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Uplink Transmission Design for Crowded Correlated Cell-Free Massive MIMO-OFDM Systems

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Appendix A Proof of Proposition 1

From the definition of $\mathbf{V}_{N\times N}$ in Proposition 1, we have

$$\left[\mathbf{V}_{N\times N}^{H}\mathbf{V}_{N\times N}\right]_{i,j}=\frac{1}{N}\sum_{n=0}^{N-1}\exp\left\{-\jmath\pi(j-i)\left(\frac{2n}{N}-1\right)\right\}\overset{N\to\infty}{=}\delta(j-i),\tag{A1}$$

i.e., $\mathbf{V}_{N\times N}^{H}\mathbf{V}_{N\times N}\overset{N\to\infty}{=}\mathbf{I}_{N\times N}$. We have $\mathbf{V}_{M\times M}^{H}\mathbf{V}_{M\times M}=\left(\mathbf{I}_{L\times L}^{H}\mathbf{I}_{L\times L}\right)\otimes\left(\mathbf{V}_{N\times N}^{H}\mathbf{V}_{N\times N}\right)\overset{N\to\infty}{=}\mathbf{I}_{M\times M}$.

From (1) and (2), we can obtain

$$\mathrm{vec}\left\{\mathbf{G}_{k}^{\beta}\right\}=\sum_{q=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\mathbf{f}_{N_c,q}\otimes\mathbf{v}_{M}(\theta)\right]\odot\mathbf{a}_{MN_c,k}^{\beta}\left(\theta,qT_s\right)\mathrm{d}\theta,\tag{A2}$$

where $\mathbf{v}_M(\theta)\triangleq\mathbf{1}_{L\times 1}\otimes\mathbf{v}_N(\theta)$, $\mathbf{f}_{N_c,q}\triangleq\left[1\ \exp\left\{-\jmath2\pi\frac{1}{N_c}q\right\}\cdots\exp\left\{-\jmath2\pi\frac{N_c-1}{N_c}q\right\}\right]^T$, $\mathbf{a}_{MN_c,k}^{\beta}\left(\theta,qT_s\right)\triangleq\mathbf{1}_{N_c\times1}\otimes\mathbf{a}_{L,k}^{\beta}(\theta,qT_s)\otimes\mathbf{1}_{N\times1}$, and $\mathbf{a}_{L,k}^{\beta}(\theta,qT_s)\triangleq\left[a_{k,0}^{\beta}(\theta,qT_s)\ a_{k,1}^{\beta}(\theta,qT_s)\cdots a_{k,L-1}^{\beta}(\theta,qT_s)\right]^T$. The space-frequency domain channel covariance matrix $\mathbf{R}_k^{\beta}$ can be obtained as

$$\begin{aligned}\mathbf{R}_{k}^{\beta}&=\mathbb{E}\left\{\mathrm{vec}\left\{\mathbf{G}_{k}^{\beta}\right\}\mathrm{vec}^{H}\left\{\mathbf{G}_{k}^{\beta}\right\}\right\}\\&=\sum_{q=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sum_{q'=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\mathbf{f}_{N_c,q}\otimes\mathbf{v}_M(\theta)\right]\left[\mathbf{f}_{N_c,q'}\otimes\mathbf{v}_M(\theta')\right]^{H}\odot\mathbb{E}\left\{\mathbf{a}_{MN_c,k}^{\beta}\left(\theta,qT_s\right)\left(\mathbf{a}_{MN_c,k}^{\beta}\left(\theta',q'T_s\right)\right)^{H}\right\}\mathrm{d}\theta\mathrm{d}\theta'.\end{aligned}\tag{A3}$$

Define $P_{k,l}^{AD}(\theta,qT_s)\triangleq P_{k,l}^{A}(\theta)P_{k,l}^{D}(qT_s)$. For an arbitrary non-negative integer d, let $n_d\triangleq\lfloor d/M\rfloor$, $m_d\triangleq\langle d\rangle_M$, $r_{m_d}\triangleq\lfloor m_d/N\rfloor$, and $s_{m_d}\triangleq\langle m_d\rangle_N$. We have

$$\begin{aligned}\left[\mathbf{R}_{k}^{\beta}\right]_{i,j}&=\sum_{q=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sum_{q'=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\mathbf{f}_{N_c,q}\otimes\mathbf{v}_M(\theta)\right]_i\left[\mathbf{f}_{N_c,q'}\otimes\mathbf{v}_M(\theta')\right]_j^{*}\mathbb{E}\left\{\left[\mathbf{a}_{MN_c,k}^{\beta}\left(\theta,qT_s\right)\right]_i\left[\mathbf{a}_{MN_c,k}^{\beta}\left(\theta',q'T_s\right)\right]_j^{*}\right\}\mathrm{d}\theta\mathrm{d}\theta'\\&\overset{(a)}{=}\sum_{q=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\mathbf{f}_{N_c,q}\right]_{n_i}\left[\mathbf{f}_{N_c,q}\right]_{n_j}^{*}\left[\mathbf{v}_N(\theta)\right]_{s_{m_i}}\left[\mathbf{v}_N(\theta)\right]_{s_{m_j}}^{*}\mathbb{E}\left\{a_{k,r_{m_i}}^{\beta}\left(\theta,qT_s\right)\left(a_{k,r_{m_j}}^{\beta}\left(\theta,qT_s\right)\right)^{*}\right\}\mathrm{d}\theta\\&=\sum_{q=0}^{N_{\mathrm{cp}}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\exp\left\{-\jmath2\pi\frac{n_i-n_j}{N_c}q\right\}\exp\left\{-\jmath\pi\left(s_{m_i}-s_{m_j}\right)\sin(\theta)\right\}\beta\left(k,r_{m_i}\right)P_{k,r_{m_i}}^{AD}(\theta,qT_s)\delta\left(r_{m_i}-r_{m_j}\right)\mathrm{d}\theta,\end{aligned}\tag{A4}$$

where (a) follows from $[\mathbf{A}\otimes\mathbf{B}]_{i,j}=[\mathbf{A}]_{n_i,n_j}[\mathbf{B}]_{m_i,m_j}$ for matrices $\mathbf{A}$ and $\mathbf{B}$. Meanwhile, we have

$$\begin{aligned}&\left[\left(\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\otimes\mathbf{V}_{M\times M}\right)\mathrm{diag}\left\{\mathrm{vec}\left\{\boldsymbol{\Upsilon}_{k}^{\beta}\right\}\right\}\left(\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\otimes\mathbf{V}_{M\times M}\right)^{H}\right]_{i,j}\\&=\sum_{d=0}^{MN_{\mathrm{cp}}-1}\left[\mathrm{vec}\left\{\boldsymbol{\Upsilon}_{k}^{\beta}\right\}\right]_d\left[\left(\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\otimes\mathbf{V}_{M\times M}\right)\right]_{i,d}\left[\left(\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\otimes\mathbf{V}_{M\times M}\right)\right]_{j,d}^{*}\\&=\sum_{n_d=0}^{N_{\mathrm{cp}}-1}\sum_{m_d=r_{m_i}}^{r_{m_i}+N-1}\left[\boldsymbol{\Upsilon}_{k,r_{m_d}}^{\beta}\right]_{s_{m_d},n_d}\left[\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\right]_{n_i,n_d}\left[\mathbf{F}_{N_c\times N_{\mathrm{cp}}}\right]_{n_j,n_d}^{*}\left[\mathbf{V}_{N\times N}\right]_{s_{m_i},s_{m_d}}\left[\mathbf{V}_{N\times N}\right]_{s_{m_j},s_{m_d}}^{*}\delta\left(r_{m_i}-r_{m_j}\right)\end{aligned}$$

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$$= \sum_{n_d=0}^{N_{\rm cp}-1}\sum_{m_d=0}^{N-1} \beta\left(k, r_{m_i}\right)\left(\theta_{m_d+1}-\theta_{m_d}\right) P^{AD}_{k,r_{m_i}}(\theta_{m_d}, n_d T_s) \exp\left\{-\bar{\jmath}2\pi\frac{n_i-n_j}{N_{\rm c}}n_d\right\}\exp\left\{-\bar{\jmath}\pi\left(s_{m_i}-s_{m_j}\right)\sin\left(\theta_{m_d}\right)\right\}\delta\left(r_{m_i}-r_{m_j}\right)$$

$$\overset{N\to\infty}{=} \sum_{q=0}^{N_{\rm cp}-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \beta\left(k, r_{m_i}\right) P^{AD}_{k,r_{m_i}}(\theta, qT_s)\exp\left\{-\bar{\jmath}2\pi\frac{n_i-n_j}{N_{\rm c}}q\right\}\exp\left\{-\bar{\jmath}\pi\left(s_{m_i}-s_{m_j}\right)\sin(\theta)\right\}\delta\left(r_{m_i}-r_{m_j}\right){\rm d}\theta. \quad \text{(A5)}$$

Since the power angle-delay spectrum is bounded [1], the limit in the first equation of (A5) exists. Since (A4) is equal to (A5), $\mathbf{R}_k^{\beta}$ can be obtained as (8). The proof of (9) is given by

$$\begin{aligned}\mathbf{R}_k^{\beta} &\overset{N\to\infty}{\overset{(a)}{=}} \left(\mathbf{F}_{N_c\times N_{\rm cp}}\otimes\mathbf{V}_{M\times M}\right)\mathbb{E}\left\{{\rm vec}\left\{\mathbf{H}_k^{\beta}\right\}{\rm vec}^H\left\{\mathbf{H}_k^{\beta}\right\}\right\}\left(\mathbf{F}_{N_c\times N_{\rm cp}}\otimes\mathbf{V}_{M\times M}\right)^H \\ &\overset{(b)}{=} \mathbb{E}\left\{{\rm vec}\left\{\mathbf{V}_{M\times M}\mathbf{H}_k^{\beta}\,\mathbf{F}^H_{N_c\times N_{\rm cp}}\right\}{\rm vec}^H\left\{\mathbf{V}_{M\times M}\mathbf{H}_k^{\beta}\,\mathbf{F}^H_{N_c\times N_{\rm cp}}\right\}\right\},\end{aligned} \quad \text{(A6)}$$

where (a) follows from (6) and (8), and (b) follows from the fact that $\left(\mathbf{C}^T\otimes\mathbf{A}\right){\rm vec}\{\mathbf{B}\} = {\rm vec}\{\mathbf{ABC}\}$. Besides, since $\mathbf{R}_k^{\beta} = \mathbb{E}\left\{{\rm vec}\left\{\mathbf{G}_k^{\beta}\right\}{\rm vec}^H\left\{\mathbf{G}_k^{\beta}\right\}\right\}$, we can obtain (9).

Appendix B Proof of Proposition 2

Considering the non-negative property of the angle-delay domain channel power distribution, it is satisfied that in (19) the term $R_{\mathcal{K}_a^{i,j},p,m,q} = \frac{\left[\boldsymbol{\Upsilon}^{\beta,0}_{\mathcal{K}_a^{i,j}}\right]_{m,q}}{\left[\boldsymbol{\Upsilon}^{\beta}_{\mathcal{K}_a^{i,j}}\right]_{m,q}+\sum\limits_{\substack{j'=0\\j'\neq j}}^{K_a-1}\delta\left(\left\langle\mathcal{P}^p_{i,j'}\right\rangle_Z-\left\langle\mathcal{P}^p_{i,j}\right\rangle_Z\right)\left[\boldsymbol{\Upsilon}^{\beta,\left\lfloor\mathcal{P}^p_{i,j'}/Z\right\rfloor-\left\lfloor\mathcal{P}^p_{i,j}/Z\right\rfloor}_{\mathcal{K}_a^{i,j'}}\right]_{m,q}+\frac{1}{\rho_p Z}} \in [0,1)$. Then, we have $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q} = \left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}\left(1-R_{\mathcal{K}_a^{i,j},p,m,q}\right)\geqslant 0$. When $\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}=0$, $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}$ can achieve the minimum value 0; when $\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}>0$, $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}$ can be minimized if and only if $R_{\mathcal{K}_a^{i,j},p,m,q}$ is maximized, i.e., $\sum\limits_{\substack{j'=0\\j'\neq j}}^{K_a-1}\delta\left(\left\langle\mathcal{P}^p_{i,j'}\right\rangle_Z-\left\langle\mathcal{P}^p_{i,j}\right\rangle_Z\right)\left[\boldsymbol{\Upsilon}^{\beta,\left\lfloor\mathcal{P}^p_{i,j'}/Z\right\rfloor-\left\lfloor\mathcal{P}^p_{i,j}/Z\right\rfloor}_{\mathcal{K}_a^{i,j'}}\right]_{m,q} = 0$. Hence, we can conclude the minimum condition of $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}$ as $\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}\left[\boldsymbol{\Upsilon}^{\beta,\left\lfloor\mathcal{P}^p_{i,j'}/Z\right\rfloor-\left\lfloor\mathcal{P}^p_{i,j}/Z\right\rfloor}_{\mathcal{K}_a^{i,j'}}\right]_{m,q} = 0$ for $\left\langle\mathcal{P}^p_{i,j'}\right\rangle_Z = \left\langle\mathcal{P}^p_{i,j}\right\rangle_Z$, $j'=0,1,\cdots,K_a-1$, and $j'\neq j$.

For any choice of $i,j,p,m,$ and q, $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}$ should be minimized. The optimal condition can be expressed as $\boldsymbol{\Upsilon}^0_k\odot\boldsymbol{\Upsilon}^0_k\odot\boldsymbol{\Upsilon}^{\beta,\lfloor\phi_{k'}/Z\rfloor-\lfloor\phi_k/Z\rfloor}_{k'} = \mathbf{0}$ for $\langle\phi_{k'}\rangle_Z = \langle\phi_k\rangle_Z$, $k'\in\mathcal{K}$, and $k\neq k'$. It is a necessary and sufficient condition, but it can be expressed as other forms. The reason why we choose $\boldsymbol{\Upsilon}^0_k\odot\boldsymbol{\Upsilon}^0_k\odot\boldsymbol{\Upsilon}^{\beta,\lfloor\phi_{k'}/Z\rfloor-\lfloor\phi_k/Z\rfloor}_{k'}$ but not $\boldsymbol{\Upsilon}^0_k\odot\boldsymbol{\Upsilon}^{\beta,\lfloor\phi_{k'}/Z\rfloor-\lfloor\phi_k/Z\rfloor}_{k'}$ is because we want to make the interference $\boldsymbol{\Upsilon}^{\beta,\lfloor\phi_{k'}/Z\rfloor-\lfloor\phi_k/Z\rfloor}_{k'}$ smaller when $\boldsymbol{\Upsilon}^0_k$ is large. In this way, $\bar{\varepsilon}^0$ can be smaller.

Based on (19) and (20), we can obtain

$$\begin{aligned}\mathbb{E}_{\mathcal{U},\mathcal{K}_a,\mathcal{P}}(\varepsilon^0) &= \sum_{i=0}^{N_{\mathcal{K}_a}-1}\sum_{j=0}^{K_a-1}\sum_{p=0}^{N_\phi-1}\frac{1}{N_{\mathcal{K}_a}K_aN_\phi N_c\left|\mathcal{B}_{\mathcal{K}_a^{i,j}}\right|}\sum_{m=0}^{M-1}\sum_{q=0}^{N_{\rm cp}-1}\left\{\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}\right\} \\ &\geqslant \sum_{i=0}^{N_{\mathcal{K}_a}-1}\sum_{j=0}^{K_a-1}\sum_{p=0}^{N_\phi-1}\frac{1}{N_{\mathcal{K}_a}K_aN_\phi N_c\left|\mathcal{B}_{\mathcal{K}_a^{i,j}}\right|}\sum_{m=0}^{M-1}\sum_{q=0}^{N_{\rm cp}-1}\left\{\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}-\frac{\left[\boldsymbol{\Upsilon}^0_{\mathcal{K}_a^{i,j}}\right]_{m,q}\left[\boldsymbol{\Upsilon}^{\beta,0}_{\mathcal{K}_a^{i,j}}\right]_{m,q}}{\left[\boldsymbol{\Upsilon}^{\beta}_{\mathcal{K}_a^{i,j}}\right]_{m,q}+\frac{1}{\rho_p Z}}\right\} \\ &\overset{(a)}{=} \sum_{k=0}^{K-1}\frac{1}{KN_c|\mathcal{B}_k|}\sum_{m=0}^{M-1}\sum_{q=0}^{N_{\rm cp}-1}\left\{\left[\boldsymbol{\Upsilon}^0_k\right]_{m,q}-\frac{\left[\boldsymbol{\Upsilon}^0_k\right]_{m,q}\left[\boldsymbol{\Upsilon}^{\beta,0}_k\right]_{m,q}}{\left[\boldsymbol{\Upsilon}^{\beta}_k\right]_{m,q}+\frac{1}{\rho_p Z}}\right\} \\ &= \left[\mathbb{E}_{\mathcal{U},\mathcal{K}_a,\mathcal{P}}(\varepsilon^0)\right]_{\min},\end{aligned} \quad \text{(B1)}$$

where (a) follows from the fact that when $\varepsilon^0_{\mathcal{K}_a^{i,j},p,m,q}$ is minimized, i.e., when the effect of pilot interference is eliminated, the average operation accounting for all possible active patterns and all types of phase shift selection is the same as the average operation over K UEs in the network [2]. Then, we have $\bar{\varepsilon}^0_{\min} = \sum\limits_{K_a=1}^{K} p(K_a|K)\left[\mathbb{E}_{\mathcal{U},\mathcal{K}_a,\mathcal{P}}(\varepsilon^0)\right]_{\min}$.

Appendix C Derivation of (30)

We can rewrite (29) as

$$r_{k,s} = \sum_{l=0}^{L-1}\nu_{k,l}r_{k,l,s} = \sqrt{\rho_u}\sqrt{\eta_k}\,\mathbb{E}\left\{\left(\mathbf{c}^{\beta,0}_{k,s}\right)^H\mathbf{g}^{\beta}_{k,s}\right\}x_{k,s}+{\rm Inf}'_{\rm sum}, \quad \text{(C1)}$$

where the interference term is given by

$${\rm Inf}'_{\rm sum} = \sqrt{\rho_u\eta_k}\left(\left(\mathbf{c}^{\beta,0}_{k,s}\right)^H\mathbf{g}^{\beta}_{k,s}-\mathbb{E}\left\{\left(\mathbf{c}^{\beta,0}_{k,s}\right)^H\mathbf{g}^{\beta}_{k,s}\right\}\right)x_{k,s}+\sqrt{\rho_u}\sum_{k'\in\,\mathcal{U}^i_{\mathcal{K}_a}\setminus\{k\}}\sqrt{\eta_{k'}}\left(\mathbf{c}^{\beta,0}_{k,s}\right)^H\mathbf{g}^{\beta}_{k',s}x_{k',s}+\left(\mathbf{c}^{\beta,0}_{k,s}\right)^H\mathbf{w}_s. \quad \text{(C2)}$$

We have $\mathbb{E}\left\{\mathrm{Inf}_{\mathrm{sum}}'\right\}=0$. Since the transmitted signal of UE k is independent of the signals of other UEs and receiver noise, the interference is uncorrelated with the transmitted signal, i.e.,

$$\mathbb{E}\left\{x_{k,s}^{*}\mathrm{Inf}_{\mathrm{sum}}'\right\}=\sqrt{\rho_u}\sqrt{\eta_k}\,\mathbb{E}\left\{\left(\mathbf{c}_{k,s}^{\beta,0}\right)^H\mathbf{g}_{k,s}^{\beta}-\mathbb{E}\left\{\left(\mathbf{c}_{k,s}^{\beta,0}\right)^H\mathbf{g}_{k,s}^{\beta}\right\}\right\}\mathbb{E}\left\{\left|x_{k,s}\right|^2\right\}=0. \tag{C3}$$

The variance of the interference term is represented as

$$\mathbb{E}\left\{\left|\mathrm{Inf}_{\mathrm{sum}}'\right|^2\right\}=\rho_u\sum_{k'\in\mathcal{U}_{\mathcal{K}_a}^{i}}\eta_{k'}\mathbb{E}\left\{\left|\left(\mathbf{c}_{k,s}^{\beta,0}\right)^H\mathbf{g}_{k',s}^{\beta}\right|^2\right\}-\rho_u\eta_k\left|\mathbb{E}\left\{\left(\mathbf{c}_{k,s}^{\beta,0}\right)^H\mathbf{g}_{k,s}^{\beta}\right\}\right|^2+\mathbb{E}\left\{\left\|\mathbf{c}_{k,s}^{\beta,0}\right\|^2\right\}. \tag{C4}$$

It follows from the independence between each of the zero-mean transmitted signals and the independence between signals and channels. Taking the OFDM CP overhead and pilot overhead into account, according to Corollary 1.3 in [3], we can obtain the SE lower bound as shown in (30).

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