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Quantized tracking control for nonlinear systems with unstable linearization

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Dear editor,

Output tracking control is currently receiving increased attention because of its applications in various fields [1–3]. Controller design for nonlinear systems with unstable linearization [4] is essential because such systems have several applications, including a class of underactuated mechanical systems. For the output tracking control for systems with unstable linearization, the global practical output tracking controller design was developed using the backstepping design method [5] and the asymptotic tracking control problem for a power-integrator planar system was addressed [6]. However, these studies did not consider quantization [5, 6]. However, signals are transmitted over communication channels and must therefore be quantized to model systems with quantization [7]. Unfortunately, a quantized tracking controller design for nonlinear systems with unstable linearizations has not been reported yet despite its potential in practical applications. This study provides a solution to address this problem.

The two main contributions of this study are as follows.

(1) This study is the first to present results on the quantized tracking control for nonlinear systems with unstable linearization. Quantization is inevitable and useful for control systems and quantization schemes that require a low communication rate; therefore, the results reported herein are more practical than the corresponding non-quantized control solutions [5, 6].

(2) This study considers systems with unstable linearization, including the linearizable nonlinear systems [7], as special cases. Because the Jacobian linearization of systems studied herein may have eigenvalues with positive real parts, some new techniques can be developed for the controller design and stability analysis.

Problem formulation. This study aims to design controllers to obtain the output of the following system:

$$\dot{x}_1 = d(t)x_2^{p_1}, \quad \dot{x}_2 = q(u^{p_2}), \quad y = x_1, \tag{1}$$

and track the signal v_1 generated by

$$\dot{v}_1 = d(t)v_2^{q_1}, \quad \dot{v}_2 = -v_1^{q_2}, \tag{2}$$

where $p_1, p_2, q_1, q_2 \in R_{\text{odd}}^{\geqslant 1} = \{s \in R\colon\ s \geqslant 1$ and s is a ratio of odd integers$\}$, $d : R^+ \to R$ is an unknown time-varying control coefficient with a known sign, and $q(u^{p_2})$ denotes the quantized input. The hysteretic quantizer [7] is used to avoid chattering considering the following expression:

$$\begin{aligned} &q(u^{p_2}) = u^{p_2} + d, \\ &d^2 \leqslant \delta^2 u^{2p_2},\ \forall |u^{p_2}| \geqslant u_{\min}^{p_2}, \\ &d^2 \leqslant u_{\min}^{2p_2},\ \forall |u^{p_2}| \leqslant u_{\min}^{p_2}, \end{aligned} \tag{3}$$

where $u_{\min} > 0$ is a parameter of the hysteretic quantizer.

For the unknown time-varying control coefficient, the following assumption must be considered.

Assumption 1. $d(t)$ is positive and

$$\lambda \leqslant d(t) \leqslant \mu,$$

where $\lambda > 0$ and $\mu > 0$ are constants.

Remark 1. $p_1, p_2 \in R_{\text{odd}}^{\geqslant 1}$ [5]; therefore, system (1) may have unstable modes with eigenvalues having positive real parts, thus rendering the existing tools for the quantized control of nonlinear systems [7] invalid. Hence, new controller design tools should be developed.

Controller design and stability analysis. Let $V = \frac{1}{2}(x_1 - v_1)^2 + \frac{1}{2}(x_2 + k_1 z_1 - v_2^{\gamma_1})^2$. The following results are for the controller design whose proof is included in Appendix A.

Theorem 1. If Assumption 1 and $q_1 \geqslant p_1$ hold for systems (1) and (2), then

$$\begin{aligned} \dot{V} \leqslant & -\frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} - \left(\frac{k_1^{p_1-1}}{2^{p_1}}\lambda + k_d\right) z_2^2 \\ & + z_2 \bar{v} + z_2 q(u^{p_2}), \end{aligned} \tag{4}$$

where $k_1 > 0$ and $k_d > 0$ are the design parameters; $\gamma_1 = \frac{q_1}{p_1} \geqslant 1$ and

$$\begin{aligned} \bar{v} = & \ \mu\beta_1(\cdot) z_2^{\frac{1}{p_1}} + \left(\frac{k_1^{p_1-1}}{2^{p_1}}\lambda + k_d\right) z_2 \\ & + \gamma_1 v_2^{\gamma_1 - 1} v_1^{q_2} + k_1 \mu |x_2^{p_1} - v_2^{q_1}| \text{sign}(z_2). \end{aligned} \tag{5}$$

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We choose the controller as

$$u = -\left[\frac{z_2\bar{v}^2}{(1-\delta)\sqrt{z_2^2\bar{v}^2+\eta^2}}\right]^{\frac{1}{p_2}}, \quad (6)$$

where η is any positive constant.

The stability analysis results of the proposed quantized tracking control for nonlinear systems are presented in the following.

Theorem 2. If Assumption 1 and the condition $q_1 \geqslant p_1$ hold for systems (1) and (2), then the following statements hold using controller expression (6) with any positive design parameters k_1, k_d and η:

(1) For any given $\delta \in (0,1)$, all the states of the closed-loop system are globally uniformly bounded;

(2) The output tracking error $|x_1 - v_1|$ can be made arbitrarily small by accurately choosing the design parameters k_1, k_d, and η.

Proof. $z_2 u^{p_2} \leqslant 0$; therefore, from $q(u^{p_2}) = u^{p_2} + d$ in (3), using the inequality $xy \leqslant \frac{x^2}{4a} + ay^2$ with $a > 0$, we have

$$\begin{aligned} z_2 d &\leqslant \delta|z_2u^{p_2}| + u_{\min}^{p_2}|z_2| \\ &\leqslant -\delta z_2 u^{p_2} + \frac{1}{4k_d}u_{\min}^{2p_2} + k_d z_2^2. \end{aligned} \quad (7)$$

Substituting (3) and (7) into (4) yields

$$\begin{aligned} \dot{V} \leqslant & -\frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} - \frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_2^2 + z_2\bar{v} \\ & + (1-\delta)z_2u^{p_2} + \frac{1}{4k_d}u_{\min}^{2p_2}. \end{aligned} \quad (8)$$

Note that

$$\begin{aligned} & -\frac{z_2^2\bar{v}^2}{\sqrt{z_2^2\bar{v}^2+\eta^2}} \\ & \leqslant -\frac{(z_2\bar{v})^2}{|z_2\bar{v}|+\eta} < -\frac{(z_2\bar{v})^2-\eta^2}{|z_2\bar{v}|+\eta} \leqslant \eta - z_2\bar{v}. \end{aligned}$$

Using this above expression and (6), we obtain

$$(1-\delta)z_2u^{p_2} = -\frac{z_2^2\bar{v}^2}{\sqrt{z_2^2\bar{v}^2+\eta^2}} < \eta - z_2\bar{v}. \quad (9)$$

Substituting (9) into (8), we get

$$\dot{V} < -\frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} - \frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_2^2 + \eta + \frac{1}{4k_d}u_{\min}^{2p_2}. \quad (10)$$

In the following proofs, two cases are discussed.

(1) The case of $p_1 > 1$. By applying Young's inequality, we have

$$\frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_1^2 1^{p_1-1} \leqslant \frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} + \beta_0,$$

which yields

$$-\frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} \leqslant -\frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_1^2 + \beta_0, \quad (11)$$

where

$$\beta_0 = \frac{p_1-1}{p_1+1}\left(\frac{2}{p_1+1}\right)^{\frac{2}{p_1-1}}\left(\frac{1}{2}\right)^{p_1}\lambda k_1^{p_1-\frac{p_1+1}{p_1-1}}. \quad (12)$$

Substituting (11) into (10) gives

$$\begin{aligned} \dot{V} &\leqslant -\frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_1^2 - \frac{k_1^{p_1-1}}{2^{p_1}}\lambda z_2^2 + \frac{1}{4k_d}u_{\min}^{2p_2} + \eta + \beta_0 \\ &\leqslant -\kappa V + b, \end{aligned} \quad (13)$$

where

$$\kappa = \left(\frac{k_1}{2}\right)^{p_1-1}\lambda,\ b = \frac{1}{4k_d}u_{\min}^{2p_2} + \eta + \beta_0. \quad (14)$$

Using (13), we have

$$0 \leqslant V(t) < \frac{b}{\kappa} + \left(V(0) - \frac{b}{\kappa}\right)\mathrm{e}^{-\kappa t},$$

which implies that

$$\begin{aligned} & V(t) < \frac{b}{\kappa} + \left|V(0) - \frac{b}{\kappa}\right|, \\ & \lim_{t\to\infty}\sup V(t) \leqslant \frac{b}{\kappa}. \end{aligned} \quad (15)$$

Using (15) and considering that $V = \frac{1}{2}(x_1 - v_1)^2 + \frac{1}{2}(x_2 + k_1 z_1 - v_2^{\gamma_1})^2$, we can verify that all the states of the closed-loop system are globally uniformly bounded.

From (12) and (14), we can confirm that

$$\begin{aligned} \frac{b}{\kappa} = & \frac{p_1-1}{2p_1+2}\left(\frac{2}{p_1+1}\right)^{\frac{2}{p_1-1}}k_1^{-\frac{2}{p_1-1}} \\ & + \frac{2^{p_1-1}}{\lambda}k_1^{-(p_1-1)}\left(\frac{1}{4k_d}u_{\min}^{2p_2} + \eta\right). \end{aligned} \quad (16)$$

Using (16), considering $p_1 > 1$ and k_1 as a design parameter independent of p_1, λ, $u_{\min}$, k_d, and η, we can find that choosing a sufficiently large k_1 value will result in an arbitrarily small $\frac{b}{\kappa}$. From Eqs. (15) and (16) and the definition of V, we can conclude that the output tracking error $|x_1 - v_1|$ can be tuned to an arbitrarily small value by choosing a sufficiently large k_1.

(2) The case of $p_1 = 1$. From (10), we have

$$\begin{aligned} \dot{V} &< -\frac{k_1}{2}\lambda z_1^2 - \frac{1}{2}\lambda z_2^2 + \eta + \frac{1}{4k_d}u_{\min}^{2p_2} \\ &= -\kappa_0 V + b_0, \end{aligned} \quad (17)$$

where

$$\kappa_0 = \min\{k_1\lambda, \lambda\},\ b_0 = \frac{1}{4k_d}u_{\min}^{2p_2} + \eta. \quad (18)$$

Similar to (15), we can prove that

$$\begin{aligned} & V(t) < \frac{b_0}{\kappa_0} + \left|V(0) - \frac{b_0}{\kappa_0}\right|, \\ & \lim_{t\to\infty}\sup V(t) \leqslant \frac{\frac{1}{4k_d}u_{\min}^{2p_2} + \eta}{\min\{k_1\lambda, \lambda\}}. \end{aligned} \quad (19)$$

From (19), we can confirm that all the states of the closed-loop system are globally uniformly bounded. Moreover, because k_d and η are independent of k_1, λ, and $u_{\min}$, the output tracking error $|x_1 - v_1|$ can be tuned to an arbitrarily small value as in the case of $p_1 > 1$ by choosing sufficiently large k_d and small η.

Remark 2. When $q(u^{p_2}) = u^{p_2}$, the results in this study reduce to the nonquantized case. In this case, $d = 0$. By choosing $u = -\overline{v}^{1/p_2}$ in (4), we obtain $\dot{V} \leqslant -\frac{k_1^{p_1}}{2^{p_1}}\lambda z_1^{p_1+1} - (\frac{k_1^{p_1-1}}{2^{p_1}}\lambda + k_d)z_2^2$, which implies that the asymptotic output tracking control of system (1) is achieved. Therefore, the results presented herein are more general than that in [5], which can not get the asymptotic output tracking but a weaker result (practical output tracking).

The simulation example is included in Appendix B.

Conclusion. We studied the quantized tracking control problem for nonlinear systems with unstable linearization. Our future work will focus on generalizing the presented results to multiagent systems [8, 9] or systems with unknown growth rate.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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