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• **Supplementary File** •

A Joint Order-Replacement Policy for Deteriorating Components with Reliability Constraint

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Appendix A Data description and model settings

Inertial platform is an important component in the inertial navigated system (INS) for space and weapon equipment. As a key role in tracking the orientation and position of the navigated systems, the inertial platform is very expensive and if there are no available spare parts when failed, the costs could be extremely high. At the same time, the INS may incur high risk. Therefore, the inertial platform pertains to the critical component concerned in this paper. Sensors fixed in an inertial platform include three gyroscopes and three accelerometers measuring angular velocity and linear acceleration, respectively. When the inertial platform is operating, the wheels of the gyroscopes rotate at very high speeds, leading to the wear of rotation axis. As the wear accumulated, the drift increases and when it reaches a threshold, the failure of gyroscope occurs. As such, we can use the gyroscopic drift to assess the health condition of an inertial platform. The drift of an inertial platform mainly includes $K_{0X}, K_{0Y}, K_{0Z}, K_{SX}, K_{SY}, K_{IZ}$, among which K_{0X}, K_{0Y}, K_{0Z} are constant drift, and K_{SX}, K_{SY}, K_{IZ} denote stochastic drift. Generally, the drift degradation measurements, K_{SX}, plays a dominant role in the health assessment of inertial platform degradation [14]. In this paper, we take the CM reading of K_{SX} as the degradation measurements and utilize it to predict the RUL of the inertial platform. The drift of the studied INS were tested under the condition that was similar to a field setting with the CM interval 2.5 hours. As illustrated in Fig. A1, 109 points of drift were obtained and the experiment terminated at about $t = 272$ hours.

The INS studied here is defined to be unstable and should be replaced when the gyroscopic drift exceeds 0.8 (degree/hour). The degradation data indicate that the drifts cross threshold at the 81th CM point, thus the INS failed at $t_k = 200$hours.

In this study, only the parameters of $\mu(t;\theta)$ in model (1) can be updated through the filtering methods such as the UKF, ignoring the diffusion parameter σ_B and variance of the measurement error σ_ε^2. To solve this problem, the MLE method is employed to get all the parameters in model (1) before the executing the filtering procedure. Specifically, degradation processes of other systems from a population can be obtained through CM technique. These degradation data will make additional contributions to the RUL prediction. Thus, we can utilize these data to estimate all the parameters by the MLE method. The estimated parameters of $\mu(t;\theta)$ can be treated as the prior input of the filtering model. In practice, the INS studied is very expensive and usually operates individually. Therefore, we consider the case that a single spare part is ordered. In addition, for illustration, we consider the model M_2 in (10) for this case study. The corresponding parameters in the cost function (14) are provided in Table A1.

Table A1 Parameters setting related to cost function

C_f	C_p	C_o	C_m	C_s	C_h	L
RMB ¥600000	RMB ¥400000	RMB ¥1000	RMB ¥500	RMB ¥10000/h	RMB ¥7/h	24 h

Appendix B Results and discussions

To show the updating results of our proposed order-replacement policy, six CM points are selected as examples. i.e. $t_k = 87.5h$, $90.0h$, $92.5h$, $95.0h$, $97.5h$, and $100.0h$, and the reliability threshold is set as $R_c = 0.95$. Note that, though an order for spare part has been placed at time $t_k = 100.3h$, the original inertial platform was still monitored to the end of its life to keep its functionality. The decision results of the chosen times are provided in Table B1, where $EC_{\min}$ is the optimal expected cost per unit time.

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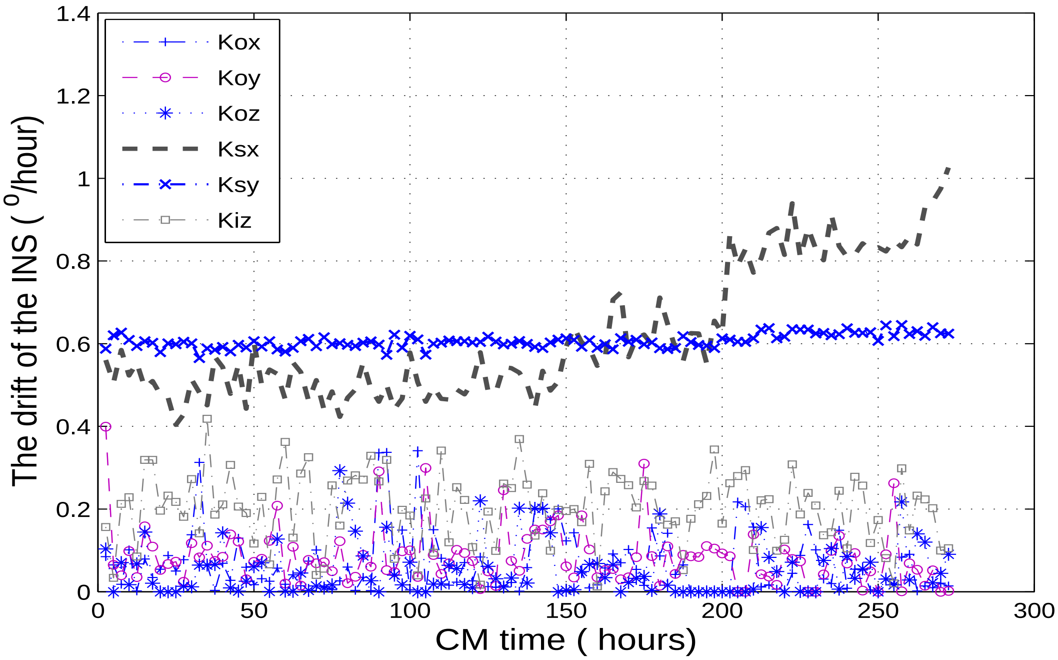


Figure A1 Drift of the platform in INS

Table B1 Optimal decision results

t_k(h)	87.5(35^{th})	90.0(36^{th})	92.5(37^{th})	95.0(38^{th})	97.5(39^{th})	100.0(40^{th})
t_o^*(h)	99.0	99.5	98.8	100.0	100.3	100.3
t_p^*(h)	170.0	182.5	172.5	195.0	187.5	160.0
$EC_{\min}$(RMB ¥/h)	2097.8	2165.4	2077.1	2221.6	2149.6	1967.6

Table B1 shows that the predicted ordering time is 100.3h at time $t_k = 100.0h$ (i.e., the 40th CM point), which is smaller than the next CM time of 102.5 hours (i.e., the 41th CM point). Therefore, the decision-making procedure is stopped at time $t_k = 100.0h$ h and accordingly an order for a spare part will be placed during this monitoring interval. The corresponding optimal replacement time is $t_k = 160h$, which is smaller but close to the actual failure time of the INS. Two further conclusions can be drawn from the Table II. First, the decision results, i.e., the ordering and replacement times can be dynamically updated at each CM time when new degradation data are available, which is an advantage making our decision policy different from the age-based decision policies. Second, the optimal replacement times made at the chosen CM points are all near the failure time. This indicates that the presented policy can make full use of the component's residual life. The decision results at times $t_k = 87.5h$(35th), and $100.0h$ (40th) are illustrated in Fig. B1. Conclusions similar to those from Table B1 can be drawn. In addition, Fig. B1 (a) shows that the reliability of the INS is always higher than 95% before optimal replacement time t_p, indicating that the component is highly reliable until replacement. For such critical component, this high reliability ensures the component's functionality during the mission phase and the success of the mission.

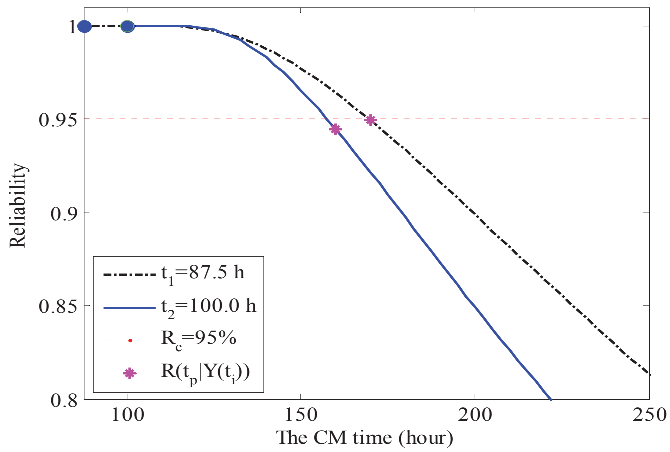
Appendix C Comparative results

In this part, The method in [1] and Louits approach in [2] are applied to this case study for comparisons to validate the superiority of the proposed decision-making policy. Notice that, in Louits approach, the system is replaced until the failure time, and the ordering time is optimized based on the time when the system reliability reaches the preset threshold. The comparative results are summarized in Table C1. The comparative results indicate that the proposed method can result in the significant cost savings in scheduling the spare part ordering and replacement. This demonstrates the advantage of the proposed method.

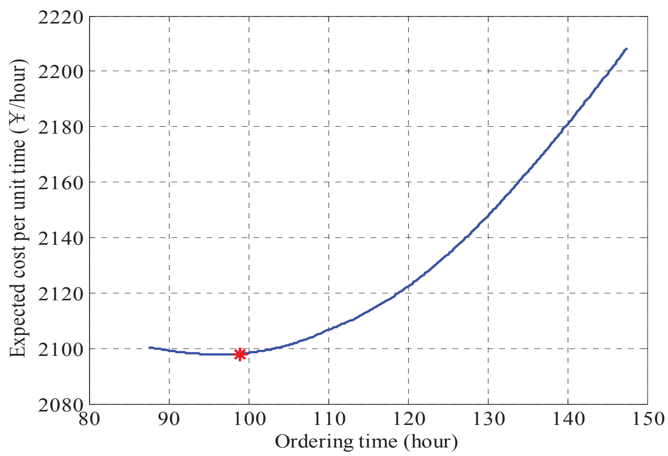
References

1 Wang Z Q, Wang W, Hu C H, et al. A prognostic-information-based order-replacement policy for a non-repairable critical system in service. IEEE Trans Rel, 2015, 64: 721-735.

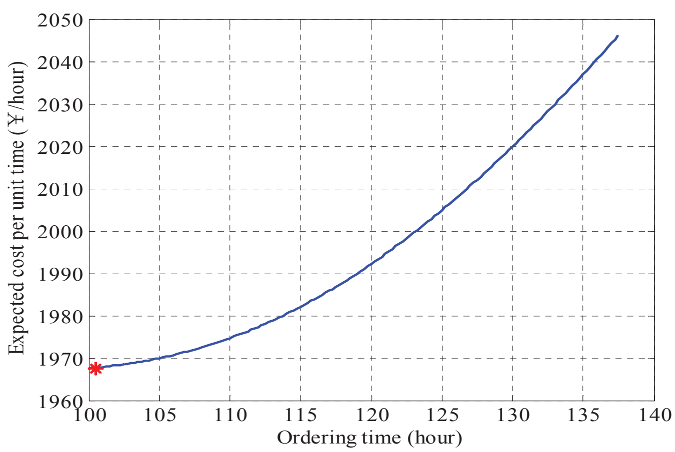
2 Louit D, Pascual R, Banjevic D, et al. Condition-based spares ordering for critical components. Mech Syst Signal Processing, 2011, 25: 1837-1848.



(a) Replacement results at two CM time



(b) Optimal ordering time at $t_k = 87.5h$



(c) Optimal ordering time at $t_k = 100.0h$

Figure B1 Illustration of the condition based order-replacement decisions at $t_k = 87.5h$(35th), and $t_k = 100h$ (40th)

Table C1 Comparative results

	t_o^*(h)	t_p^*(h)	$EC_{\min}$(RMB ¥/h)
Method in [1]	125.6	164.5	2306.8
Louts policy in [2]	165.0	Until failed	3034.8
Proposed method	100.3	160	1967.6