SCIENCE CHINA
Information Sciences

• LETTER •

August 2021, Vol. 64 189203:1–189203:3
https://doi.org/10.1007/s11432-018-9804-9

A joint order-replacement policy for deteriorating components with reliability constraint

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Received 23 October 2018/Revised 18 December 2018/Accepted 26 February 2019/Published online 2 June 2020

Citation Si X S, Hu C H, Li T M, et al. A joint order-replacement policy for deteriorating components with reliability constraint. Sci China Inf Sci, 2021, 64(8): 189203, https://doi.org/10.1007/s11432-018-9804-9

Dear editor,

Condition-based maintenance (CBM) is a popular maintenance program for degrading components and can recommend maintenance decisions based on the condition monitoring (CM) information [1]. In CBM applications, a widely adopted assumption is that spare parts are always available or can be delivered without any lead time. However, the lead time for ordering spare parts requires that they should be ordered in advance of the occurrence of system failure or maintenance. Therefore, it is well recognized that the inventory provisioning and maintenance scheduling should be jointly concerned. Some studies have been conducted to optimize the ordering and replacement policy [2, 3]. However, these studies are basically performed based on the time-to-failure data, ignoring the real-time CM information.

In the CBM framework, the CM data can provide real-time information of the component's health and thus are usually used in predicting the remaining useful life (RUL) via degradation modeling. Once the prognostic information is available, the relevant logistics or maintenance activities can be jointly scheduled. Recent advances in this study can be found in [4–6]. However, most of the associated degradation models are linear and the degradation signals are error-free. Additionally, for some critical components in vital systems, the relevant costs (e.g., CM costs, ordering costs, replacement costs, and spare related holding costs), are not the only considerations during the order-replacement policy. This is particularly true for the system performing certain important missions. In these cases, it is noted that the critical component reliability must satisfy the requirement of specific missions. If the component has low reliability and fails during the mission phase, the associated results may be severe or unacceptable. Therefore, it is necessary to consider not only the costs but also the reliability constraint during the order-replacement policy.

In this study, we propose a joint order-replacement method based on prognostic information for partially observed degrading components with reliability constraint.

Degradation mode and RUL distribution. In this study, we consider the following degradation model with the state-space description:

$$\begin{cases} X(t) = \int_0^t \mu(t;\theta)\mathrm{d}t + \sigma_B B(t), \\ Y(t) = X(t) + \varepsilon(t), \end{cases} \tag{1}$$

where $X(t)$ denotes the degradation state of a degrading component at time t, $B(t)$ is a standard Brownian motion, and $Y(t)$ is the measurement of the degradation state affected by the measurement error $\varepsilon(t)$, with $\varepsilon(t) \sim N(0, \sigma_\varepsilon^2)$ (see [1] for more details). In addition, we use $\theta = (a, b)$ to denote a row parameter vector, where a is a random-effect parameter representing variability between different components, and b is a fixed effect that is common to all units. Further, we assume a follows a commonly used normal distribution, i.e., $a \sim N(\mu_a, \sigma_a)$, and independent with $B(t)$ and $\varepsilon(t)$. The model parameters in (1) can be initialized by the historical data using the maximum likelihood estimation method [7].

Then, RUL L_k at time t_k can be defined by $L_k = \inf\{l_k | X(t_k + l_k) \geqslant w | X(t_k) < w, t_k, l_k > 0\}$, where w denotes the failure threshold.

Here, we consider two examples to illustrate the general model (1). By letting $\mu(t;\theta) = a$ in Model M_1 and $\mu(t;\theta) = abt^{b-1}$ in Model M_2, a linear model and a power law model can be respectively obtained. The probability density functions (PDFs) of the RUL at t_k under M_1 and M_2 can be derived respectively as [7]

$$f_{M_1}(l_k | X_k) = \frac{w - X_k}{\sqrt{2\pi l_k^3(\sigma_a^2 l_k + \sigma_B^2)}} \cdot \exp\left[\frac{-(w - X_k - \mu_a l_k)^2}{2l_k(\sigma_a^2 l_k + \sigma_B^2)}\right], \tag{2}$$

$$f_{M_2}(l_k | X_k) = \frac{1}{\sqrt{2\pi l_k^2(\sigma_a^2 \eta(l_k)^2 + \sigma_B^2 l_k)}}$$

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$$\times \exp\left[-\frac{(w_k-\mu_a\eta(l_k))^2}{2(\sigma_a^2\eta(l_k)^2+\sigma_B^2 l_k)}\right] \times (w_k-[(\eta(l_k)-bl_k(l_k+t_k)^{b-1})(w_k\sigma_a^2\eta(l_k) + \mu_a\sigma_B^2 l_k)]/(\sigma_a^2\eta(l_k)^2+\sigma_B^2 l_k)), \quad (3)$$

where $\eta(l_k)=(l_k+t_k)^b-t_k^b, w_k=w-X_k$, and X_k is the degradation state at t_k represented as $X_k=X(t_k)$.

Parameters and hidden state estimation. To estimate the hidden degradation state of the component in service, a discrete state-space model is constructed as

$$X_k=X_{k-1}+g(t_k;\theta)-g(t_{k-1};\theta)+W_k,\quad Y_k=X_k+\varepsilon_k,$$

where $g(t_k;\theta)$ denotes $\int_0^{t_k}\mu(t;\theta)\mathrm{d}t$, $X_k=X(t_k)$, and $Y_k=Y(t_k)$. In addition, $W_k\sim N(0,\sigma_B^2\cdot\Delta t_k)$ and $\varepsilon_k\sim N(0,\sigma_\varepsilon^2)$.

To update the model parameters, we consider a random walk model for θ, i.e., $\theta_k=\theta_{k-1}+W_{\theta,k-1}$, where $W_{\theta,k-1}$ is assumed to be random noise, following a multivariate normal distribution $\mathrm{MVN}(0,\Sigma_\theta)$. As such, the new state vector can be written as $X^\Delta=[X,\theta]^{\mathrm{T}}$. Then we get the following discrete state-space model:

$$\begin{aligned} &X_k^\Delta=h(X_{k-1}^\Delta,W_{k-1},W_{\theta,k-1}),\\ &Y_k=HX_k^\Delta+\varepsilon_k, \end{aligned}\quad (4)$$

where $h(\cdot)$ is the function of the hidden state and the unknown parameters, and $H=[1\ 0\ 0]$. For M_1 and M_2, the model (4) can be respectively specified as

$$M_1:\ \begin{aligned} &X_k^\Delta=AX_{k-1}^\Delta+W_{k-1}^\Delta,\\ &Y_k=HX_k^\Delta+\varepsilon_k, \end{aligned}\quad (5)$$

where $A=\left[\begin{smallmatrix}1 & \Delta t_k\\ 0 & 1\end{smallmatrix}\right]$, $H=[\,1\ 0\,]$, $X_k^\Delta=[\,X_k\ \ a_k\,]^{\mathrm{T}}$, and $W_{k-1}^\Delta=[\,W_{k-1}\ \ W_{\theta,k-1}\,]^{\mathrm{T}}$, and

$$M_2:\ \begin{aligned} &X_k=X_{k-1}+a_k t_k^{b_k}-a_{k-1}t_{k-1}^{b_{k-1}}+W_{k-1},\\ &\begin{bmatrix}a_k\\ b_k\end{bmatrix}=\begin{bmatrix}a_{k-1}\\ b_{k-1}\end{bmatrix}+W_{\theta k},\\ &Y_k=HX_k^\Delta+\varepsilon_k, \end{aligned}\quad (6)$$

where $X_k^\Delta=[\,X_k\ \ a_k\ \ b_k\,]^{\mathrm{T}}$ and $H=[\,1\ 0\ 0\,]$.

Based on the degradation measurements up to t_k, denoted as $Y_{1:k}$, are available at time t_k, we can compute the estimated state $X_k^\Delta=[X_k,\theta_k]^{\mathrm{T}}$ using the filtering method. In existing filtering methods, the unscented Kalman filter (UKF) has been shown generally better performing than other filters in nonlinear estimation problems. Thus, we use the UKF in this study and the detailed implementation can be found in [8].

Based on the filtering methods and the observations $Y_{1:k}$, the posterior estimate of the hidden state X_k^Δ can be approximated as $X_k^\Delta|Y_{1:k}\sim N(\hat{X}_k^\Delta,\hat{P}_k)$, where $\hat{X}_k^\Delta$ and $\hat{P}_k$ are the point estimate and the associated covariance of the estimate, respectively. The real-time predicted PDFs of the RUL under M_1 and M_2 can be formulated as

$$f_{M_i}(l_k|Y_{1:k})=f_{M_i}(l_k|\hat{X}_k^\Delta),\quad (7)$$

where $f_{M_i}(l_k|\hat{X}_k^\Delta)$ can be obtained by replacing the model parameters and X_k in (2) and (3) with the estimated $\hat{X}_k^\Delta$ for the model M_i, $i=1,2$.

Joint order-replacement policy. In this joint policy, the decision variables are the ordering time t_o of a spare part and the preventive replacement time t_p. To take the reliability into consideration when constructing the joint order-replacement policy, a preventive replacement is carried out when reliability reaches the preset threshold R_c, i.e., $R(t\geqslant t_p|t_k,Y_{1:k})\leqslant R_c$, where $R(t\geqslant t_p|t_k,Y_{1:k})$ can be formulated as $R(t\geqslant t_p|t_k,Y_{1:k})=1-\int_0^{t_p}f(t_k+l_k|Y_{1:k})\mathrm{d}l_k$.

Then, the joint decision-making problem at time t_k can be formulated based on the renewal-reward theory as

$$\begin{aligned} &\min\quad \mathrm{E}[C(t_o,t_p)]=\frac{\mathrm{E}U}{\mathrm{E}V},\\ &\mathrm{s.t.}\quad R(t\geqslant t_p|t_k,Y_{1:k})\leqslant R_c,\ t_o\geqslant t_k, \end{aligned}\quad (8)$$

$$\begin{aligned} \frac{\mathrm{E}U}{\mathrm{E}V}=&\left[C_p\left(1-\int_0^{t_p}f(l_k|Y_{1:k})\mathrm{d}l_k\right)\right.\\ &\left.+C_f\int_0^{t_p}f(l_k|Y_{1:k})\mathrm{d}l_k+kC_m+C_o\right]\\ &\Big/\left\{t_k+\int_{t_o}^{t_o+L}F(l_k|Y_{1:k})\mathrm{d}l_k\right.\\ &\left.+\int_0^{t_p}[1-F(l_k|Y_{1:k})]\,\mathrm{d}l_k\right\}\\ &+\left\{C_s\int_{t_o}^{t_o+L}F(l_k|Y_{1:k})\,\mathrm{d}l_k\right.\\ &\left.+C_h\int_{t_o+L}^{t_p}[1-F(l_k|Y_{1:k})]\mathrm{d}l_k\right\}\Big/\left\{t_k\right.\\ &+\int_{t_o}^{t_o+L}F(l_k|Y_{1:k})\mathrm{d}l_k\\ &\left.+\int_0^{t_p}[1-F(l_k|Y_{1:k})]\,\mathrm{d}l_k\right\}, \end{aligned}\quad (9)$$

where $\mathrm{E}[C(t_o,t_p)]$ is the expected cost rate, C_m,C_p,C_f,C_h,C_S, and L are the fixed monitoring cost, preventive replacement cost, failure replacement cost, holding cost per unit time, the shortage cost per unit time, and the lead time, respectively. In addition, there is $C_m<C_p<C_f$.

Optimization procedure.

Step 1. When new observation Y_k comes, we use state space model (4) and the observations $Y_{1:k}$ to update the parameters and estimate the hidden state X_k^Δ.

Step 2. The real-time RUL distribution is updated by X_k^Δ through (7).

Step 3. Based on the predicted RUL, the decisions can be made by solving (8).

Step 4. If the optimized ordering time is less than the next CM time, stop and schedule the maintenance activities based on the optimized t_o and t_p. Otherwise, do nothing but repeat Steps 1–4 until the next observation comes.

Case study. A case study for the inertial platform is provided for demonstration. The data description and the model settings can be found in Appendix A. To show the updating results of our proposed order-replacement policy, six CM points are selected as examples and the reliability threshold is set as $R_c=0.95$. Table B1 shows that the predicted ordering time is 100.3 h at time t_k =100.0 h, which is smaller than the next CM time of 102.5 h. Therefore, the decision-making procedure is stopped at time t_k=100.0 h. The corresponding optimal replacement time is t_k=160 h, which is smaller but close to the actual failure time. These results indicate that the decision results can be dynamically updated at each CM time when new degradation data are available. In addition, the optimal replacement times are all near the failure time, which implies that the presented policy can make full use of the component's residual life. The decision results at t_k=87.5 h (35th) and t_k=100.0 h (40th) are illustrated in Figure B1. Figure B1(a) shows that the reliability is always higher than 95% before optimal replacement time t_p, indicating that the component is highly reliable until replacement. Comparative studies with the methods in [2, 6] are summarized in Table C1, and indicate the significant cost savings of the proposed method. Thus, the effectiveness of the developed method is verified.

Conclusion. This study proposes a joint order-replacement policy for components subjected to the gradual degradation and reliability constraint. The main advantage is that the cost and the reliability are jointly considered in the optimization objective. Results in the case study validate that, by using the proposed method, the component is highly reliable until replacement and the cost savings are significant compared with other methods without reliability constraint.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61773386, 61573366), and National Key R&D Program of China (Grant No. 2018YFB1306100).

Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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