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Information Sciences

• **Supplementary File** •

Supporting Information: Appendices A, B and C

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Appendix A Proof of Proposition 1

The following proceeds on arbitrarily fixed $[t_k,\, t_{k+1})$. By (5) and $A_c^{\mathrm{T}}P + PA_c \leqslant -I_{n+nl}$ in Switching Logic 1, we have

$$\begin{aligned}\dot{V}(\bar{\xi}(t)) \;&=\; \bar{\xi}(t)\big(A_c^{\mathrm{T}}P + PA_c\big)\bar{\xi}(t) + 2\bar{\xi}^{\mathrm{T}}(t)PD_c\big(x(t_k) - x(t)\big) + 2\bar{\xi}^{\mathrm{T}}(t)PG_c\big(z(t_k) - z(t)\big)\\ &\leqslant\; -\|\bar{\xi}(t)\|^2 + 2\|\bar{\xi}(t)\|\cdot\|PD_c\|\cdot\|x(t_k) - x(t)\| + 2\|\bar{\xi}(t)\|\cdot\|PG_c\|\cdot\|z(t_k) - z(t)\|. \end{aligned} \tag{A1}$$

Now appropriate estimations need be given for the last two terms on the right-hand side of (A1).

Substituting the first equality of (3) into (1) yields

$$\dot{x}(t) \;=\; Ax(t) + BK_1z(t_k) + BK_2x(t_k) + Ev(t),$$

by which and the definition of $\bar{\xi}$, we have

$$\begin{aligned}\|\dot{x}(t)\| \;&\leqslant\; \|A\|\cdot\|x(t)\| + \|BK_1\|\cdot\|z(t_k)\| + \|BK_2\|\cdot\|x(t_k)\| + \|E\|\cdot\|v(t)\|\\ &\leqslant\; \|A\|\cdot\|\bar{\xi}(t)\| + \|A\|\cdot\|X_c\|\cdot\|v(t)\| + \|BK_1\|\cdot\|\bar{\xi}(t_k)\| + \|BK_1\|\cdot\|X_c\|\cdot\|v(t_k)\|\\ &\quad + \|BK_2\|\cdot\|\bar{\xi}(t_k)\| + \|BK_2\|\cdot\|X_c\|\cdot\|v(t_k)\| + \|E\|\cdot\|v(t)\|\\ &\leqslant\; c_0\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\| + \theta_2,\end{aligned}$$

where $c_0 = \|A\| + \|BK_1\| + \|BK_2\|$ and $\theta_2 = \big(\|A\|\cdot\|X_c\| + \|E\| + \|BK_1\|\cdot\|X_c\| + \|BK_2\|\cdot\|X_c\|\big)\cdot\sup_{t\geqslant t_0}\|v(t)\|$. Then, it follows that

$$\|x(t_k) - x(t)\| \;\leqslant\; \int_{t_k}^{t}\|\dot{x}(\tau)\|\mathrm{d}\tau \;\leqslant\; c_0\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \theta_2(t - t_k).$$

Then, by the technique of completing squares, the second term on the right-hand side of (A1) can be estimated as follows:

$$\begin{aligned}&2\|\bar{\xi}(t)\|\cdot\|PD_c\|\cdot\|x(t_k) - x(t)\|\\ &\quad\leqslant\; 2\|\bar{\xi}(t)\|\cdot\|PD_c\|\cdot\Big(c_0\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \theta_2(t - t_k)\Big)\\ &\quad\leqslant\; 2c_0\|PD_c\|\cdot\|\bar{\xi}(t)\|\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \frac{1-\mu h_0}{4}\|\bar{\xi}(t)\|^2 + \frac{4\theta_2^2\|PD_c\|^2}{1-\mu h_0}(t - t_k)^2.\end{aligned} \tag{A2}$$

Moreover, by the second equality in (3), we have

$$\begin{aligned}\|\dot{z}(t)\| \;&\leqslant\; \|G_1\|\cdot\|z(t)\| + \|G_2\|\cdot\|x(t_k)\|\\ &\leqslant\; \|G_1\|\cdot\|\bar{\xi}(t)\| + \|G_1\|\cdot\|X_c\|\cdot\|v(t)\| + \|G_2\|\cdot\|\bar{\xi}(t_k)\| + \|G_2\|\cdot\|X_c\|\cdot\|v(t_k)\|\\ &\leqslant\; c_1\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\| + \theta_3,\end{aligned}$$

where $c_1 = \|G_1\| + \|G_2\|$ and $\theta_3 = \big(\|G_1\|\cdot\|X_c\| + \|G_2\|\cdot\|X_c\|\big)\cdot\sup_{t\geqslant t_0}\|v(t)\|$. Then, there holds

$$\|z(t_k) - z(t)\| \;\leqslant\; \int_{t_k}^{t}\|\dot{z}(\tau)\|\mathrm{d}\tau \;\leqslant\; c_1\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \theta_3(t - t_k),$$

which, together with the technique of completing squares, implies

$$\begin{aligned}&2\|\bar{\xi}(t)\|\cdot\|PG_c\|\cdot\|z(t_k) - z(t)\|\\ &\quad\leqslant\; 2\|\bar{\xi}(t)\|\cdot\|PG_c\|\cdot\Big(c_1\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \theta_3(t - t_k)\Big)\\ &\quad\leqslant\; 2c_1\|PG_c\|\cdot\|\bar{\xi}(t)\|\sup_{\tau\in[t_k,t)}\|\bar{\xi}(\tau)\|(t - t_k) + \frac{1-\mu h_0}{4}\|\bar{\xi}(t)\|^2 + \frac{4\theta_3^2\|PG_c\|^2}{1-\mu h_0}(t - t_k)^2.\end{aligned} \tag{A3}$$

Substituting (A2) and (A3) into (A1) can directly yield Proposition 1 with $\theta_0 = \frac{4}{1-\mu h_0}\big(\theta_2^2\|PD_c\|^2 + \theta_3^2\|PG_c\|^2\big)$. ◇

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Appendix B Proof of Proposition 2

It suffices to show Proposition 2 with $\theta_1 = h_0\sqrt{\frac{2\theta_0}{1-\mu h_0}} + 1$, where μ and θ_0 are the same as in Proposition 1.

Suppose for contradiction that there exists k_1 such that $\sup_{\tau\in[t_{k_1}, t_{k_1+1})} \|\bar{\xi}(\tau)\| > \|\xi(t_0)\| + \theta_1$, which implies $\bar{\xi}(t') > \|\xi(t_0)\| + \theta_1$ for some $t' \in [t_{k_1}, t_{k_1+1})$. Then, certain $k_2 \leqslant k_1$ and $t'' \in (t_{k_2}, t_{k_2+1})$ can be chosen to satisfy

$$\begin{cases} \|\bar{\xi}(t'')\| \ > \ \|\bar{\xi}(t_0)\| + h_0\sqrt{\frac{2\theta_0}{1-\mu h_0}} + 1, \\ \|\bar{\xi}(t_{k_2})\| \ \leqslant \ \|\bar{\xi}(t_0)\| + h_0\sqrt{\frac{2\theta_0}{1-\mu h_0}} + 1. \end{cases}$$

Furthermore, by the continuous differentiability of $\bar{\xi}(t)$ on $[t_{k_2}, t'']$, there exists $t''' \in [t_{k_2}, t'')$ such that

$$\begin{cases} \|\bar{\xi}(t''')\| \ = \ \|\bar{\xi}(t_0)\| + h_0\sqrt{\frac{2\theta_0}{1-\mu h_0}} + 1, \\ \dot{V}(\bar{\xi}(t''')) \ \geqslant \ 0, \\ \|\bar{\xi}(t)\| \ \leqslant \ \|\bar{\xi}(t''')\|, \quad \forall t \in [t_{k_2}, t'''), \end{cases} \tag{B1}$$

which, together with $h_{k_2} \leqslant h_0 < \frac{1}{\mu}$ and Proposition 1, implies

$$\begin{aligned} \dot{V}(\bar{\xi}(t''')) \ &\leqslant \ -\frac{1+\mu h_0}{2}\|\bar{\xi}(t''')\|^2 + \mu\|\bar{\xi}(t''')\| \sup_{\tau\in[t_{k_2}, t''')} \|\bar{\xi}(\tau)\|(t''' - t_{k_2}) + \theta_0 h_{k_2}^2 \\ &\leqslant \ -\frac{1+\mu h_0}{2}\|\bar{\xi}(t''')\|^2 + \mu h_0\|\bar{\xi}(t''')\|^2 + \theta_0 h_0^2 \\ &= \ -\frac{1-\mu h_0}{2}\left(\|\bar{\xi}(t_0)\| + h_0\sqrt{\frac{2\theta_0}{1-\mu h_0}} + 1\right)^2 + \theta_0 h_0^2 \ < \ 0, \end{aligned}$$

a contradiction to the second inequality in (B1), and Proposition 2 is thus proved. $\Diamond$

Appendix C A Numerical Example

To illustrate the effectiveness of the theoretical results, we consider the sampled-data stabilization of system (1) with

$$A = 0, \quad B = 1, \quad E = [1, 0, 5]$$

and

$$A_v = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

Let $\varepsilon = 0.1$ be the pre-specified convergence precision for the system state. Then, we design the sampled-data controller (3) with

$$K_1 = [-1, 0, -5], \qquad K_2 = -4, \qquad G_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \qquad G_2 = [0, 0, 1]^{\mathrm{T}},$$

as well as the switching adjustment mechanism for the variable sampling periods with the initial sampling period $h_0 = 0.08$ and $H = \left\{\frac{640}{7999+k} : k = 1, 2, \ldots\right\}$.

Let the initial condition be $x(0) = 1$, $z(0) = [0.8, 0, 1]^{\mathrm{T}}$ and $v(0) = [0.8, -1, 1.5]^{\mathrm{T}}$. By MATLAB, Figures 1–4 are obtained to exhibit the trajectories of all the signals of the resulting closed-loop system. Particularly, Figure 1 shows that the system state $x(t)$ converges to the pre-specified neighborhood of the origin. Moreover, Figure 3 demonstrates that the sampling period remains unchanged after $9s$, which means that the sampling period is adjusted only a finite number of times.

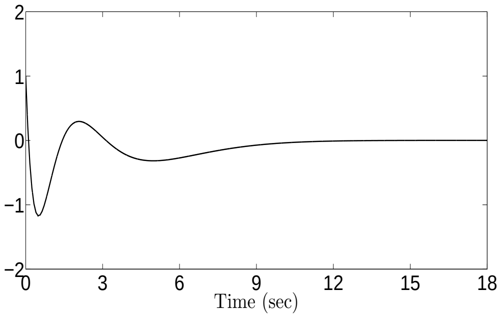

Fig. 1 The trajectory of the system state x.

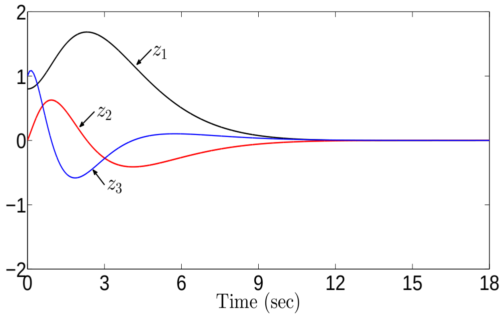

Fig. 2 The trajectories of the internal state z.

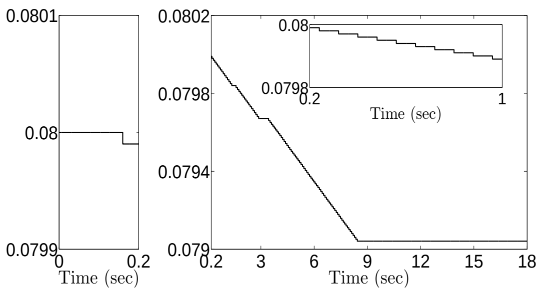

Fig. 3 The adjustment of the sampling periods.

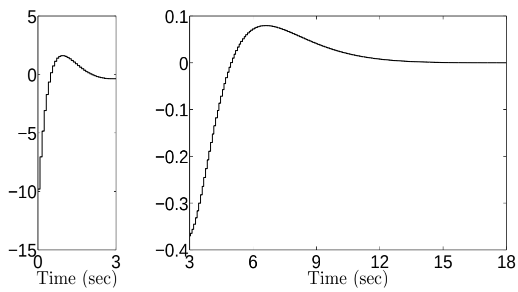

Fig. 4 The trajectory of the system input u.