SCIENCE CHINA
Information Sciences

• **Supplementary File** •

Containment Control of Multi-agent Systems with Nonvanishing Disturbance via Topology Reconfiguration

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Appendix A Proof of Theorem 1

Theorem 1 Assume that the communication digraph G has a directed spanning forest. For the ith agent, the jth convex parameter is the average of ones of ith agent's neighbors.

Proof. Since the communication digraph G has a directed spanning forest, L_2 is a nonsingular matrix. Therefore, it can be obtained by the fact that $L_1 + L_2\Lambda = \mathbf{0}_{(n-m)\times m}$, where $\Lambda = -L_2^{-1}L_1$ formed by $\lambda_{(m+i)j}$ and $\mathbf{1}_{pq}$ or $\mathbf{0}_{pq}$ denotes $p\times q$ matrix and all its elements are 1 or 0. We can conclude that

$$\begin{bmatrix} a_{(m+1)1} & \cdots & a_{(m+1)m} \\ \cdots & \cdots & \cdots \\ a_{(m+i)1} & \cdots & a_{(m+i)m} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}_{(n-m)\times m} + \begin{bmatrix} -\sum_{k_1=1}^{n-m} a_{(m+1)k_1} & \cdots & a_{(m+1)n} \\ \cdots & \cdots & \cdots \\ a_{(m+i)(m+1)} & \cdots & a_{(m+i)n} \\ \cdots & \cdots & \cdots \\ a_{n(m+1)} & \cdots & -\sum_{k_{n-m}=1}^{n-m} a_{nk_{n-m}} \end{bmatrix}_{(n-m)\times(n-m)}$$

$$\times \begin{bmatrix} \lambda_{(m+1)1} & \cdots & \lambda_{(m+1)m} \\ \cdots & \cdots & \cdots \\ \lambda_{(m+i)1} & \cdots & \lambda_{(m+i)m} \\ \cdots & \cdots & \cdots \\ \lambda_{n1} & \cdots & \lambda_{nm} \end{bmatrix}_{(n-m)\times m} = \begin{bmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{bmatrix}_{(n-m)\times m}. \tag{A1}$$

We choose any element denoted as $\lambda_{(m+i)j}$. Noting that the jth row of Λ, $\{\lambda_{(m+1)j}, \ldots, \lambda_{nj}\}^T$, the element in $(m+i)$th line jth row of L_1, $a_{(m+i)j}$, and the $(m+i)$th line of L_2, $\{a_{(m+i)(m+1)}, \ldots, -\sum_{k=1}^{n} a_{(m+i)k}, \ldots, a_{(m+i)n}\}$, we can obtain that,

$$a_{(m+i)j} + \lambda_{(m+1)j}a_{(m+i)(m+1)} + \cdots - \lambda_{(m+i)j}\sum_{k_i=1}^{n-m} a_{(m+i)k_i} + \cdots + \lambda_{nj}a_{(m+i)n} = 0. \tag{A2}$$

As a result,

$$a_{(m+i)j} + \sum_{q\neq m+i,\ q=1,\ldots,n-m} \lambda_{(m+q)j}a_{(m+i)q} - \lambda_{(m+i)j}\sum_{k=1}^{n} a_{(m+i)k} = 0. \tag{A3}$$

The parameter λ_{ij} reflects the position in the convex hull formed by leaders. As a result, without loss of generality, we denote

$$\lambda_{pq} = \begin{cases} 0 & p\neq q \\ 1 & p = q \end{cases} \qquad p,q = 1,\ldots,m. \tag{A4}$$

Then, from (A3), (A4) and $a_{(m+i)i} = 0$, ones can obtain that

$$\lambda_{(m+i)j} = \frac{1}{\sum_{k=1}^{n} a_{(m+i)k}}\left[\sum_{q=1}^{n} a_{(m+i)q}\lambda_{(m+q)j}\right]. \tag{A5}$$

That implies that for the ith agent, the jth convex parameter is the average of ones of its neighbors.

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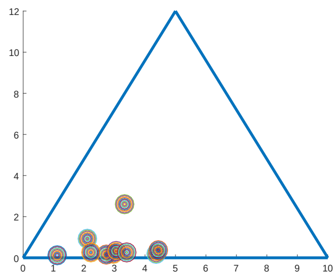

Fig. 1 (a)

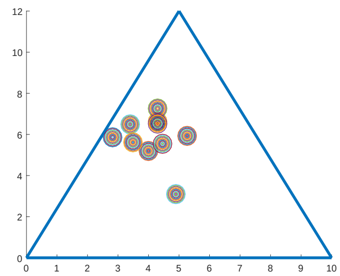

Fig. 1 (b)

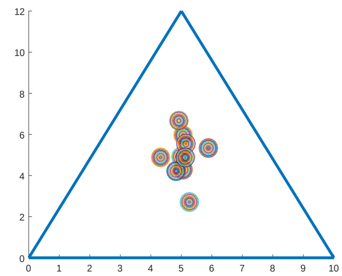

Fig. 1 (c)

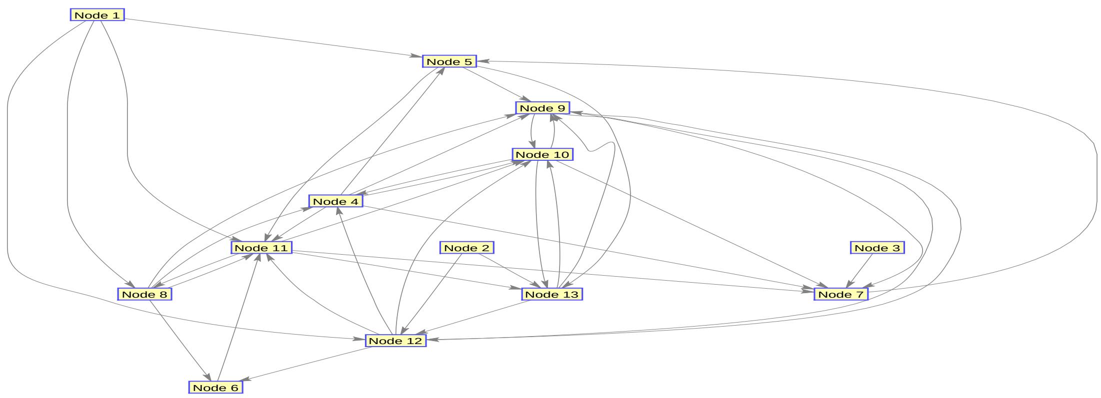

Fig. 2 (a)

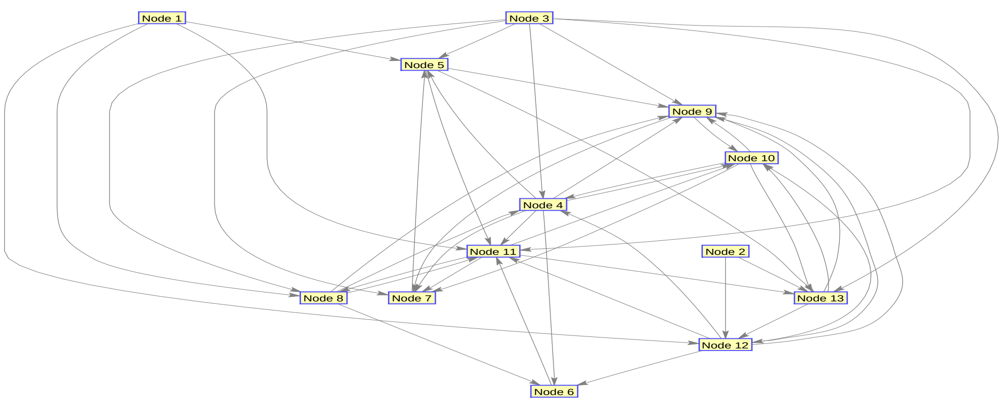

Fig. 2 (b)

Appendix B Proof of Corollary 1

Corollary 1 Assume that the communication digraph G has a directed spanning forest. The sum of each row of $-L_2^{-1}L_1$ is 1 and the element of $-L_2^{-1}L_1$ λ_{ij} is positive if and only if the ith leader has a directed path to jth follower.

Proof. Since the communication digraph G has a directed spanning forest, ones can conclude that the sum of each row of $-L_2^{-1}L_1$ is 1 and all the follower agents have neighbours.

Sufficiency: The apagoge is utilized. Assume that one of the element of $-L_2^{-1}L_1$ λ_{ij} is zero. From (A5) in Theorem 1, since $\lambda_{qj} \geqslant 0$, $q = 1, \ldots, n$, it can be obtained that all $\lambda_{qj} = 0$, which means that there is no connection between the jth leader to any follower agent. That is in contradiction to the condition that the ith leader has a directed path to jth follower.

Necessity: Noting (A5), $\lambda_{(m+i)j} > 0$ implies that there are some $\lambda_{q_1 j} > 0$, where we denote $q_1 \in N_1$ without loss of generality. Then check the neighbours of $\lambda_{q_1 j} > 0$, and there are some $\lambda_{q_2 j} > 0$ where $q_2 \in N_2$ from (A5). Continue this operation, since the communication digraph G has a directed spanning forest, $\{1, \ldots, n+m\} = N_1 \cup N_2 \cup \cdots \cup N^*$. As a result, the path $i \to q_1 \to q_2 \to \cdots \to q^*$ is the directed path from ith leader to jth follower.

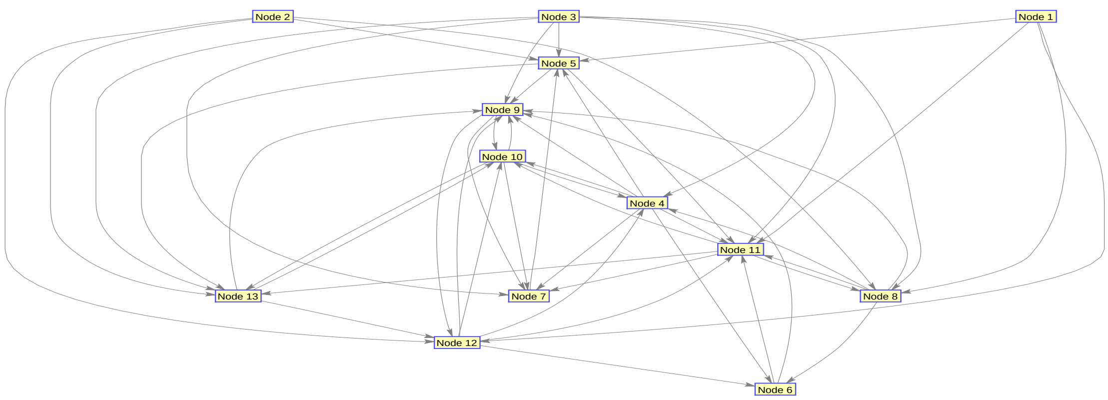
Node 2
Node 3
Node 1
Node 5
Node 9
Node 10
Node 4
Node 11
Node 13
Node 7
Node 8
Node 12
Node 6

Fig. 2 (c)