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A variable-period scheme for dynamic sampled-data stabilization

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Dear editor,

With the popularity of computers and networks in control systems, sampled-data control is becoming increasingly important and has attracted intensive attention over the past decades [1, 2]. However, for systems with serious uncertainties (without any known upper bounds), particularly additive uncertainties, it is rather challenging to achieve sampled-data control: the classical schemes with constant sampling rate usually fail to work, while schemes with variable sampling rate are complex and difficult to implement. We specially mention [3], wherein an adaptive adjustment mechanism was introduced to the variable sampling rate to handle the serious uncertainties. However, the scheme in [3] is inapplicable to cases with additive uncertainties/disturbances. Although studies in [4, 5] considered cases with additive disturbances, these authors provided only sampled-data control schemes without any compensation mechanism to additive disturbances. Thus, they established only certain quantitative relations between the system states and the additive disturbances (e.g., input-to-state stability in [4] and $\mathcal{L}_p$ in [5]). In a separate development, [6] proposed an event-triggered scheme of output regulation for single-input single-output linear systems.

This study is devoted to discovering a dynamic sampled-data scheme for a continuous-time system subjected to additive disturbance. In particular, we address additive disturbance generated from an exosystem that is neutrally stable with unknown initial value. This means that the disturbance is nonvanishing and has a significant and persistent effect on the system; this scenario makes the classical schemes inapplicable. Inspired by [7], we resort to a switching scheme, which, while complex and difficult to synthesize, is ultimately more robust. We introduce a switching adjustment mechanism to a dynamic scheme to specifically counteract the additive disturbance. In this scheme, the sampling periods are variable and adjusted via the adjustment mechanism to guarantee that the sampling information of the system states is suitable for feedback compensation. Combining this scheme with the internal model principle, we propose a dynamic sampled-data controller with variable sampling periods for the system. The controller guarantees the system states to ultimately converge to an arbitrary pre-specified neighborhood of the origin, and the sampling periods remain unchanged after a finite number of switchings.

Problem formulation. We consider stabilization via sampled-data feedback for the following continuous-time system with external disturbance:

$$\dot{x} \;=\; Ax + Bu + Ev, \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state with the initial condition $x(t_0) = x_0$; $u \in \mathbb{R}^m$ is the system input; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ ($\mathrm{rank}B = m$), and $E \in \mathbb{R}^{n\times l}$ are constant matrices; and $v \in \mathbb{R}^l$ is an unknown external disturbance, which is generated by the following l-dimensional linear exosystem:

$$\dot{v} \;=\; A_v v, \tag{2}$$

with unknown initial condition $v(t_0) = v_0$ and $A_v \in \mathbb{R}^{l\times l}$ being neutrally stable (the eigenvalues of A_v are distinct and have zero-valued real parts).

We have the following important lemma, whose proof is based on the internal model principle (see the proofs of Theorem 1 and Corollary 2 in [8]).

Lemma 1. For any $E \in \mathbb{R}^{n\times l}$, there exists a linear dynamic continuous-time controller for system (1) such that $\lim_{t\to+\infty} x(t) = 0$ and the closed-loop system is stabilizable if and only if $m = n$.

This lemma shows that we should have the condition $\mathrm{rank}B = m = n$, under which we should search for a dynamic sampled-data controller rather than a static one.

The control objective of this study is for system (1) with $\mathrm{rank}B = m = n$, to achieve stabilization in the sense: all the signals of the resulting closed-loop system are well-defined and bounded on $[t_0, +\infty)$, and particularly, $\limsup_{t\to+\infty} \|x(t)\| < \varepsilon$, where ε is a pre-specified positive constant representing the ultimate precision.

Controller design. The intensity and persistence of the disturbance in system (1) force us to design a dynamic sampled-data controller with variable sampling periods. In what follows, we assume that sampling and computation occur synchronously, and we ignore the computation time.

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Motivated by Lemma 1, we design the following dynamic sampled-data controller for system (1):

$$\begin{cases} u(t) = K_1 z(t_k) + K_2 x(t_k), \\ \dot{z}(t) = G_1 z(t) + G_2 x(t_k), \end{cases} \tag{3}$$

for $\forall t \in [t_k,\ t_{k+1})$, $k = 0, 1, 2, \ldots$, where $z \in \mathbb{R}^{nl}$ with the initial condition $z(t_0) = z_0$, and t_k are sampling times that will be determined by a switching adjustment mechanism (see below) for the variable sampling periods $h_k = t_{k+1} - t_k$.

The matrices K_1, K_2, G_1, and G_2 in (3) are suitably chosen such that $A_c = \left[\begin{smallmatrix} A+BK_2 & BK_1 \\ G_2 & G_1 \end{smallmatrix}\right]$ is Hurwitz, and the following equations admit a solution X_c:

$$\begin{cases} X_c A_v = A_c X_c + B_c, \\ C_c X_c = 0, \end{cases} \tag{4}$$

where $B_c = \left[E^{\mathrm{T}},\ 0\right]^{\mathrm{T}}$ and $C_c = [I_n, 0]$. Notably, such K_1, K_2, G_1, and G_2 can be chosen in terms of the scheme in [9] (see Remark 1.23 in [9]).

The sampling times t_k in controller (3) are determined online through the following switching logic on the variable sampling periods h_k.

Switching logic 1. Adjustment mechanism of the variable sampling periods h_k.

(i) Initialization. Let t_0 and $t_1 = t_0 + h_0$ be the first two sampling times, and the first sampling period h_0 satisfies $h_0 < \frac{1}{\mu}$, where $\mu = 2\big((\|A\| + \|BK_1\| + \|BK_2\|)\|PD_c\| + (\|G_1\| + \|G_2\|)\|PG_c\|\big)$ with $D_c = \left[K_2^{\mathrm{T}} B^{\mathrm{T}}, G_2^{\mathrm{T}}\right]^{\mathrm{T}}$, $G_c = \left[K_1^{\mathrm{T}} B^{\mathrm{T}},\ 0\right]^{\mathrm{T}}$, and P being a symmetric positive-definite matrix such that $A_c^{\mathrm{T}} P + PA_c \leqslant -I_{n+nl}$. Choose $H = \{\bar{h}_k : k = 1, 2, \ldots\}$ as a strictly-decreasing positive sequence with $\bar{h}_k < h_0$, $\lim_{k\to+\infty} \bar{h}_k = 0$, and $\sum_{k=1}^{+\infty} \bar{h}_k = +\infty$. (Clearly, $\{\frac{h_0}{k+1} : k = 1, 2, \ldots\}$ is a suitable choice of H.).

(ii) Switching logic. Sampling times $t_{k+1} = t_k + h_k$, $k = 1, 2, \ldots$ are specified with the variable sampling periods h_k which are recursively defined as

$$h_k = \begin{cases} h_{k-1}, & \text{if } \|x(t)\| < \varepsilon,\ \forall t \in [t_{k-1}, t_k), \\ \max\{h \in H \,|\, h < h_{k-1}\}, & \text{otherwise.} \end{cases}$$

The solution of the closed-loop system, comprising (1)–(3), is unique and does not escape on each $[t_k, t_{k+1})$, due to the linear nature of the system on each $[t_k, t_{k+1})$. Meanwhile, noting $\sum_{k=1}^{+\infty} \bar{h}_k = +\infty$, the switchings in the variable sampling periods would not occur an infinite number of times in any finite interval. Therefore, for any initial value, the resulting closed-loop system has a unique solution on $[t_0, +\infty)$, regardless of its stability.

Stability analysis. Let $\xi = \left[x^{\mathrm{T}},\ z^{\mathrm{T}}\right]^{\mathrm{T}}$ and set $\bar{\xi} = \xi - X_c v$. Then, by (1)–(4), we have

$$\begin{aligned} \dot{\bar{\xi}}(t) = {} & A_c \bar{\xi}(t) + D_c\big(x(t_k) - x(t)\big) \\ & + G_c\big(z(t_k) - z(t)\big), \quad \forall t \in [t_k,\ t_{k+1}), \end{aligned} \tag{5}$$

where A_c, D_c, and G_c are the same as those in (4) and Switching logic 1.

For system (5), we have the following two propositions, whose proofs are given in Appendixes A and B.

Proposition 1. Let $V(\bar{\xi}) = \bar{\xi}^{\mathrm{T}} P \bar{\xi}$. Along trajectories of system (5), the following relation holds:

$$\begin{aligned} \dot{V}(\bar{\xi}(t)) \leqslant {} & -\frac{1+\mu h_0}{2}\|\bar{\xi}(t)\|^2 + \theta_0 (t - t_k)^2 \\ & + \mu\|\bar{\xi}(t)\| \sup_{\tau\in[t_k,t)} \|\bar{\xi}(\tau)\|(t - t_k),\ \forall t \in [t_k,\ t_{k+1}), \end{aligned}$$

where μ is the same as in Switching logic 1, and θ_0 is an unknown positive constant.

Proposition 2. Because $h_0\mu < 1$, there exists an unknown positive constant θ_1 such that

$$\sup_{\tau\in[t_k,\,t_{k+1})} \|\bar{\xi}(\tau)\| \leqslant \|\bar{\xi}(t_0)\| + \theta_1,\ k = 0, 1, 2, \ldots.$$

Now, we are ready to establish the concluding theorem on adaptive sampled-data stabilization.

Theorem 1. Consider system (1) with exosystem (2) and $\operatorname{rank} B = m = n$. The controller (3), with Switching logic 1 to generate the variable sampling periods h_k, guarantees that for any initial value $(x_0, v_0, z_0) \in \mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R}^{nl}$, (i) the switchings on the variable sampling periods occur only a finite number of times, after which the variable sampling periods would remain constant, i.e., $h_k = h_{\bar{k}}$, $\forall k \geqslant \bar{k}$ for some integer $\bar{k}$; (ii) all the signals of the resulting closed-loop system (i.e., x, z, and u) are well-defined and bounded on $[t_0, +\infty)$, and furthermore, $\|x(t)\| < \varepsilon, \forall t \geqslant t_0 + \sum_{i=0}^{\bar{k}} h_i$, where $\varepsilon > 0$, which can be arbitrarily small, denotes the pre-specified ultimate precision.

Proof. As discussed in the above, for any initial value, the closed-loop system comprising (1)–(3) has a unique solution $\big(x(t), v(t), z(t)\big)$ on $[t_0, +\infty)$.

We subsequently prove that regardless of whether the switchings occur a finite number of times, all the signals $x(t), z(t)$, and $u(t)$) of the closed-loop system are bounded on $[t_0, +\infty)$. By Propositions 1 and 2, and noting $h_k \leqslant h_0$, we have that $\forall t \in [t_k, t_{k+1}), k = 0, 1, 2, \ldots$,

$$\dot{V}(\bar{\xi}(t)) \leqslant -\frac{1}{2\lambda_{\max}(P)} V(\bar{\xi}(t)) + \Theta h_0^2, \tag{6}$$

where $\lambda_{\max}(P)$ is the maximum eigenvalue of P and $\Theta = \frac{2\mu(\|\bar{\xi}(t_0)\|+\theta_1)^2 + \theta_0 h_0}{h_0} > 0$ is unknown.

From (6), it follows that

$$V(\bar{\xi}(t)) \leqslant V(\bar{\xi}(t_0)) + 2\Theta h_0^2 \lambda_{\max}(P), \forall t \in [t_0,\ +\infty).$$

Together with the definition of $V(\bar{\xi})$, the boundeness of $v(t)$, and $\xi = \bar{\xi} + X_c v$, this immediately yields the boundedness of $\xi(t) = [x^{\mathrm{T}}(t),\ z^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $u(t)$ on $[t_0, +\infty)$.

We next show by contradiction that the switchings on the variable sampling periods indeed occur a finite number of times, which, together with Switching logic 1, means that $\|x(t)\| < \varepsilon$, $\forall t \geqslant t_0 + \sum_{i=0}^{\bar{k}} h_i$ for the pre-given ultimate precision ε and some integer $\bar{k}$.

We suppose that the switchings occur an infinite number of times, such that for any $t' \geqslant t_0$, there is always $t > t'$ at which $\|x(t)\| \geqslant \varepsilon$. Then, $\lim_{k\to+\infty} h_k = 0$ holds, which implies that for some integer k', $\Theta h_k^2 \leqslant \frac{\varepsilon^2 \lambda_{\min}(P)}{4\lambda_{\max}(P)}, \forall k \geqslant k'$, with $\lambda_{\min}(P)$ denoting the minimum eigenvalue of P. From this and (6), it follows that

$$\dot{V}(\bar{\xi}(t)) \leqslant -\frac{1}{2\lambda_{\max}(P)} V(\bar{\xi}(t)) + \frac{\varepsilon^2 \lambda_{\min}(P)}{4\lambda_{\max}(P)},$$

which implies

$$\begin{aligned} V(\bar{\xi}(t)) \leqslant {} & V(\bar{\xi}(t_{k'})) \exp\left(-\frac{1}{2\lambda_{\max}(P)}(t - t_{k'})\right) \\ & + \frac{\varepsilon^2 \lambda_{\min}(P)}{2}, \qquad \forall t \geqslant t_{k'}. \end{aligned} \tag{7}$$

Let $\bar{t} = t_{k'} + 2\lambda_{\max}(P)\ln(\frac{2V(\bar{\xi}(t_{k'}))+\varepsilon^2\lambda_{\min}(P)}{\varepsilon^2\lambda_{\min}(P)})$. Then, by (7), we have $V(\bar{\xi}(t)) < \varepsilon^2\lambda_{\min}(P)$, $\forall t \geqslant \bar{t}$, which, together with the definition of $V(\bar{\xi})$, yields

$$\|\bar{\xi}(t)\| < \varepsilon, \quad \forall t \geqslant \bar{t}. \tag{8}$$

Noting $x = [I_n, 0]\xi$, $[I_n, 0]X_c = C_cX_c = 0$ (from (4)), and $\bar{\xi} = \xi - X_cv$, we obtain $x = [I_n, 0]\xi - [I_n, 0]X_cv = [I_n, 0]\bar{\xi}$, which, along with (8), allows us to establish $\|x(t)\| < \varepsilon$, $\forall t \geqslant \bar{t}$. This clearly contradicts our above hypothesis.

Conclusion. In this study, a new dynamic sampled-data control scheme with variable sampling periods has been proposed for systems with additive disturbance. To deal with the disturbance, a dynamic compensator is incorporated based on the internal model principle in addition to introducing a switching adjustment mechanism for the variable sampling periods. In future, we aim to extend the sampled-data scheme to more general systems with inherent nonlinearities.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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