed as

$$\begin{aligned}\Xi &= \int_0^{\pi/2} \frac{1}{\cos^2(\theta)} e^{-\sqrt{x\left(\vartheta_2 \tan^2(\theta)+1\right)/\vartheta_1}-\tan(\theta)} d\theta \\ &= \frac{1}{2}\int_{-1}^{1} \frac{1}{\sqrt{1-z^2}} \frac{1}{1-z^2} e^{-\sqrt{x\left(\frac{\vartheta_2 z^2}{1-z^2}+1\right)/\vartheta_1}-\frac{|z|}{\sqrt{1-z^2}}} dz \\ &\approx \frac{\pi}{2K}\sum_{k=1}^{K} a_k e^{-b_k\sqrt{x}},\end{aligned} \tag{B3}$$

where $a_k = \frac{1}{1-z_k^2} e^{-\frac{|z_k|}{\sqrt{1-z_k^2}}}$, $b_k = \sqrt{\left(\frac{\vartheta_2 z_k^2}{1-z_k^2}+1\right)/\vartheta_1}$, and $z_k = \cos\frac{(2k-1)\pi}{2K}$, in which K denotes the cheybshev approximation parameter.

By taking derivative to (B2) with respect to x, the pdf of $\gamma_{S,x_1'}^{UP}$ can be derived as

$$f_{\gamma_{S,x_1'}^{UP}}(x) = \frac{\pi}{4K}\sum_{k=1}^{K}\frac{a_k b_k}{\sqrt{x}} e^{-\sqrt{x}b_k}. \tag{B4}$$

By applying [1, Eq.4.338.1], we can express R_1^{UP} and R_2^{UP} as

$$\begin{aligned}R_1^{UP} &= \int_0^{+\infty} \log_2(1+x) f_{\gamma_{S,x_1'}^{UP}}(x)\, dx \\ &= \frac{\pi}{2K\ln 2}\sum_{k=1}^{K} a_k b_k \int_0^{+\infty} \ln\left(1+t^2\right) e^{-b_k t} dt \\ &= \frac{\pi}{K\ln 2}\sum_{k=1}^{K} a_k \left[-\sin(b_k)\,\mathtt{si}(b_k) - \cos(b_k)\,\mathtt{ci}(b_k)\right]\end{aligned} \tag{B5}$$

and

$$\begin{aligned}R_2^{UP} &= \int_0^{+\infty} \log_2\left(1+\vartheta_2 x^2\right) e^{-x} dx \\ &= \frac{2}{\ln 2}\left[-\sin\left(\frac{1}{\sqrt{\vartheta_2}}\right)\mathtt{si}\left(\frac{1}{\sqrt{\vartheta_2}}\right)\right. \\ &\quad \left.-\cos\left(\frac{1}{\sqrt{\vartheta_2}}\right)\mathtt{ci}\left(\frac{1}{\sqrt{\vartheta_2}}\right)\right],\end{aligned} \tag{B6}$$

respectively.

Finally, by substituting (B5) and (B6) into (B1), we can complete the proof of Proposition 2.

References

1 Gradshteyn I S, Ryzhik I M. Table of Integrals, Series, and Products. 7th ed. New York, NY, USA: Academic, 2007