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Exponential stabilization of memristor-based neural networks with unbounded time-varying delays

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Dear editor,

As a consequence of symmetry arguments, the memristor was predicted by Chua [1]. As the fourth basic circuit element, its memory characteristic and nanometer dimensions are devoid of resistors, capacitors, and inductors. In the field of the dynamical behavior analysis for memristive neural networks (MNNs), information exchange and signal transmission among different neurons are time-varying activities and discrete time delays are frequently supposed to be bounded, which implies that the current state of a neuron depend only on a part of its history. Actually, the current behavior of a neuron depends upon all its historical information. Consequently, discrete time delays in MNNs should be assumed to be time-varying and unbounded, which can exhibit the characteristics of the neurons in human brains. Many outstanding achievements on MNNs have already been investigated, but the discrete time delays of the investigated MNNs were all assumed to be bounded [2–4].

In this study, pth moment exponential stabilization for a class of MNNs with unbounded discrete time-varying delays under a designed controller is investigated. With the help of inequality techniques and theories of exponential stabilization, a sufficient condition is presented to ensure the stabilization of MNNs. A numerical example is given to illustrate the effectiveness of the theoretical results.

Preliminaries. Consider a class of MNNs with discrete time-varying delays described by

$$\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -a_i y_i(t) + \sum_{j=1}^{n} b_{ij}(y_i(t)) f_j(y_j(t)) + \sum_{j=1}^{n} c_{ij}(y_i(t)) g_j(y_j(t-d_j(t))), \quad (1)$$

where $t \geqslant 0, i = 1,2,\ldots,n$, $y_i(t)$ is the state variable of the ith neuron, $a_i > 0$ denotes the self-feedback coefficient, and $d_j(t) > 0$ is a discrete time-varying delay that is unbounded and differentiable. Additionally, there exists a real value d such that $\dot{d}_j(t) \leqslant d < 1$, and $f_j(\cdot)$ and $g_j(\cdot)$ correspond to activation functions without and with time delays, respectively. Measurable functions $b_{ij}(y_i(t))$ and $c_{ij}(y_i(t))$ are connection weight coefficient and delayed connection weight coefficient, respectively.

$$b_{ij}(y_i(t)) = \begin{cases} \hat{b}_{ij}, & \text{if } |y_i(t)| \leqslant \Gamma_i, \\ \check{b}_{ij}, & \text{if } |y_i(t)| > \Gamma_i, \end{cases}$$

$$c_{ij}(y_i(t)) = \begin{cases} \hat{c}_{ij}, & \text{if } |y_i(t)| \leqslant \Gamma_i, \\ \check{c}_{ij}, & \text{if } |y_i(t)| > \Gamma_i, \end{cases}$$

where $\hat{b}_{ij}$, $\check{b}_{ij}$, $\hat{c}_{ij}$, and $\check{c}_{ij}$ are real values, and switching jump $\Gamma_i > 0$. For details about MNN models, please refer to [5].

The initial value condition in MNN (1) is $y_i(s)=\psi_i(s) \in \mathcal{C}((-\infty,0];\mathbb{R})$, $i = 1,2,\ldots,n$. By applying theories of set-valued maps and differential inclusions in the sense of Filippov, we can obtain that $\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} \in -a_i y_i(t) + \sum_{j=1}^{n} \mathrm{co}[b_{ij}(y_i(t))] f_j(y_j(t)) + \sum_{j=1}^{n} \mathrm{co}[c_{ij}(y_i(t))] g_j(y_j(t - d_j(t)))$ for almost everywhere (a.e.) $t \geqslant 0, i = 1,2,\ldots,n$, where

$$\mathrm{co}\big[b_{ij}(y_i(t))\big] = \begin{cases} \hat{b}_{ij}, & \text{if } |y_i(t)| < \Gamma_i, \\ [\underline{b}_{ij}, \overline{b}_{ij}], & \text{if } |y_i(t)| = \Gamma_i, \\ \check{b}_{ij}, & \text{if } |y_i(t)| > \Gamma_i, \end{cases}$$

$$\mathrm{co}\big[c_{ij}(y_i(t))\big] = \begin{cases} \hat{c}_{ij}, & \text{if } |y_i(t)| < \Gamma_i, \\ [\underline{c}_{ij}, \overline{c}_{ij}], & \text{if } |y_i(t)| = \Gamma_i, \\ \check{c}_{ij}, & \text{if } |y_i(t)| > \Gamma_i, \end{cases}$$

with $\overline{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\overline{c}_{ij} = \max\{\hat{c}_{ij}, \check{c}_{ij}\}$, and $\underline{c}_{ij} = \min\{\hat{c}_{ij}, \check{c}_{ij}\}$ for $i,j = 1,2,\ldots,n$. Equivalently, there exist $\tilde{b}_{ij} \in \mathrm{co}\big[b_{ij}(y_i(t))\big]$ and $\tilde{c}_{ij} \in \mathrm{co}\big[c_{ij}(y_i(t))\big]$, $i,j = 1,2,\ldots,n$ such that for a.e. $t \geqslant 0, i = 1,2,\ldots,n$,

$$\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -a_i y_i(t) + \sum_{j=1}^{n} \tilde{b}_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} \tilde{c}_{ij} g_j(y_j(t-d_j(t))). \quad (2)$$

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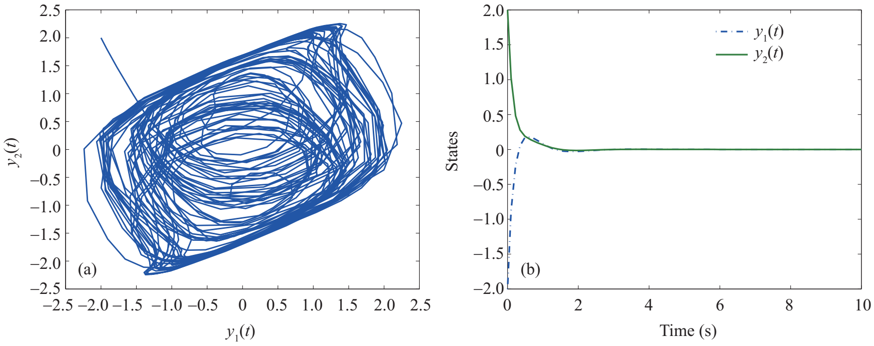

Figure 1 (Color online) Phase plot of MNN (10); State curves of MNN (10).

To present the main results of this study, an assumption and a definition are first given.

Assumption 1. The activation functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 1, 2, \ldots, n$ in MNN (2) are bounded and there exist positive real constants F_i, G_i, δ such that for any $y_j \in \mathbb{R}$, $j = 1, 2, 3, 4$, $|f_i(y_1) - f_i(y_2)| \leqslant F_i|y_1 - y_2|$, $|g_i(y_3) - g_i(y_4)| \leqslant G_i|y_3 - y_4|\mathrm{e}^{-\delta d_i(t)}$. Additionally, assume $f_i(0) = g_i(0) = 0$, $i \in \mathbb{N}$.

Definition 1. MNN (2) is pth moment exponentially stable if there exist real values $\Upsilon > 0$ and $\epsilon > 0$ such that

$$|y(t)|^p \leqslant \Upsilon \sup_{-\infty < s \leqslant t_0} |\Psi(s)|^p \mathrm{e}^{-\epsilon(t-t_0)}, \quad t \geqslant t_0,$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ and $\Psi(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_n(s)]^{\mathrm{T}}$.

A suitable controller should be designed to stabilize MNN (2). To do this, the following controlled MNN is introduced:

$$\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -a_i y_i(t) + \sum_{j=1}^{n} \tilde{b}_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} \tilde{c}_{ij} g_j(y_j(t - d_j(t))) + u_i(t), \tag{3}$$

in which $t \geqslant 0, i = 1, 2, \ldots, n$, $u_i(t)$ is the designed controller described by

$$u_i(t) = \sum_{j=1}^{n} K_{ij} y_j(t), \tag{4}$$

where K_{ij} is a real constant and $K_{ii} < 0$. The aim of this study is to design controller (4) such that controlled MNN (3) is pth moment exponentially stable.

Main results. In the following, $b^*_{ij} = \max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\}$, $c^*_{ij} = \max\{|\hat{c}_{ij}|, |\check{c}_{ij}|\}$, and $d_0 = \max_{1 \leqslant i \leqslant n} d_i(0)$ is a finite real value.

Theorem 1. Given positive real constants $p \geqslant 2$, F_i, G_i, K_{ij}, d, d_0, and δ, $i, j = 1, 2, \ldots, n$, under Assumption 1, MNN (3) is pth moment exponentially stabilizable via the designed controller (4) if there exist real values $m_i > 0$ such that

$$\frac{1}{1-d} \sum_{j=1}^{n} m_j c^*_{ji} G_i - \Lambda_{1i} < 0, \quad i \in \mathbb{N}, \tag{5}$$

where $\Lambda_{1i} = pm_i a_i - pm_i K_{ii} - \sum_{j=1}^{n} (p-1) m_i b^*_{ij} F_j - \sum_{j=1}^{n} m_j b^*_{ji} F_i - \sum_{j=1}^{n} (p-1) m_i c^*_{ij} G_j - \sum_{j=1, j \neq i}^{n} (p-1) m_i |K_{ij}| - \sum_{j=1, j \neq i}^{n} m_j |K_{ji}|$.

Proof. Construct the following function:

$$V(t) = \sum_{i=1}^{n} m_i |y_i(t)|^p. \tag{6}$$

It follows from (3) and Assumption 1 that

$$\begin{aligned} \dot{V}(t) \leqslant & -\sum_{i=1}^{n} pm_i a_i |y_i(t)|^p \\ & + \sum_{i=1}^{n} \sum_{j=1}^{n} pm_i b^*_{ij} F_j |y_i(t)|^{p-1} |y_j(t)| \\ & + \sum_{i=1}^{n} \sum_{j=1}^{n} pm_i c^*_{ij} G_j |y_i(t)|^{p-1} |y_j(t - d_j(t))| \mathrm{e}^{-\delta d_j(t)} \\ & + \sum_{i=1}^{n} pm_i |y_i(t)|^{p-2} y_i(t) \sum_{j=1}^{n} K_{ij} y_j(t). \end{aligned} \tag{7}$$

Applying Young's inequality to the cross-product terms in (7) yields

$$\begin{aligned} \dot{V}(t) \leqslant & -\sum_{i=1}^{n} \Lambda_{1i} |y_i(t)|^p \\ & + \sum_{i=1}^{n} \Lambda_{2i} |y_j(t - d_j(t))|^p \mathrm{e}^{-\delta d_j(t)}, \end{aligned} \tag{8}$$

where $\Lambda_{2i} = \sum_{j=1}^{n} m_i c^*_{ij} G_j$.

Choosing a real constant $\varepsilon \in (0, \delta)$ to be determined later, then $\mathrm{e}^{\varepsilon t} V(t) = V(0) + \int_0^t [\varepsilon \mathrm{e}^{\varepsilon s} V(s) + \mathrm{e}^{\varepsilon s} \dot{V}(s)] \mathrm{d}s \leqslant V(0) + \sum_{i=1}^{n} \int_0^t \mathrm{e}^{\varepsilon s} [\Lambda_{2i} |y_j(s - d_j(s))|^p \mathrm{e}^{-\delta d_j(s)} - \frac{1}{1-d} \Lambda_{2i} |y_j(s)|^p + \Lambda_{3i} |y_i(s)|^p + \varepsilon m_i |y_i(s)|^p] \mathrm{d}s$ in which $\Lambda_{3i} = \frac{1}{1-d} \sum_{j=1}^{n} m_j c^*_{ji} G_i - \Lambda_{1i} < 0$, $i = 1, 2, \ldots, n$. Combining with (5), there surely exist positive constants ε_i such that $\Lambda_{3i} + \varepsilon_i m_i = 0$. Choosing $\varepsilon = \min_{1 \leqslant i \leqslant n} \{\varepsilon_i\}$, then $\Lambda_{3i} + \varepsilon m_i \leqslant 0$, $i = 1, 2, \ldots, n$. Therefore,

$$\mathrm{e}^{\varepsilon t} V(t) \leqslant V(0) + \sum_{i=1}^{n} \int_{-d_0}^{0} \frac{\mathrm{e}^{\varepsilon s} \Lambda_{2i}}{1-d} |y_j(s)|^p \mathrm{d}s. \tag{9}$$

In light of (6) and (9), there surely exists a real constant $\Upsilon > 0$ such that $|y(t)|^p \leqslant \Upsilon \sup_{-\infty < s \leqslant 0} |\Psi(s)|^p \mathrm{e}^{-\varepsilon t}$, $t > 0$, which implies that MNN (3) is pth moment exponentially stabilizable according to Definition 1.

Remark 1. The developed theoretical results hold for bounded and unbounded discrete time delays. Hence, our result enhances those reported in [2, 3].

Remark 2. Because $d_j(t)$ in the second term of (8) is unbounded, the general methods (such as Halanay inequality) are invalid.

Numerical example.

Example 1. Consider MNN (3) with $i = 1, 2$ and parameters as follows:

$$
\begin{aligned}
&a_1 = 1.5,\ a_2 = 1.5,\\
&b_{11}\left(y_1(t)\right) = \begin{cases} 2.5, & |y_1(t)| \leqslant 1,\\ 2.4, & |y_1(t)| > 1,\end{cases}\\
&b_{12}\left(y_2(t)\right) = \begin{cases} -2.1, & |y_1(t)| \leqslant 1,\\ -2.3, & |y_1(t)| > 1,\end{cases}\\
&b_{21}\left(y_1(t)\right) = \begin{cases} 1, & |y_2(t)| \leqslant 1,\\ 1.1, & |y_2(t)| > 1,\end{cases}\\
&b_{22}\left(y_2(t)\right) = \begin{cases} 2.8, & |y_2(t)| \leqslant 1,\\ 3, & |y_2(t)| > 1,\end{cases}\\
&c_{11}\left(y_1(t)\right) = \begin{cases} 1.5, & |y_1(t)| \leqslant 1,\\ 1.4, & |y_1(t)| > 1,\end{cases}\\
&c_{12}\left(y_2(t)\right) = \begin{cases} 1.7, & |y_1(t)| \leqslant 1,\\ 1.8, & |y_1(t)| > 1,\end{cases}\\
&c_{21}\left(y_1(t)\right) = \begin{cases} -0.2, & |y_2(t)| \leqslant 1,\\ -0.15, & |y_2(t)| > 1,\end{cases}\\
&c_{22}\left(y_2(t)\right) = \begin{cases} -0.7, & |y_2(t)| \leqslant 1,\\ -0.6, & |y_2(t)| > 1.\end{cases}
\end{aligned} \tag{10}
$$

For $i = 1, 2$, taking $f_i(y_i) = \tanh(y_i)$ and $g_i(y_i) = \tanh(y_i)\mathrm{e}^{-d_i(t)}$, then $F_i = G_i = 1$. By computing, $b^*_{11} = 2.5, b^*_{12} = 2.3, b^*_{21} = 1.1, b^*_{22} = 3, c^*_{11} = 1.5, c^*_{12} = 1.8, c^*_{21} = 0.2, c^*_{22} = 0.7$. Let $p = 2, m_1 = m_2 = 1$ and $d_i(t) = 0.5t + 0.3\sin t$, and then $\dot{d}_i(t) \leqslant 0.8 < 1$. Because the delays are unbounded, the methods in [2–4] are invalid. MNN (10) is unstable with the initial condition $[-2\ 2]^{\mathrm{T}}$. Figure 1(a) shows the phase plot of model (10). Let controller parameters $K_{11} = -10,\ K_{22} = -7$, and $K_{12} = K_{21} = 0$, and then the conditions in Theorem 1 are satisfied. Now, MNN (10) is pth moment exponentially stabilizable according to Theorem 1. The state trajectories are shown in Figure 1(b).

Conclusion. The exponential stabilization of MNNs with unbounded time delays was investigated. A new algebraic criterion with pth moment exponential stabilization was obtained using Lyapunov stability theory and inequality techniques.

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