SCIENCE CHINA
Information Sciences

• **Supplementary File** •

A novel strategy to solve communication constraints for formation control of multi-AUVs

Zhenyu GAO[1] & Ge GUO[1,2,3*]

[1]*Department of Automation, Dalian Maritime University, Dalian,* 116026*, China;*
[2]*School of Control Engineering, Northeastern University Qinhuangdao, Qinhuangdao* 066004*, China;*
[3]*Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang* 110819*, China*

Appendix A The proof of Theorem 1

Subtracting (12) from (11) yields the following error dynamics:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 - k_2 e_1^{\alpha} - k_3 e_1^{\beta} + e_2 \\ \dot{e}_2 = -\lambda_1 e_1 - \lambda_2 e_1^{\bar{\alpha}} - \lambda_3 e_1^{\bar{\beta}} + e_f \end{cases} \tag{A1}$$

with $e_1 = p - \hat{p}$, $e_2 = \zeta - \hat{\zeta}$ and $e_f = f(p,\zeta) - f(p,\hat{\zeta})$.

In this paper, the proof contain three steps. First, we prove that system (A1) with the parameters provided in (13) is GAS. Then, we show that its approximating systems in $0-$ limit and $\infty-$ limit are also GAS, respectively.

Step 1: In this step, to show the error system (A1) is GAS, a candidate Lyapunov function for system (A1) is constructed as follows:

$$V = V_1 + V_2, \tag{A2}$$

where

$$V_1 = \frac{2\lambda_2}{1+\bar{\alpha}}(e_1^T e_1)^{\frac{1+\bar{\alpha}}{2}} + \frac{2\lambda_3}{1+\bar{\beta}}(e_1^T e_1)^{\frac{1+\bar{\beta}}{2}} + \frac{1}{2}(e_2 - k_1 e_1)^T (e_2 - k_1 e_1), \tag{A3}$$

and

$$V_2 = \frac{1}{2} e_2^T e_2. \tag{A4}$$

We calculate the time derivative of V_1 and V_2 along the trajectories of the augmented system (A1), respectively.

$$\begin{aligned}
\dot{V} =& \dot{V}_1 + \dot{V}_2 \\
=& 2\lambda_2 (e_1^T)^{\bar{\alpha}} \dot{e}_1 + 2\lambda_3 (e_1^T)^{\bar{\beta}} \dot{e}_1 + (e_2 - k_1 e_1)^T (\dot{e}_2 - k_1 \dot{e}_1) + e_2^T \dot{e}_2 \\
=& 2\lambda_2 (e_1^T)^{\bar{\alpha}} (-k_1 e_1 - k_2 e_1^{\alpha} - k_3 e_1^{\beta} + e_2) + 2\lambda_3 (e_1^T)^{\bar{\beta}} (-k_1 e_1 - k_2 e_1^{\alpha} - k_3 e_1^{\beta} + e_2) \\
&+ (e_2 - k_1 e_1)^T ((-\lambda_1 e_1 - \lambda_2 e_1^{\bar{\alpha}} - \lambda_3 e_1^{\bar{\beta}} + e_f) - k_1(-k_1 e_1 - k_2 e_1^{\alpha} - k_3 e_1^{\beta} + e_2)) + e_2^T (-\lambda_1 e_1 - \lambda_2 e_1^{\bar{\alpha}} - \lambda_3 e_1^{\bar{\beta}} + e_f) \\
=& -2k_2\lambda_2 (e_1^T e_1)^{\frac{\alpha+\bar{\alpha}}{2}} - k_1\lambda_2 (e_1^T e_1)^{\frac{\bar{\alpha}+1}{2}} - 2k_3\lambda_2 (e_1^T e_1)^{\frac{\beta+\bar{\alpha}}{2}} - 2k_3\lambda_3 (e_1^T e_1)^{\frac{\bar{\alpha}+\bar{\beta}}{2}} - k_1\lambda_3 (e_1^T e_1)^{\frac{\bar{\beta}+1}{2}} \\
&- 2k_3\lambda_3 (e_1^T e_1)^{\frac{\beta+\bar{\beta}}{2}} - 2\lambda_3 k_2 (e_1^T e_1)^{\frac{\bar{\beta}+\alpha}{2}} - \lambda_1 (2e_2 - k_1 e_1)^T e_1 + (2e_2 - k_1 e_1)^T e_f - k_1 (e_2 - k_1 e_1)^T (e_2 - k_1 e_1) \\
&+ k_1 (e_2 - k_1 e_1)^T (k_2 e_1^{\alpha} + k_3 e_1^{\beta}) \\
=& -2k_2\lambda_2 (e_1^T e_1)^{\frac{\alpha+\bar{\alpha}}{2}} - k_1\lambda_2 (e_1^T e_1)^{\frac{\bar{\alpha}+1}{2}} - 2k_3\lambda_2 (e_1^T e_1)^{\frac{\beta+\bar{\alpha}}{2}} - 2k_3\lambda_3 (e_1^T e_1)^{\frac{\bar{\alpha}+\bar{\beta}}{2}} - k_1\lambda_3 (e_1^T e_1)^{\frac{\bar{\beta}+1}{2}} - 2k_3\lambda_3 (e_1^T e_1)^{\frac{\beta+\bar{\beta}}{2}} \\
&- 2\lambda_3 k_2 (e_1^T e_1)^{\frac{\bar{\beta}+\alpha}{2}} - \lambda_1 (e_2 - k_1 e_1)^T e_1 - \lambda_1 e_2^T e_1 + (2e_2 - k_1 e_1)^T e_f + k_1 (e_2 - k_1 e_1)^T (e_2 - k_1 e_1) \\
&- k_1 (e_2 - k_1 e_1)^T (k_2 e_1^{\alpha} + k_3 e_1^{\beta}).
\end{aligned} \tag{A5}$$

On the basis of (12), we have

$$\begin{aligned}
&k_1 (e_2 - k_1 e_1)^T (k_2 e_1^{\alpha} + k_3 e_1^{\beta}) = k_1 k_2 (e_2 - k_1 e_1)^T e_1^{\alpha} + k_1 k_3 (e_2 - k_1 e_1)^T e_1^{\beta} \\
&\leqslant \frac{k_1(k_2+k_3)}{2} (e_2 - k_1 e_1)^T (e_2 - k_1 e_1) + \frac{k_1 k_2}{2} (e_1^T e_1)^{\frac{\bar{\alpha}+1}{2}} + \frac{k_1 k_3}{2} (e_1^T e_1)^{\frac{\bar{\beta}+1}{2}}.
\end{aligned} \tag{A6}$$

* Corresponding author (email: geguo@yeah.net)

Based on the class Cauchy inequality, one obtains

$$\begin{aligned}
&-\lambda_1(e_2-k_1e_1)^Te_1=\frac{\lambda_1}{k_1}(e_2-k_1e_1)^T(e_2-k_1e_1-e_2)\\
\leqslant&\frac{\lambda_1}{k_1}(e_2-k_1e_1)^T(e_2-k_1e_1)+\frac{\lambda_1}{2k_1}(e_2-k_1e_1)^T(e_2-k_1e_1)+\frac{\lambda_1}{2k_1}e_2^Te_2
\end{aligned}\tag{A7}$$

and

$$-\lambda_1e_2^Te_1=\frac{-\lambda_1}{k_1}e_2^Te_2+\frac{\lambda_1}{k_1}e_2^T(e_2-k_1e_1)\leqslant\frac{-\lambda_1}{k_1}e_2^Te_2+\frac{\lambda_1}{2k_1}(e_2-k_1e_1)^T(e_2-k_1e_1)+\frac{\lambda_1}{2k_1}e_2^Te_2.\tag{A8}$$

Under **Assumption 1**, one has

$$\begin{aligned}
&(2e_2-k_1e_1)^Te_f\leqslant(2e_2-k_1e_1)^T\varrho e_2=\varrho(e_2+e_2-k_1e_1)^Te_2\\
&=\varrho e_2^Te_2+\varrho(e_2-k_1e_1)^Te_2\leqslant\varrho e_2^Te_2+\varrho(\frac{1}{2}((e_2-k_1e_1)^T(e_2-k_1e_1)+e_2^Te_2)\\
&=\frac{3}{2}\varrho e_2^Te_2+\frac{\varrho}{2}(e_2-k_1e_1)^T(e_2-k_1e_1).
\end{aligned}\tag{A9}$$

On the light of (A2)-(A9), we have

$$\begin{aligned}
\dot V\leqslant&-2k_2\lambda_2(e_1^Te_1)^{\frac{\alpha+\bar\alpha}{2}}-(k_1\lambda_2-\frac{k_1k_2}{2})(e_1^Te_1)^{\frac{\bar\alpha+1}{2}}-2k_3\lambda_2(e_1^Te_1)^{\frac{\beta+\bar\alpha}{2}}-(k_1\lambda_3-\frac{k_1k_3}{2})(e_1^Te_1)^{\frac{\bar\beta+1}{2}}-2k_2\lambda_3(e_1^Te_1)^{\frac{\alpha+\bar\beta}{2}}\\
&-2k_3\lambda_3(e_1^Te_1)^{\frac{\beta+\bar\beta}{2}}-(\frac{\lambda_1}{4k_1}-\frac{3}{2}\varrho)e_2^Te_2-(k_1-\frac{\varrho}{2}-\frac{k_1(k_2+k_3)}{2}-\frac{2\lambda_1}{k_1})(e_2-k_1e_1)^T(e_2-k_1e_1)\\
=&-2k_2\lambda_2(e_1^Te_1)^{\frac{\alpha+\bar\alpha}{2}}-(k_1\lambda_2-\frac{k_1k_2}{2})(e_1^Te_1)^{\frac{\bar\alpha+1}{2}}-2k_3\lambda_2(e_1^Te_1)^{\frac{\beta+\bar\alpha}{2}}-(k_1\lambda_3-\frac{k_1k_3}{2})(e_1^Te_1)^{\frac{\bar\beta+1}{2}}-2k_2\lambda_3(e_1^Te_1)^{\frac{\alpha+\bar\beta}{2}}\\
&-2k_3\lambda_3(e_1^Te_1)^{\frac{\beta+\bar\beta}{2}}-\epsilon_1e_2^Te_2-\epsilon_2(e_2-k_1e_1)^T(e_2-k_1e_1)
\end{aligned}\tag{A10}$$

where $\epsilon_1=\frac{\lambda_1}{4k_1}-\frac{3}{2}\varrho$, $\epsilon_2=k_1-\frac{2\lambda_1}{k_1}-\frac{\varrho}{2}-\frac{k_1(k_2+k_3)}{2}$. According to parameters k_i, λ_i selected in (14), we can obtain $\epsilon_1>0$ and $\epsilon_2>0$. Then from (A10), it can be proved that the error dynamics system (A1) is GAS.

Step 2: In this step, we will show that the approximating system of (A1) in the ∞-limit is homogenous of positive degree l_∞ and is GAS. System (A1) can be written as

$$\begin{aligned}
\dot e_1&=e_2-k_3e_1^{\beta}+\hat f_1(e_1,e_2)\\
\dot e_2&=-\lambda_3e_1^{\bar\beta}+\hat f_2(e_1,e_2)
\end{aligned}\tag{A11}$$

where $\hat f_1(e_1,e_2)=-k_1e_1-k_2e_1^{\alpha}$ and $\hat f_2(e_1,e_2)=-\lambda_1e_1-\lambda_2e_1^{\bar\alpha}+e_f$.

Since $\beta>1$, $\bar\beta=2\beta-1$, one can obtain that system (A11) is homogeneous of degree $l_\infty=1$ with respect to the weight vector $[d_1,d_2]^T=[\frac{1}{\beta-1},\frac{\beta}{\beta-1}]^T$.

$$\begin{aligned}
\dot e_1&=e_2-k_3e_1^{\beta}\\
\dot e_2&=-\lambda_3e_1^{\bar\beta}.
\end{aligned}\tag{A12}$$

In addition, due to the fact that $0<\alpha,\bar\alpha<1$, $\beta>1$, $\bar\beta>1$, we have that $l_\infty+d_1=d_2=\beta d_1>d_1>\alpha 1_1$, $l_\infty d_2=\bar\beta d_1>d_1>\bar\alpha d_1$. Hence, we have that

$$\begin{aligned}
&\lim_{\varepsilon\longrightarrow0}\frac{\hat f_1(\varepsilon^{d_1}e_1,\varepsilon^{d_2}e_2)}{\varepsilon^{d_1+l_\infty}}\leqslant\lim_{\varepsilon\longrightarrow0}\frac{-k_2\varepsilon^{\alpha d_1}e_1^{\alpha}-k_1\varepsilon^{d_1}e_1}{\varepsilon^{d_1+l_\infty}}=0\\
&\lim_{\varepsilon\longrightarrow0}\frac{\hat f_2(\varepsilon^{d_1}e_1,\varepsilon^{d_2}e_2)}{\varepsilon^{d_2+l_\infty}}\leqslant\lim_{\varepsilon\longrightarrow0}\frac{|\lambda_2\varepsilon^{\bar\alpha d_1}e_1^{\bar\alpha}+\lambda_1\varepsilon^{d_1}e_1|+\varrho|\varepsilon^{d_2}e_2|}{\varepsilon^{d_2+l_\infty}}=0.
\end{aligned}\tag{A13}$$

This means that system (A12) is the approximating system of (A1), which is also homogenous. Here, to show that the approximating system is stable, we choose the Lyapunov function candidate $V_\infty=\frac{\lambda_3}{1+\beta}(e_1^Te_1)^{\frac{1+\bar\beta}{2}}+\frac{1}{2}e_2^Te_2$ and taking the derivative of V_∞ yields $\dot V_\infty\leqslant-k_3\lambda_3(e_1^Te_1)^{\frac{\beta+\bar\beta}{2}}$. It can be easily verified that the system (A12) is GAS.

Step 3: Similar to the previous step, we will show the approximating system of (A1) in the 0-limit is homogeneous of negative degree l_0 and is GAS in this step. Similar to Step 2, System (A1) is written as

$$\begin{aligned}
\dot e_1&=e_2-k_2e_1^{\alpha}+\hat g_1(e_1,e_2)\\
\dot e_2&=-\lambda_2e_1^{\bar\alpha}+\hat g_2(e_1,e_2)
\end{aligned}\tag{A14}$$

where $\hat g_1(e_1,e_2)=-k_1e_1-k_3e_1^{\beta}$ and $\hat g_2(e_1,e_2)=-\lambda_1e_1-\lambda_3e_1^{\bar\beta}+e_f$.

According to the fact that $\frac{1}{2}<\alpha<1$, $\bar\alpha=2\alpha-1$, we can obtain (A13) is homogeneous of degree $l_0=-1$ with respect to the weight vector $[r_1,r_2]^T=[\frac{1}{1-\alpha},\frac{\alpha}{1-\alpha}]^T$.

$$\begin{aligned}
\dot e_1&=e_2-k_2e_1^{\alpha}\\
\dot e_2&=-\lambda_2e_1^{\alpha}
\end{aligned}\tag{A15}$$

In the light of $0 < \alpha, \bar{\alpha} < 1$, $\beta_1 > 1$, $\bar{\beta} > 1$, we have that $l_0 + r_1 = r_2 = \alpha r_1 < r_1 < \beta r_1$, $l_0 r_2 = \bar{\alpha} r_1 < r_1 < \bar{\beta} r_1$. According to *Assumption 1*, one can obtain that

$$\begin{aligned}
&\lim_{\varepsilon \longrightarrow 0} \frac{\hat{g}_1(\varepsilon^{r_1} e_1, \varepsilon^{r_2} e_2)}{\varepsilon^{r_1 + l_0}} \leqslant \lim_{\varepsilon \longrightarrow 0} \frac{-k_1 \varepsilon^{r_1} e_1 - k_3 \varepsilon^{\beta r_1} e_1^{\beta}}{\varepsilon^{r_1 + l_0}} = 0 \\
&\lim_{\varepsilon \longrightarrow 0} \frac{\hat{g}_2(\varepsilon^{r_1} e_1, \varepsilon^{r_2} e_2)}{\varepsilon^{r_2 + l_0}} \leqslant \lim_{\varepsilon \longrightarrow 0} \frac{|\lambda_1 \varepsilon^{r_1} e_1 + \lambda_3 \varepsilon^{\bar{\beta} r_1} e_1^{\beta}| + \varrho |\varepsilon^{r_2} e_2|}{\varepsilon^{r_2 + l_0}} = 0.
\end{aligned} \tag{A16}$$

Therefore, it is concluded that system (A15) is the approximating system of (A1), which is homogenous with negative degree. We now show the approximating system (A15) is GAS. A candidate lyapunov function is choosen as $V_0 = \frac{\lambda_2}{1+\bar{\alpha}} (e_1^T e_1)^{\frac{1+\bar{\alpha}}{2}} + \frac{1}{2} e_2^T e_2$ and its time derivative with (A15) can be put into $\dot{V}_0 \leqslant -k_2 \lambda_2 (e_1^T e_1)^{\frac{\alpha + \bar{\alpha}}{2}}$. By analyzing, the system (A15) is GAS, too.

Combining **Lemma 1** and the results of **Steps 1, 2, 3**, we can conclude that the proposed velocity observer (12) is fixed-time stable. This completes the proof.

Appendix B Simulation

The initial states of the AUV are $p(0) = [0, -2, -4]$ and $\upsilon(0) = [0, 0, 0]$.

The parameters of velocity observer are selected as: $k_1 = \text{diag}(2, 2, 2)$, $k_2 = \text{diag}(5, 5, 5)$, $k_3 = \text{diag}(5, 5, 5)$, $\lambda_1 = \text{diag}(8, 8, 8)$, $\lambda_2 = \text{diag}(8, 8, 8)$, $\lambda_3 = \text{diag}(8, 8, 8)$, $\alpha = 0.8$, $\beta = 1.2$, $\bar{\alpha} = 0.6$ and $\bar{\beta} = 1.4$.

The simulation results are shown in Figs. 1-4.

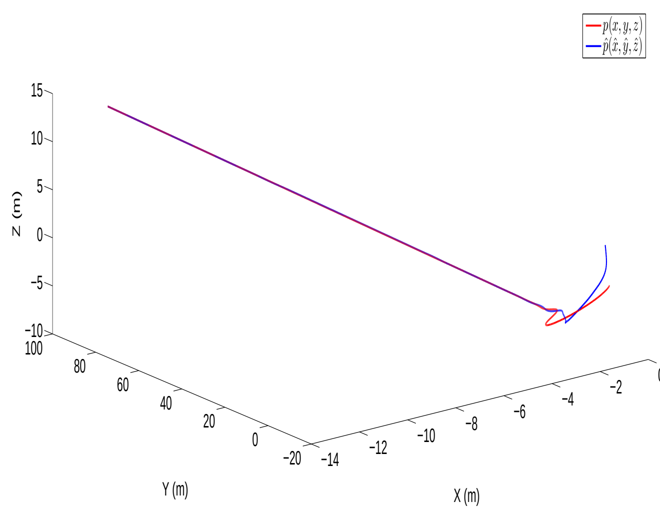
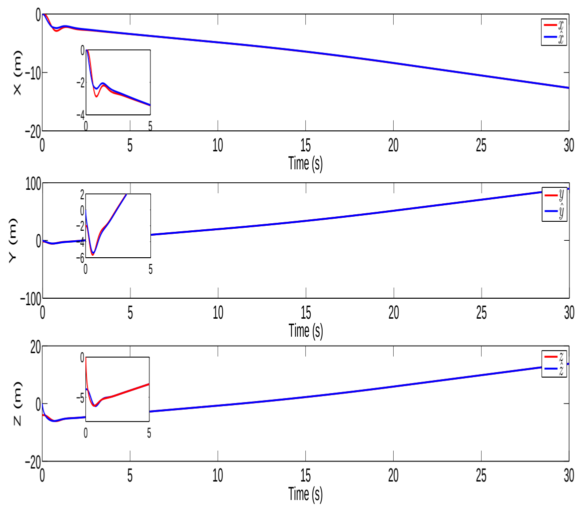


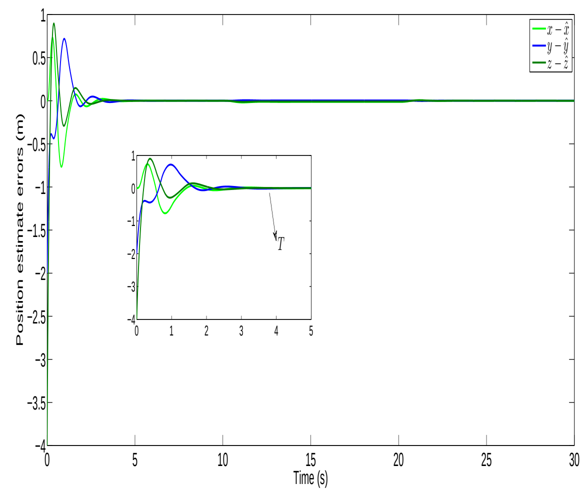
Fig. 1. Position vector $p(x, y, z)$ and $\hat{p}(\hat{x}, \hat{y}, \hat{z})$ of the AUV.

Fig. 1 depicts the position vector p and $\hat{p}$ of the AUV. Figs. 2-3 depict the state estimate results (i.e. position and velocity) of the designed fixed-time observer (12). It can be seen that the state estimate errors converge to zeros about 4s, which verifies that the proposed observer (12) is fixed-time convergent.

To prove the effectiveness of given communication mechanism, here, by defineε, Θ as the amount of data transferred per

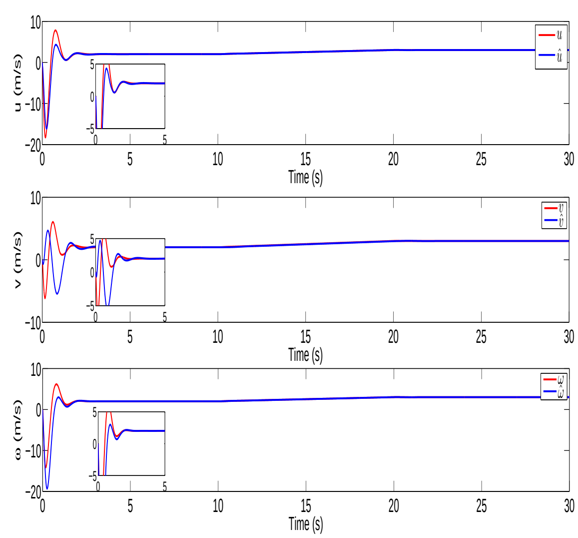


(a) Position vector $p(x, y, z)$ and $\hat{p}(\hat{x}, \hat{y}, \hat{z})$ of the AUV in each direction of motion.

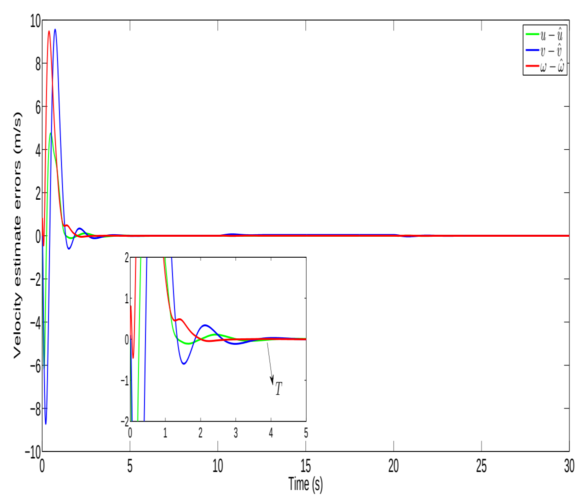


(b) Position estimate errors $p - \hat{p}$

Fig. 2. Position estimation results of the observer.



(c) Velocity vector $v(u, v, \omega)$ and $\hat{v}(\hat{u}, \hat{v}, \hat{\omega})$ of the AUV in each direction of motion.



(d) Velocity estimate errors $v - \hat{v}$

Fig. 3. Velocity estimation results of the observer.

unit time and the amount of data in the whole formation process, respectively. The simulation result is shown in Fig. 4.

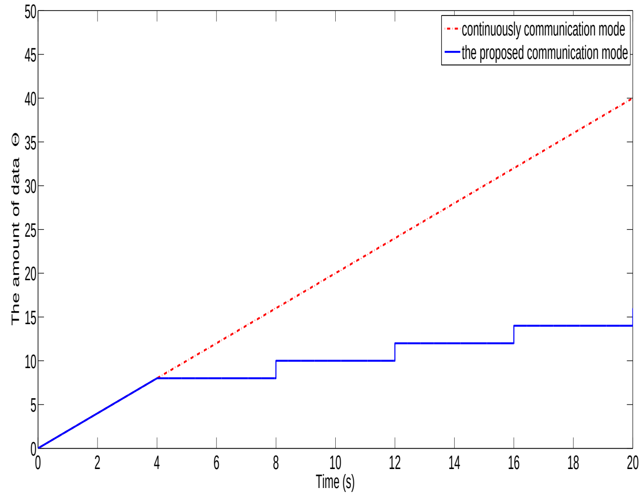


Fig. 4. The amount of data Θ with $\varepsilon = 2$.

As shown in Fig. 4, the proposed event-triggered communication scheme can reduce amount of data significantly.