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Quantized tracking control for nonlinear systems with unstable linearization

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Appendix A Proof of Theorem 1

The following lemmas are the key tools for the proof of Theorem 1, which can be found in [4,6].

Lemma A.1 For any $p \in R^{\geqslant 1}_{odd} := \{s \in R\colon\ s \geqslant 1$ and s is a ratio of odd integers$\}$, the following inequality holds:

$$e[(-e+a)^p - a^p] \ \leqslant\ -2^{1-p}e^{p+1}, \forall e, a \in R.$$

Lemma A.2 Let x, y be real variables. For any positive real numbers m and n, and positive real numbers a and b, the following inequality holds:

$$ax^m y^n \ \leqslant\ b|x|^{m+n} + \frac{n}{m+n}\left(\frac{m+n}{m}\right)^{-m/n} a^{(m+n)/n} b^{-m/n} |y|^{m+n}.$$

Lemma A.3 Let $p \in R^{\geqslant 1}_{odd}$, and x, y be real-valued functions. Then

$$|x^p - y^p| \ \leqslant\ p|x-y|(x^{p-1} + y^{p-1}).$$

Proof of Theorem 1. The proof includes two steps.

Step 1. Define $z_1 = x_1 - v_1$ and choose $V_1 = \frac{1}{2}z_1^2$, then from (1)-(2) we have

$$\begin{aligned}\dot{V}_1 \ &=\ d(t)z_1(x_2^{*p_1} - v_2^{q_1}) + d(t)z_1(x_2^{p_1} - x_2^{*p_1})\\ &=\ d(t)z_1(x_2^{*p_1} - (v_2^{\gamma_1})^{p_1}) + d(t)z_1(x_2^{p_1} - x_2^{*p_1}),\end{aligned} \tag{M.1}$$

where $\gamma_1 = \frac{q_1}{p_1} \geqslant 1$.

Choosing the virtual controller x_2^* as

$$x_2^* = -k_1 z_1 + v_2^{\gamma_1}, \tag{M.2}$$

where $k_1 > 0$ is a constant. By $(M.1)$-$(M.2)$ and Lemma A.1 we obtain

$$\begin{aligned}\dot{V}_1 \ &\leqslant\ -2^{1-p_1}d(t)k_1^{p_1}z_1^{p_1+1} + d(t)z_1(x_2^{p_1} - x_2^{*p_1})\\ &=\ -\frac{k_1^{p_1}}{2^{p_1-1}}d(t)z_1^{p_1+1} + d(t)z_1(x_2^{p_1} - x_2^{*p_1}).\end{aligned} \tag{M.3}$$

Step 2. Define $z_2 = x_2 - x_2^*$, from (1)-(2) and $(M.2)$ we have

$$\dot{z_2} \ =\ \dot{x_2} - \dot{x_2^*} = q(u^{p_2}) + \gamma_1 v_2^{\gamma_1 - 1} v_1^{q_2} + k_1 d(t)(x_2^{p_1} - v_2^{q_1}). \tag{M.4}$$

Choosing $V = V_1 + \frac{1}{2}z_2^2$, from $(M.3)$-$(M.4)$ we get

$$\begin{aligned}\dot{V} \ &=\ \dot{V}_1 + z_2\dot{z_2}\\ &\leqslant\ -\frac{k_1^{p_1}}{2^{p_1-1}}d(t)z_1^{p_1+1} + d(t)z_1(x_2^{p_1} - x_2^{*p_1})\\ &\quad + z_2(q(u^{p_2}) + \gamma_1 v_2^{\gamma_1-1}v_1^{q_2} + k_1 d(t)(x_2^{p_1} - v_2^{q_1})).\end{aligned} \tag{M.5}$$

By Lemmas A.2 and A.3, there exists a real function $\rho_1(x_2, x_2^*) \geqslant 0$ such that

$$\begin{aligned}z_1(x_2^{p_1} - x_2^{*p_1}) \ &\leqslant\ \rho_1(x_2, x_2^*)|z_1||z_2|\\ &\leqslant\ \frac{k_1^{p_1}}{2^{p_1}}z_1^{p_1+1} + \beta_1(k_1, \rho(\cdot))z_2^{\frac{1}{p_1}+1},\end{aligned} \tag{M.6}$$

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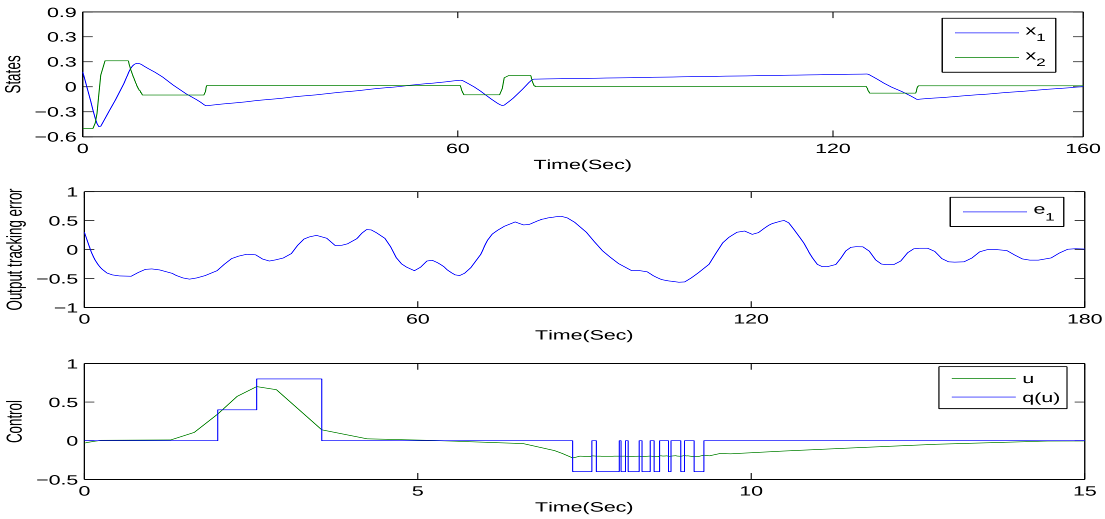

Figure B1 The responses of closed-loop system $(M.10)$-$(M.12)$.

where

$$\begin{aligned} \rho_1(\cdot) &= p_1(x_2^{p_1-1} + x_2^{*p_1-1}), \\ \beta_1(\cdot) &= \frac{2}{k_1}\frac{p_1}{p_1+1}\left(\frac{1}{p_1+1}\right)^{\frac{1}{p_1}} \rho_1^{\frac{p_1+1}{p_1}}(\cdot). \end{aligned}$$

Substituting $(M.6)$ into $(M.5)$ yields

$$\begin{aligned} \dot{V} \leqslant & -\tfrac{k_1^{p_1}}{2^{p_1-1}} d(t) z_1^{p_1+1} + \tfrac{k_1^{p_1}}{2^{p_1}} d(t) z_1^{p_1+1} + \beta_1(\cdot) d(t) z_2^{\frac{1}{p_1}+1} + z_2(q(u^{p_2}) + \gamma_1 v_2^{\gamma_1-1} v_1^{q_2} \\ & + k_1 d(t)(x_2^{p_1} - v_2^{q_1})) \\ = & -\tfrac{k_1^{p_1}}{2^{p_1}} d(t) z_1^{p_1+1} + \beta_1(\cdot) d(t) z_2^{\frac{1}{p_1}+1} + z_2(q(u^{p_2}) + \gamma_1 v_2^{\gamma_1-1} v_1^{q_2} + k_1 d(t)(x_2^{p_1} - v_2^{q_1})). \end{aligned} \tag{M.7}$$

By $(M.7)$ and Assumption 1 we have

$$\begin{aligned} \dot{V} \leqslant & -\tfrac{k_1^{p_1}}{2^{p_1}} \lambda z_1^{p_1+1} + \mu \beta_1(\cdot) z_2^{\frac{1}{p_1}+1} + z_2(q(u^{p_2}) + \gamma_1 v_2^{\gamma_1-1} v_1^{q_2} + k_1 \mu |x_2^{p_1} - v_2^{q_1}| sign(z_2)) \\ \leqslant & -\tfrac{k_1^{p_1}}{2^{p_1}} \lambda z_1^{p_1+1} - \left(\tfrac{k_1^{p_1-1}}{2^{p_1}}\lambda + k_d\right) z_2^2 + z_2 \bar{v} + z_2 q(u^{p_2}), \end{aligned} \tag{M.8}$$

where $k_d > 0$ is a design parameter and

$$\bar{v} = \mu \beta_1(\cdot) z_2^{\frac{1}{p_1}} + \left(\tfrac{k_1^{p_1-1}}{2^{p_1}}\lambda + k_d\right) z_2 + \gamma_1 v_2^{\gamma_1-1} v_1^{q_2} + k_1 \mu |x_2^{p_1} - v_2^{q_1}| sign(z_2). \tag{M.9}$$

Appendix B A Simulation Example

Consider the following system

$$\dot{x}_1 = (\tfrac{1}{2} + \tfrac{1}{4}\sin(t)) x_2, \quad \dot{x}_2 = q(u), \quad y = x_1. \tag{M.10}$$

The reference signal v_1 is generated by the following exosystem

$$\dot{v}_1 = (\tfrac{1}{2} + \tfrac{1}{4}\sin(t)) v_2^3, \quad \dot{v}_2 = -v_1. \tag{M.11}$$

Obviously, $0 < \lambda = \frac{1}{4} \leqslant d(t) \leqslant \frac{3}{4} = \mu$, Assumption 1 is satisfied. Let $e_1 = x_1 - v_1$ denote the output tracking error. By following the design procedure developed in Theorem 1, we get

$$\begin{aligned} \bar{v} &= \left(\tfrac{3\rho_1^2}{8k_1} + \tfrac{k_1}{8} + k_d\right) z_2 + 3v_2^2 v_1 + \tfrac{3}{4} k_1 \sqrt{(x_2 - v_2^3)^2} sign(z_2), \\ u &= -\tfrac{z_2 \bar{v}^2}{(1-\delta)\sqrt{z_2^2 \bar{v}^2 + \eta^2}}. \end{aligned} \tag{M.12}$$

In the simulation, we choose $\delta = 0.4, \eta = \sqrt{40}, k_1 = 2, \rho_1 = 1, k_d = 1$ and the initial values $x_1(0) = 0.18, x_2(0) = -0.5, v_1(0) = -0.12, v_2(0) = 0.05$. Fig. 1 describe the responses of closed-loop system $(M.10)$-$(M.12)$, which proves the effectiveness of the proposed scheme.