SCIENCE CHINA
Information Sciences

• LETTER •

July 2021, Vol. 64 179203:1–179203:3
https://doi.org/10.1007/s11432-018-9695-2

Containment control of multi-agent systems with nonvanishing disturbance via topology reconfiguration

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Received 27 August 2018/Revised 31 October 2018/Accepted 10 December 2018/Published online 28 May 2020

Citation Shan Q H, Teng F, Li T S, et al. Containment control of multi-agent systems with nonvanishing disturbance via topology reconfiguration. Sci China Inf Sci, 2021, 64(7): 179203, https://doi.org/10.1007/s11432-018-9695-2

Dear editor,

Multi-agent systems (MASs) are ubiquitous in natural as well as artificial systems. Over the past several decades, an increasing number of researchers have devoted attention to distributed cooperative control problems of MASs such as mobile robots [1] and unmanned aircraft [2]. Recently, different containment control problems with multiple leaders have arisen, including finite-time coordination [3, 4], formation producing [5], and heterogeneous MASs [6]. In previous studies, the objective of containment control has been to make the followers converge to the convex hull formed by the leaders [7, 8]. However, in real-world applications for containment control of MASs, disturbances from the environment make it more reasonable to change the aforementioned convex hull into its interior points because the followers sometimes are not allowed to converge to the boundaries. They should be constrained to converge to interior points of a leaders-formed convex hull.

Consider an MAS comprising n agents among existing studies on containment control problem. A weighted digraph G=($\mathscr{V}$, $\mathscr{E}$, $\mathcal{A}$) comprises a node set $\mathscr{V} = \{1, \ldots, n\}$, an edge set $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ satisfying $a_{ij} > 0$ if $(j, i) \in \mathscr{E}$, otherwise $a_{ij} = 0$. Here, we assume that $(i, i) \notin \mathscr{E}$; hence, $a_{ii} = 0$ for all $i = 1, \ldots, n$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The Laplacian matrix $L = [l_{ij}]_{n\times n}$ of a weighted digraph G is defined as $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. Certainly, L satisfies $L\mathbf{1}_n = 0$. Each agent is regarded as a node and the element a_{ij} of the adjacent matrix denotes the weight on information link (j, i). The dynamics of the ith agent is described by

$$\dot{p}_i(t) = \sum_{i=1}^{n} a_{ij}(p_j(t) - p_i(t)), \tag{1}$$

where $p_i(t) \in \mathbb{R}^N$.

Assume that there are m leaders and $n-m$ followers. The Laplacian matrix L related to the communication digraph G can be partitioned as

$$L = \begin{bmatrix} \mathbf{0}_{m\times m} & \mathbf{0}_{m\times(n-m)} \\ L_1 & L_2 \end{bmatrix}, \tag{2}$$

where $L_1 \in \mathbb{R}^{(n-m)\times m}$ and $L_2 \in \mathbb{R}^{(n-m)\times(n-m)}$.

On the basis of [3], we assume that the communication digraph G has a directed spanning forest. All the eigenvalues of L_2 then have positive real parts, and each element of $-L_2^{-1}L_1$ is nonnegative. The sum of each row of $-L_2^{-1}L_1$ is 1.

The containment control problem is solved for MASs under the aforementioned communication digraph G. The followers converge to the convex hull formed by the leaders, including the boundaries. However, when the concerned MAS is disturbed by nonvanishing perturbation, it is necessary that followers cannot remain on the boundary of the convex hull formed by the leaders of MASs. The agents on the boundary will escape the convex hull because of the nonvanishing disturbance. Consider the containment control of MASs with a nonvanishing disturbance,

$$\dot{p}_i(t) = \sum_{i=1}^{n} a_{ij}(p_j(t) - p_i(t)) + d_i(t), \tag{3}$$

where $d_i(t)$ satisfying $\|d_i(t)\| \leqslant h_i$(constant) is a bounded disturbance occurring on the ith agent. From the nonlinear system analysis [9], the following inequations are concluded to hold under some conditions:

$$\|p_i(t)\| \leqslant k_i \mathrm{e}^{-\gamma_i(t-t_0)}\|x(t_0)\| \quad \text{and} \quad \|p_i(t)\| \leqslant b_i,$$

where k_i, γ_i, and b_i are some suitable constants. The authors of previous studies have proposed variable results for this problem. However, with increasing h, the bound of the norm of the MASs' states increases sharply. Hence, we notice the disturbance-tolerant character of the convex hull

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formed by multiple leaders in containment control problem of MASs to deal with the disturbance.

First, to use this disturbance tolerant character, an assumption is needed.

Assumption 1. There are at least $N+1$ leaders that are not on the same $(N-1)$-D hyperplane.

This assumption ensures that the convex hull formed by the leaders is an N-D convex set, which means that the concerned convex hull is disturbance-tolerant.

Furthermore, the relation between the position of an agent and its neighbors' is considered.

Definition 1. λ_{ij} is called the jth convex parameter of the ith follower agent, where λ_{ij} is the ith row and jth column element of $-L_2^{-1}L_1$.

This relation of the specific values of the elements in $-L_2^{-1}L_1$ is significant for the disturbance-tolerant character of the concerned MAS. Thus, we propose the following theorem.

Theorem 1. Assume that the communication digraph G has a directed spanning forest. For the ith agent, the jth convex parameter is the average of the jth convex parameters of the ith agent's neighbors.

From Theorem 1, we can easily conclude that the following relation exists between the element of $-L_2^{-1}L_1$ and the topology of the concerned MAS.

Corollary 1. Assume that the communication digraph G has a directed spanning forest. The sum of each row of $-L_2^{-1}L_1$ is 1 and the element of $-L_2^{-1}L_1$, λ_{ij}, is positive if and only if the ith leader has a directed path to the jth follower.

Corollary 1 reveals an evident and reasonable fact that if an agent just obtains the information from the leaders in the same boundary, it will converge to this boundary. Under the existing conditions of the topology of MASs with multiple leaders, the convex parameters of an agent may be zero; therefore, it lies on the boundary. From Corollary 1, we can easily change the position of the followers on the boundary of the convex hull formed by the leaders via a few steps. For instance, we can simply add a connection between the concerned agent and a follower agent within the convex hull to prevent it from converging to the boundary.

The next problem is how to adjust the convex parameters of agents to make their positions sufficiently far from the boundaries based on the variable bounds of agents' nonvanishing disturbance b_i.

Before a topology reconfiguration algorithm is proposed, an assumption should be noted.

Assumption 2. The upper bounds of nonvanishing disturbance of all agents, h_i, are known.

In view of the given h_i of the followers, their order from largest to smallest can be obtained as $h_{k_1} \geqslant h_{k_2} \geqslant \cdots \geqslant h_{k_{n-m}}$. We propose the following algorithm to solve the containment control problem with topology reconfiguration.

Step 1. Calculate the center O_L and the radius R_L of the inscribed ball of the convex hull formed by the leaders. Thus, the corresponding convex parameters $(\lambda_{r_1}, \ldots, \lambda_{r_m})$ of the center can be obtained as the optimal objective for topology reconfiguration.

Step 2. Eliminate the convex parameters of agents whose values are zeros by redesigning the topology of the MAS. In particular, add the communication between the agent with the zero convex and nonzero convex parameters based on proposed Corollary 1.

Step 3. The objective of the topology reconfiguration is to make undesirable convex parameters converge to the optimal one. In particular, based on $h_{k_1} \geqslant h_{k_2} \geqslant \cdots \geqslant h_{k_{n-m}}$ with priority from h_{k_1} to $h_{k_{n-m}}$, two options exist for a concerned agent whose neighbors are to be selected.

Adding a connection. Find an agent in the set of $\mathscr{V} - N_i$, which can be assumed as the kth agent. And add a connection between the ith and the kth agent to minimize the following value:

$$|\lambda_{ij} + \lambda_{kj} - 2\lambda_{r_j}|. \tag{4}$$

Through the aforementioned process, the sum of the concerned convex parameter and λ_{ij} will approach $2\lambda_{r_j}$ step by step.

Cutting a connection. If the set of $\mathscr{V} - N_i$ contains few agents, which may be a consequence of the large number of the ith agent's neighbors, we will search for the kth agent in the neighbor set N_i and cut it, which makes $\frac{\lambda_{ij}-\lambda_{kj}}{\lambda_{r_j}-\lambda_{ij}}$ the largest.

Condition to stop adjusting the convex parameters. Assume that the distance between the concerned follower agent and the boundary is R_{ip} ($i = 1, \ldots, n-m$; p is the ordinal number of the step of adjusting the convex parameters dealing with the ith follower agent). Then we will stop adjusting the convex parameters for the ith follower agent if the inequality $R_{ip} > h_{k_i}$ holds, or $R_{ip} - h_{k_i} > e$ in practical operation, where e is a small reasonable constant.

In the application of the aforementioned algorithm, we always use "cutting" to deal with MASs with a large number of communications and "adding" to deal with MASs with small number of communications.

Simulations. Consider the MAS with three leaders that form an equilateral triangle in a two dimensional (2-D) plane and with ten followers. Using the proposed method, we design the communication topology of the concerned MAS with a nonvanishing disturbance to solve the containment control problem.

$$\dot{p}_i(t) = \sum_{i=1}^{n} a_{ij}(p_j(t) - p_i(t)) + d_i(t), \tag{5}$$

where the nonvanishing disturbance is chosen as a sinusoidal or cosine function randomly:

$$\begin{aligned}
&d_i(t) = 0.5\sin(t-6), \quad d_i(t) = 0.5\sin(2t),\\
&d_i(t) = 0.5\sin(t-2), \quad d_i(t) = 0.5\cos(t),\\
&d_i(t) = 0.5\cos(t-4), \quad d_i(t) = 0.5\sin(2t-1),\\
&d_i(t) = 0.5\cos(t-1), \quad d_i(t) = 0.5\sin(3t),\\
&d_i(t) = 0.5\sin(t-1), \quad d_i(t) = 0.5\sin(4t-3).
\end{aligned}$$

Given an initial topology of the MAS (Figure S2(a)), the followers may not be containment controlled with nonvanishing disturbance (Figure S1(a)). Using our proposed algorithm for topology reconfiguration twice, we give the positions and the relative topology of the MAS in Figures S1(b) and (c) and S2(b) and (c) (see supporting information).

We conclude that redesigning the communication topology structure can effectively avoid the followers escaping the boundary of the convex hull formed by the concerned agents.

Conclusion and future work. In this study, containment control problems for leader-followers MASs with a nonvanishing disturbance were considered. A topology redesign method was proposed to handle this problem. Instead of constructing a highly required anti-disturbance distributed controller, we redesigned the communication topology structure on the basis of the norm of the concerned agents' disturbance to avoid the followers escaping the boundary of the convex hull formed by the agents. However, adjusting the convex parameters quantificationally through a distributed algorithm remains difficult and only a range of the concerned group of convex parameters can be obtained. Narrowing this range will be the main objective of our future work.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 61751202, 61751205, 61572540, 61803064), Science & Technology Innovation Funds of Dalian (Grant No. 2018J11CY022), Natural Science Foundation of Liaoning Province (Grant No. 20180550082), and Fundamental Research Funds of Dalian Maritime University (Grant No. 3132018157).

Supporting information Appendixes A and B, and Figures S1 and S2. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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