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New health-state assessment model based on belief rule base with interpretability

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Abstract Health-state assessment is the foundation of optimal-maintenance decision-making for complex systems to maintain reliability and safety. Generating the assessment results in a convincing and interpretable way to avoid potential risks is of great importance. Belief rule base (BRB) as an interpretable model performs well in health-state assessment. However, the interpretability of a BRB-based model may be lost during the optimization process, which is expressed mainly as three problems: expert knowledge is not effectively used in the optimization process; the optimized rules of BRB may be in conflict with real systems; and some parameters get over-optimized, which may affect experts' initial judgment. Three concepts — "searching intensity", "interpretability constraint of belief distribution", and "rule-activation factor" — are defined to address these problems. Using these concepts, we propose a new health-state assessment model based on the interpretable BRB and a new optimization method to improve the accuracy and preserve the interpretability of the new model. To demonstrate the effectiveness of the proposed model, we conducted an aero-engine case study.

Keywords health-state assessment, belief rule base, interpretability, complex systems

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1 Introduction

Complex systems such as the aerospace engine and the power grid play an important role in a nation's economy, and therefore their reliability and safety have received a lot of attention [1, 2]. Health-state assessment methods can identify the weak links in complex systems and thereby help to further improve system reliability by the application of various resources. However, unreliable assessment results can lead to potential risks, such as irrational maintenance decisions and delayed alarm information [3, 4]. Therefore, it is important to generate reliable and convincing assessment results in support of optimal decision-making for complex-system maintenance.

In health-state assessment, there are three types of models: (1) A data-driven model is based on a large sample population, such as a support vector machine or an artificial neural network. However, the rationality of a data-driven model's results may be unconvincing owing to the nontransparent modelling process [5, 6]. (2) A white-box model can provide a transparent modelling process and an interpretable result. However, it may be impractical owing to the challenge of extracting precise mathematical relationships from complex systems [7]. (3) A gray-box model, as a compromise between the data-driven and white-box models, is established by using limited expert knowledge and data samples, thereby preserving both modelling accuracy and model interpretability to a certain degree. Our proposed belief rule base (BRB) model, a typical gray-box model, relies on generalizing the fuzzy rule base (FRB) with the belief

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structure [8, 9]. Compared with FRB, BRB can handle various uncertainties with a better knowledge-representation scheme to explain real-world systems in a more transparent way [8, 9]. Thus, it has been widely applied in fault diagnosis and state assessment [10–12].

Engineering practice shows that an accurate model for health-state assessment can be constructed by using a data-driven model when a lot of data samples can be obtained. Generally, the more data samples that are used, the more accurate the constructed model can be. However, a more frequently encountered situation is that a complex system cannot be tested frequently and therefore yields only limited test-data samples. For instance, because the aero-engine is expensive and has only a short useful life, data samples representing the whole life cycle are hard to obtain. Fortunately, a lot of experience and knowledge can be accumulated in long-term practice. For this situation, the BRB-based health-state assessment model may be a sound choice, as it can use expert knowledge and limited data samples effectively, with good modelling accuracy and model interpretability [9, 13].

Because of the limitations of expert knowledge, the initial BRB-based health-state assessment model is inaccurate, which may reduce the credibility of the assessment process. To improve its performance, an offline optimization model was first proposed by Yang et al. [14] and then was further extended and analyzed by other researchers [15–17]. To satisfy real-time requirements in certain applications of the BRB model, Zhou et al. [10, 18, 19] proposed the online updating approach for BRB parameters and structure. Yang et al. [14] argued that the initial BRB model should be locally optimized by using both expert judgment and data samples. In effect, this helps ensure that a rational-optimization process does not destroy the interpretability of the initial BRB model [14]. Addressing this problem, the expert intervention was first introduced by Zhou et al. [10] into the online updating process to guarantee the rationality of the updated BRB model. However, the same problem has been ignored in the offline optimization process in many previous studies. Taking the safety assessment of the WD615 diesel engine as an example [20], the belief degree in one optimized rule is $\{0.2370, 0.0244, 0.7386\}$, which indicates that the support degrees for the normal state, the minor fault, and the serious fault are 0.237, 0.0244, and 0.7386, respectively. This is an irrational result that conflicts with reality. The loss of the interpretability and the rationality behind rules may affect the understanding of real systems and could lead to some risks in further decision-making.

This paper focuses on three problems of interpretability in the BRB-based health-state assessment model: expert knowledge is not effectively used in the optimization process; the optimized rules of the BRB model may be in conflict with real systems; and some parameters get over-optimized, which may affect experts' initial judgment. To address the first problem, searching intensity is introduced to achieve local optimization based on expert knowledge. To address the second problem, an interpretability constraint of belief distribution is constructed to guarantee the effectiveness and the rationality of rules. To address the third problem, a rule-activation factor is defined to determine key parameters to use in optimization. Accordingly, we propose a new health-state assessment model based on the interpretable BRB model.

The remainder of this paper is organized as follows. In Section 2, the BRB-based health-state assessment model is introduced, and the interpretability of the model is summarized. In Section 3, the problems of BRB-based model interpretability and their solutions are analyzed. In Section 4, we propose a new health-state assessment model based on the interpretable BRB model. In Section 5, we discuss a case study conducted to verify the effectiveness of the proposed model. In Section 6, we summarize our conclusion.

2 BRB-based health-state assessment model

For some complex systems, it may be expensive to obtain adequate data samples while the expert knowledge accumulated in long-term practice is available. In this situation, the BRB-based health-state assessment model can effectively fuse the limited data samples and the expert knowledge to generate accurate results. The BRB-based health-state assessment model is introduced briefly in Subsection 2.1 and the model interpretability is discussed in Subsection 2.2.

2.1 The description and reasoning of the health-state assessment model based on BRB

To describe the relationship between features and system health status, the kth belief rule in BRB-based health-state assessment is constructed as [8]

$$R_k : \text{IF } X_1 \text{ is } A_1^k \wedge \cdots \wedge X_{T_k} \text{ is } A_{T_k}^k, \text{ THEN}\{(D_1, \beta_{1k}), (D_2, \beta_{2k}), \ldots, (D_N, \beta_{Nk})\}, \left(\sum_{n=1}^{N} \beta_{nk} \leqslant 1\right)$$
$$\text{with a rule weight } \theta_k \text{ and attribute weights } \delta_i \ (k = 1, 2, \ldots, L; i = 1, 2, \ldots, T_k), \quad (1)$$

where $X_1, \ldots, X_{T_k}$ are the features that can reflect the system health states. A_i^k $(i = 1, \ldots, T_k)$ denotes the referential value of the ith feature X_i $(i = 1, \ldots, T_k)$. θ_k is the weight of the kth rule representing the relative importance to other rules. δ_i $(i = 1, 2, \ldots, T_k)$ denotes the weight of ith feature representing the relative importance to other features. L denotes the number of rules. β_{nk} $(n = 1, 2, \ldots, N)$ represents the belief degree of the health state D_n.

In the application, the model input in different forms is first transformed into a belief distribution, which is further used to calculate the activation weights of belief rules. Then, these rules are fused by the evidential reasoning (ER) approach to generate the final result [8]. The specific reasoning process is expressed as follows:

Step1. Transform quantitative and qualitative information into the belief distribution as

$$S(x_i) = \{(A_{i,j}, a_{i,j}), i = 1, \ldots, M; j = 1, \ldots, J_i\}. \quad (2)$$

Step2. Calculate the rule activation weight as follows:

$$w_k = \theta_k \prod_{i=1}^{T} (a_i^k)^{\bar{\delta}_i} \Bigg/ \left(\sum_{l=1}^{L} \theta_l \prod_{i=1}^{T} (a_i^k)^{\bar{\delta}_i} \right), \quad \bar{\delta}_i = \delta_i \Big/ \max_{i=1,\ldots,T_k} \{\delta_i\}, \quad (3)$$

where $\theta_k \in [0, 1]$. $\bar{\delta}_i$ denotes the normalized weight of the ith antecedent attribute.

Step3. Generate the final belief degree using the analytical ER algorithm as follows:

$$\beta_n = \frac{\left[\prod_{k=1}^{L} (w_k \beta_{n,k} + \lambda_{n,i}^k) - \prod_{k=1}^{L} (\lambda_{n,i}^k)\right]}{\left[\sum_{n=1}^{N} \prod_{k=1}^{L} (w_k \beta_{n,k} + \lambda_{n,i}^k) - (N-1) \prod_{k=1}^{L} (\lambda_{n,i}^k)\right] - \left[\prod_{k=1}^{L} (1 - w_k)\right]},$$
$$\lambda_{n,i}^k = 1 - w_k \sum_{i=1}^{N} \beta_{i,k}. \quad (4)$$

Step4. The final belief distribution is expressed as

$$S(A^*) = \{(D_n, \beta_n); n = 1, \ldots, N\}, \quad (5)$$

where A^* denotes the actual input vector. Let $u(D_n)$ denote the utility of D_n, and the expected utility of $S(A^*)$ can be expressed as

$$u(S(A^*)) = \sum_{n=1}^{N} u(D_n) \beta_n. \quad (6)$$

2.2 The interpretability of the BRB-based health-state assessment model

The health-state assessment is the basis to realize the optimal maintenance decision-making of complex systems, which requires the assessment process to be interpretable. The interpretability refers to the ability of models to express system behaviors in an understandable way [21, 22]. For the health-state assessment, the interpretability is conductive to providing more information about the system.

Different from black box models, the expert knowledge accumulated in long-term practice can be introduced into the BRB-based model intuitively, which is the crucial source of interpretability [11,21,23]. The BRB-based model can be summarized as follows:

• **Model structure** refers to the structure of the hierarchical BRB. There are two types, the mechanism-information-based BRB (MIBRB) and the expert-knowledge-based BRB (EKBRB) [16]. The structure of MIBRB is established by using clear design principles, which corresponds to the structure of the real system. The structure of EKBRB is determined according to expertise, which is friendly to human understanding.

• **Rules** describe the relationship between the attributes and the consequence in a semantic form. Moreover, the parameters of rules such as attribute weights, referential values, and belief degrees are obtained by using system principles, which have certain meanings [8]. For example, the nominal output of the constant voltage source (type: NPS306W) is 24±0.003 V, which can be used as the referential value.

• **Reasoning engine** is transparent and reliable. As a typical reasoning engine, ER approach is logical and comprehensible, which is consistent with the human thinking process [5, 8].

The transparent frame and the ability to use expert knowledge make the interpretability an inherent characteristic of the BRB-based health-state assessment model [8]. However, it may be destroyed by the irrational optimization process. In Section 3, the problem is formulated and the corresponding solution is discussed in detail.

3 The problems and solutions for preserving the interpretability of BRB

To overcome the limitation of the expert knowledge, the initial BRB-based health-state assessment model should be optimized by using data samples [14]. The optimization of BRB is usually conducted by using two types of optimization algorithms: the gradient-descent-based algorithm and the intelligent optimization algorithm [13, 16, 17, 24]. In current studies, the intelligent optimization algorithm is more widely adopted due to its fast convergence speed and high search accuracy [13, 16, 17]. Focusing on the optimization process by the intelligent optimization algorithm, three problems of model interpretability are discussed in this paper.

Problem 1. The expert knowledge is not effectively used in the model optimization process.

The expert knowledge accumulated in long-term practice is the foundation of model interpretability. Therefore, the BRB-based assessment model should be locally optimized by using the initial judgment from experts, which aims at improving its modelling accuracy under interpretability requirements. However, in many related studies [17, 20, 25, 26], the initial judgment of experts is ignored in the model optimization. For example, in the safety assessment of the diesel engine conducted by Li et al. [20], the BRB model is optimized by using the differential evolution (DE) algorithm, whose initial population is generated by

$$\boldsymbol{\Omega}_i = \boldsymbol{\Omega}_i^L + \mathrm{rand}(n_X, m_X)(\boldsymbol{\Omega}_i^U - \boldsymbol{\Omega}_i^L)\ (i = 1, \ldots, \mathrm{NP}), \tag{7}$$

where $\boldsymbol{\Omega}$ denotes the parameter vector of BRB. $\boldsymbol{\Omega}_i^U$ and $\boldsymbol{\Omega}_i^L$ are the upper and lower bounds of BRB parameters respectively. NP is the population size. It can be seen that the initial population is generated randomly in the whole solution space without considering the initial judgment information, which is irrational for the health-state assessment.

Aiming at Problem 1, a new concept "searching intensity" is defined as follows.

Definition 1 (searching intensity)**.** Suppose that the searching domain near to the initial judgement is denoted by $\boldsymbol{S}$. The searching intensity is denoted by $P(\boldsymbol{\Omega})$, $\boldsymbol{\Omega} \in \boldsymbol{S}$, which is used to describe the sampling possibility in $\boldsymbol{S}$. The larger the search intensity is, the larger the probability of sampling in $\boldsymbol{S}$ is.

To achieve the local searching, $P(\boldsymbol{\Omega})$ should satisfy two properties: (1) regularity: $\sum P(\boldsymbol{\Omega}) = 1, \boldsymbol{\Omega} \in \boldsymbol{S}$; (2) unimodal distribution: there are only one peak value (denoted by $P(\boldsymbol{\Omega}_q)$). As shown in Figure 1, the value of $P(\boldsymbol{\Omega})$ and the distance (denoted by $D(\boldsymbol{\Omega}, \boldsymbol{\Omega}^0)$) between $\boldsymbol{\Omega}$ and the initial point $\boldsymbol{\Omega}^0$ have the negative correlation, which means that a larger sampling probability is set to the domain close to $\boldsymbol{\Omega}^0$.

Due to the limitation of experts, the credibility of the initial judgement should also be considered. The lower the credibility is, the larger the searching domain is. In view of this, Euclidean distance is introduced as follows to quantify $D(\boldsymbol{\Omega}, \boldsymbol{\Omega}^0)$.

$$D(\boldsymbol{\Omega}, \boldsymbol{\Omega}^0) = \sqrt{(\boldsymbol{\Omega} - \boldsymbol{\Omega}^0)^{\mathrm{T}} \sigma^{-1} (\boldsymbol{\Omega} - \boldsymbol{\Omega}^0)}, \tag{8}$$

where σ is the covariance matrix. It can be used to reflect the credibility of the initial judgement. The lower σ is, the higher the credibility is, the smaller the max value of $D(\boldsymbol{\Omega}, \boldsymbol{\Omega}^0)$ could be, which means that the searching domain should be closer to $\boldsymbol{\Omega}^0$. To describe the distribution of $P(\boldsymbol{\Omega})$, the normal distribution is selected as Eq. (9), which satisfies the properties mentioned above.

$$P(\boldsymbol{\Omega}; \boldsymbol{\Omega}^0, \sigma) = \frac{1}{\sqrt{(2\pi)^n |\sigma|}} \exp\left(-\frac{1}{2} D(\boldsymbol{\Omega}, \boldsymbol{\Omega}_0)^2\right). \tag{9}$$

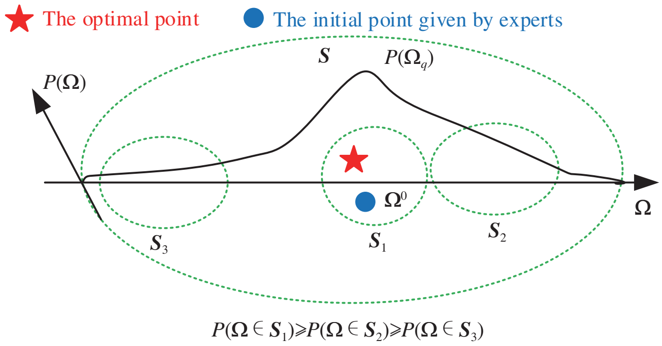

Figure 1 (Color online) The relationship between $P(\boldsymbol{\Omega})$ and $D(\boldsymbol{\Omega}, \boldsymbol{\Omega}^0)$.

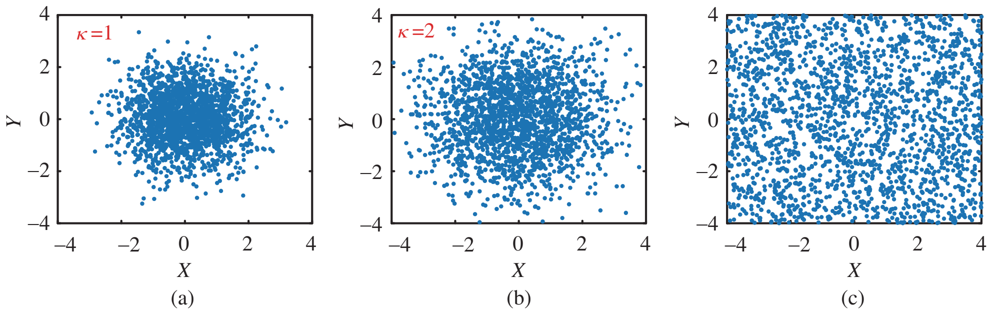

Figure 2 (Color online) 2000 points generated by using (9) with $\kappa = 1, 2$ and (7). (a) The point distribution when $\kappa = 1$; (b) the point distribution when $\kappa = 2$; (c) the point distribution generated by (7).

Based on Eq. (9), a clear explanation is shown in Figure 2, where the characteristic of $P(\boldsymbol{\Omega})$ is analyzed by using a bivariate normal distribution. Suppose $\boldsymbol{\Omega}^0 = (0, 0)$ and $\sigma = \kappa \times \boldsymbol{I}$ $(\kappa = 1, 2)$, where $\boldsymbol{I}$ is the identity matrix. It can be seen from Figures 2(a) and (b) that the searching range of $\boldsymbol{\Omega}$ increases while σ increases. That is to say, when the credibility becomes lower, the searching range of $\boldsymbol{\Omega}$ becomes larger. Figure 2(c) shows the $\boldsymbol{\Omega}$ generated by Eq. (7), which indicates that it is an equal-probability random process different from Figures 2(a) and (b).

Remark 1. In current studies, there is a lack of the unified definition of the credibility of expert knowledge due to its subjectivity. It is well accepted that the credibility of expert knowledge belongs to the concept of statistics, which is determined according to historical data and adjusted by long-term practice. σ can also be determined in this way.

Problem 2. The optimized rules may be conflicted with the real system.

The belief rule provides a semantic description of the relationship between system features and health states, which is a crucial presentation of the model interpretability. Based on the rule, expert knowledge can be introduced intuitively into the model as parameters so that health-state assessment results can be generated reasonably. However, in current researches, many wrong rules conflicted with the real system are generated in the optimization process. Taking the health assessment of central navigation computer (CNC) milling machine as an example [4], the optimized rule is presented as

$$\begin{aligned}
&\text{IF Mean is Middle} \wedge \text{Kurtosis is High,}\\
&\text{THEN the health condition is } \{(\text{normal}, 0.5573), (\text{medium}, 0.1352), (\text{serious}, 0.3076)\},\\
&\text{with } \ \theta = 0.86,\ \delta_1 = 1,\ \text{and}\ \delta_2 = 0.8,
\end{aligned} \tag{10}$$

where the belief degrees of "the health state is normal" and "the health state is serious" are both larger than "the health state is medium". It is contradictory and impractical for the health assessment of CNC milling machine. Thus, more attention should be paid to the rationality of the optimized rule.

The interpretability of a belief rule is mainly reflected in its parameters such as the attribute weight $\delta_t \in [0, 1]$, the rule weight $\theta_k \in [0, 1]$ and the belief degree $\beta_{nk}, \sum_n^N \beta_{nk} \in [0, 1]$. Generally, the initial values of these parameters are given based on the analysis to the real system. In the optimization process, these parameters can be well constrained by the feasible interval from experts while the distribution of

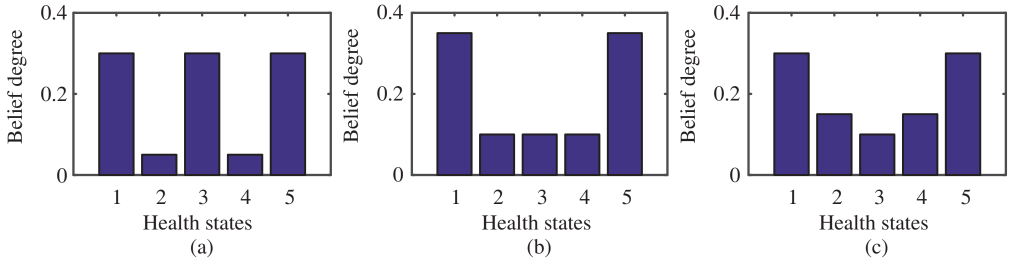

Figure 3 (Color online) Conflict belief distributions of (a) W-shape, (b) U-shape, and (c) V-shape in the health-state assessment.

belief degrees cannot. It should be further considered.

Aiming at this problem, the interpretability constraint of the belief distribution is defined as follows.

Definition 2 (interpretability constraint of belief distribution). The interpretability constraint of the distribution is expressed as

$$\beta_k \sim C_k \quad (k = 1, \ldots, L), \tag{11}$$

where C_k is the interpretability constraint of belief distribution in the kth rule. The interpretability constraint of belief distribution differs in different engineering practices. It is flexible without a unified form, which should be an acceptable distribution consistent with common sense and real system principles.

C_k is determined by analyzing the real system. In the health-state assessment, the basic constraint is that the conflict states should not be supported with a high belief degree concurrently, which can be expressed as

$$C_k \to \beta_k \notin \{\beta | (\beta_n \leqslant \beta_{n-1}) \& (\beta_n \leqslant \max(\beta_{n+1}, \ldots, \beta_N)),\ n = 2, \ldots, N-1\}, \quad k = 1, \ldots, L, \tag{12}$$

where β_n is the belief degree of D_n in the kth rule. Some examples of wrong belief distributions are shown in Figure 3.

Actually, the above basic constraint can be further enhanced by using the system principle when necessary. Taking the pipeline leak detection as an example, Zhou et al. [10] set different constraints on the belief distribution of different rules according to the historical information, the mass balance principle, and the running patterns of a pipeline leak. This preserved the initial judgment information and the rationality in the optimized rules.

Remark 2. In the process of establishing the initial rules, the belief distribution is determined by the knowledge and data of the system, which satisfies the interpretability constraint. Moreover, the local searching process mentioned above is also conductive to making the optimized belief distribution satisfy the interpretability constraint.

Problem 3. Some parameters of BRB are over-optimized.

Generally, the accumulated expert knowledge contains the information of all working states of the real system. However, the limited data samples may only contain part of the information of the working states. That is to say, the initial BRB-based model is established by considering all the working states of the system while the optimization process can only adjust the parameters associated with part of the working states. Thus, it can be inferred that only the parameters of the activated rule should be adjusted slightly by using the data samples. In this paper, the over-optimization means the process of optimizing the not-activated parameters, which exist in most current studies [10, 11, 13–20, 24–26].

For further explanation, suppose the parameter vector of BRB is denoted by

$$\begin{aligned} &\boldsymbol{\Omega} = (\delta_1, \ldots, \delta_t, \ldots, \delta_T, \theta_1, \ldots, \theta_k, \ldots, \theta_L, \beta_{11}, \ldots, \beta_{nk}, \ldots, \beta_{NL}), \\ &t = 1, \ldots, T,\ k = 1, \ldots, L,\ n = 1, \ldots, N. \end{aligned} \tag{13}$$

If the ith rule is activated, the activated parameters to be adjusted are $(\delta_1, \ldots, \delta_t, \ldots, \delta_T, \theta_i, \beta_{1i}, \ldots, \beta_{Ni})$ rather than the whole parameter vector $\boldsymbol{\Omega}$.

The over-optimization may affect the modeling performance of BRB in two aspects: (1) It may change the initial judgment information from experts and lead to the loss of interpretability. (2) It may increase the burden of the optimization algorithm especially when there are high-dimensional parameters.

According to Problem 3, to achieve an efficient optimization of BRB parameters, it is necessary to discriminate the activated rules from all rules, which leads to Definition 3.

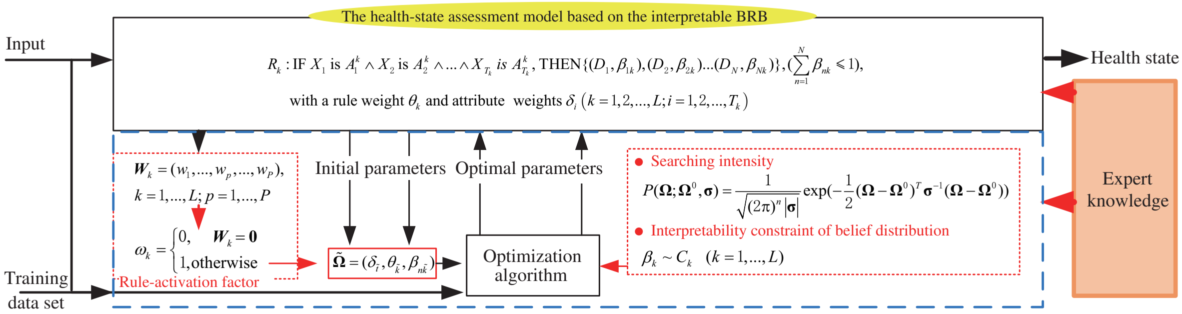

Figure 4 (Color online) The structure of the health-state assessment model based on interpretable BRB.

Definition 3 (rule-activation factor). The rule-activation factor ω is used to mark activated rules, which can be expressed as

$$\omega_k = \begin{cases} 0, & \boldsymbol{W}_k = \boldsymbol{0}, \\ 1, & \text{otherwise}, \end{cases} \tag{14a}$$

where $\boldsymbol{W}_k$ denotes the activation weight vector of the kth rule calculated by the data set. Suppose the size of the data set is P. $\boldsymbol{W}_k$ can be expressed as

$$\boldsymbol{W}_k = (w_1, \ldots, w_p, \ldots, w_P),\ k = 1, \ldots, L,\ p = 1, \ldots, P. \tag{14b}$$

After the activated rules have been determined, the corresponding parameters are selected to be optimized, which can be presented as

$$\tilde{\boldsymbol{\Omega}} = (\delta_{\tilde{t}}, \theta_{\tilde{k}}, \beta_{n\tilde{l}}),\ \tilde{t} \in \{1, 2, \ldots, T\},\ \tilde{k} \in \{1, \ldots, L\},\ n = 1, \ldots, N, \tag{15}$$

where $\delta_{\tilde{t}}$, $\theta_{\tilde{k}}$ and $\beta_{n\tilde{k}}$ denote the activated parameters.

4 The health-state assessment model based on the interpretable BRB

Based on the concepts defined in Section 3, the structure of the health-state assessment model based on the interpretable BRB is proposed in Subsection 4.1. A new optimization method for the health-state assessment is proposed in Subsection 4.2, followed by the summary of the new model in Subsection 4.3.

4.1 The structure of the health-state assessment model based on the interpretable BRB

As shown in Figure 4, the initial BRB should be established according to expert knowledge and optimized in an appropriate way to improve its modelling accuracy under the interpretability requirement. To realize the optimization, a new optimization method is developed, where the rule-activation factor is used to construct the vector $\boldsymbol{\Omega}$. The searching intensity and the interpretability constraint are adopted to keep the meanings of the parameters.

4.2 The new optimization method for the health-state assessment model

To realize a reasonable optimization, both the objective function and the optimization algorithm should be modified.

The new objective function is constructed as

$$\begin{aligned} &\min\{\psi(\theta, \delta, \beta)\} \\ &\text{s.t. } 0 \leqslant \theta_k(\delta_i) \leqslant 1,\ 0 \leqslant \beta_{n,k} \leqslant 1,\ \sum_{n=1}^{N} \beta_{n,k} \leqslant 1,\ \beta_k \sim C_k \\ &\quad (k = 1, \ldots, L;\ i = 1, \ldots, T;\ n = 1, \ldots, N), \end{aligned} \tag{16}$$

where $\psi(\theta, \delta, \beta)$ denotes the error between model outputs and the actual values. C_k is the interpretability constraint of the belief distribution in the kth rule.

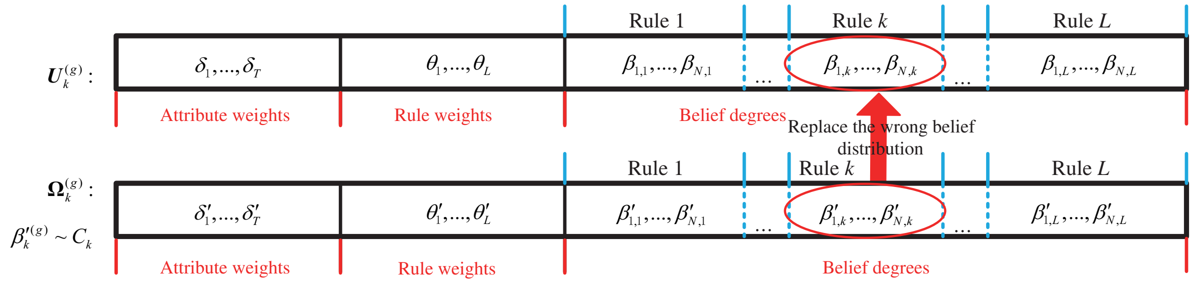

Figure 5 (Color online) The replacement operation.

The existing optimization algorithm should be modified in two aspects: (1) The searching intensity should be considered during the generation of a new population. (2) The interpretability constraint should be implemented in the optimization process. Taking the original DE algorithm as an example, the searching intensity is introduced to the initial operation while the interpretability constraint is used in the newly added controlling operation. The modified DE algorithm is denoted by P-DE-I, shown as follows.

Step 1 (initial operation). Generate the initial population considering the searching intensity.

$$\tilde{\boldsymbol{\Omega}}_k^{(0)} = N(\boldsymbol{\mu}^{(0)}, \boldsymbol{\sigma}),\ k = 1, \ldots, \lambda, \tag{17}$$

where $\boldsymbol{\mu}^{(0)} = \tilde{\boldsymbol{\Omega}}^0$ denotes the initial parameter vector. The covariance matrix is $\boldsymbol{\sigma}$. λ denotes the population size. $N(\cdot)$ denotes the normal distribution.

Step 2 (mutation operation). Mutation operation is employed to generate new solutions, which is expressed as

$$\boldsymbol{H}_k^{(g)} = \boldsymbol{\mu}^{(g)} + F \times (\tilde{\boldsymbol{\Omega}}_{\text{best}}^{(g)} - \tilde{\boldsymbol{\Omega}}_{p1}^{(g)} + \tilde{\boldsymbol{\Omega}}_{p2}^{(g)} - \tilde{\boldsymbol{\Omega}}_{p3}^{(g)}),\ k = 1, \ldots, \lambda,\ g = 1, \ldots, G, \tag{18}$$

where $p_1 \neq p_2 \neq p_3 \neq k$, $F \in [0, 1]$ is the zoom factor. $\boldsymbol{\Omega}_{\text{best}}^{(g)}$ is the best individual in the gth generation. $\boldsymbol{\mu}$ denotes the center of the population, which is calculated by

$$\boldsymbol{\mu}^{(g)} = \sum_{k=1}^{\lambda} \tilde{\boldsymbol{\Omega}}_k^{(g)} / \lambda,\ g = 1, \ldots, G. \tag{19}$$

Step 3 (crossover operation). The crossover operation is introduced to increase the population diversity, which is expressed as

$$\boldsymbol{U}_k^{(g)} = \begin{cases} \boldsymbol{H}_k^{(g)} & (\text{rand}(0,1) \leqslant \text{Cr}), \\ \boldsymbol{\Omega}_k^{(g)} & (\text{rand}(0,1) > \text{Cr}), \end{cases} \tag{20}$$

where $\text{Cr} \in [0, 1]$ is the crossover rate. $\boldsymbol{U}_k^{(g)}$ is the updated population.

Step 4 (controlling operation). Resample the wrong belief distributions in the kth solution of the gth generation until all of them can satisfy the distribution constraint, shown as follows:

$$\boldsymbol{U}_k^{(g)} \Leftarrow \boldsymbol{\Omega}_k^{(g)} = N(\boldsymbol{\mu}^{(g)}, \boldsymbol{\sigma}),\ {\beta'_k}^{(g)} \sim C_k,\ k = 1, 2, \ldots, \lambda, \tag{21}$$

where $\boldsymbol{U}_k^{(g)}$ is the kth solution of the gth generation, which may contain wrong belief distributions. ${\beta_k}^{(g)}$ represents the newly created belief distribution that satisfy the constraint. $\Leftarrow$ denotes the replacement operation.

For further explanation, the specific process is shown in Figure 5.

Step 4.1. Check the wrong distribution in $\boldsymbol{U}_k^{(g)}$ by using the interpretability distribution C_k. If there is no wrong distribution, go to Step 5. Otherwise, go to Step 4.2.

Step 4.2. Generate the new parameter vector $\boldsymbol{\Omega}_k^{(g)}$ until ${\beta'_k}^{(g)}$ in it satisfies C_k.

Step 4.3. Replace the wrong distributions in $\boldsymbol{U}_k^{(g)}$ by ${\beta'_k}^{(g)}$ in $\boldsymbol{\Omega}_k^{(g)}$.

Step 5 (projection operation). To satisfy the equality constraint, the projection operation is used to transform the equality constraint to the one in a hyperplane:

$$\boldsymbol{A}_e \boldsymbol{U}_k^{(g)}(1+n_e\times(j-1):n_e\times j)=1,\ j=1,2,\ldots,N+1. \tag{22}$$

The projection operation is implemented as follows:

$$\begin{aligned}&\boldsymbol{U}_k^{(g)}(1+n_e\times(j-1):n_e\times j)\\&=\boldsymbol{U}_k^{(g)}(1+n_e\times(j-1):n_e\times j)-\boldsymbol{A}_e^{\mathrm{T}}\times(\boldsymbol{A}_e\times\boldsymbol{A}_e^{\mathrm{T}})^{-1}\times\boldsymbol{U}_k^{(g)}(1+n_e\times(j-1):n_e\times j)\times\boldsymbol{A}_e.\end{aligned} \tag{23}$$

Step 6 (selection operation). The selection operation is conducted to update the best individual and the population as

$$\boldsymbol{\Omega}_k^{(g+1)}=\begin{cases}\boldsymbol{U}_k^{(g)}\ (\psi(\boldsymbol{U}_k^{(g)})<\psi(\boldsymbol{\Omega}_k^{(g)})),\\ \boldsymbol{\Omega}_k^{(g)}\ (\psi(\boldsymbol{U}_k^{(g)})\geqslant\psi(\boldsymbol{\Omega}_k^{(g)})),\end{cases}\quad \boldsymbol{\Omega}_{\text{best}}^{(g+1)}=\begin{cases}\boldsymbol{U}_k^{(g)}\ (\psi(\boldsymbol{\Omega}_k^{(g)})<\psi(\boldsymbol{\mu}^{(g)})),\\ \boldsymbol{\mu}^{(g)}\ (\psi(\boldsymbol{\Omega}_k^{(g)})\geqslant\psi(\boldsymbol{\mu}^{(g)})).\end{cases} \tag{24}$$

Step 7 (termination criterion). Go to Step 2 until the maximum evolution generation is achieved.

4.3 The health-state assessment by using the new model

By using the new health-state assessment model, the assessment process can be summarized as follows:

Step 1. Determine the referential values and establish the initial model by expert knowledge.

Step 2. Generate the rule-activation factor by using Eqs. (3), (13), and (14), and determine the final parameter vector.

Step 3. Determine the parameters of searching intensity and the interpretability constraint of belief distribution according to the expert knowledge.

Step 4. Construct the objective function as Eq. (16) by using the interpretability constraint of belief distribution.

Step 5. Optimize the initial model by using the modified optimization method to realize the local searching process.

5 Case study

In this section, a case study of the health-state assessment on the aero-engine is conducted to verify the effectiveness of the proposed method. An aero-engine is a precise and complex dynamic system composed of electromagnetic and mechanical units, whose health state is full of uncertainties and affected by high temperatures, high pressures, and strong stress variations. Owing to the speciality of the aero-engine, it cannot be tested frequently so that the testing data samples obtained are limited. It is necessary to estimate the health state in an interpretable and reliable way by using the limited data samples and knowledge, which can reduce the potential safety risk in engineering practice. In this experiment, 5000 groups of vibration data are collected by using the internal vibration sensor of the aero-engine when the high-altitude valve spring pressures are decreased by 0%, 5%, 10%, and 20%, respectively. The corresponding health states are normal state (N), small fault state (S), middle fault state (M), and serious fault state (SE). The original vibration data are shown in Figure 6(a). Kurtosis and skewness of the vibration data are employed as features to describe the health state of the aero-engine, which are expressed as

$$\text{Kurtosis: } K=\mathrm{E}[(x_n^2-\bar{x})^4]/\sigma^4,\quad \text{Skewness: } S=\mathrm{E}[(x_n^2-\bar{x})^3]/\sigma^3, \tag{25}$$

where $\mathrm{E}(\cdot)$ is the expectation and σ is the standard deviation. For every 100 groups of data, the features are calculated by using Eq. (25), which are shown in Figures 6(b) and (c). 30 groups of the data in each condition are selected as the training set and all the data are treated as the testing set.

5.1 Construction of the initial BRB

According to "Guidelines for failure modes, effects, and hazards analysis of Aero-engines (HB/Z 281-1995)", the degradation of the key component of the aero-engine such as the valve spring may lead to a slight change of the engine vibration signal, which can be indirectly reflected in the change of kurtosis

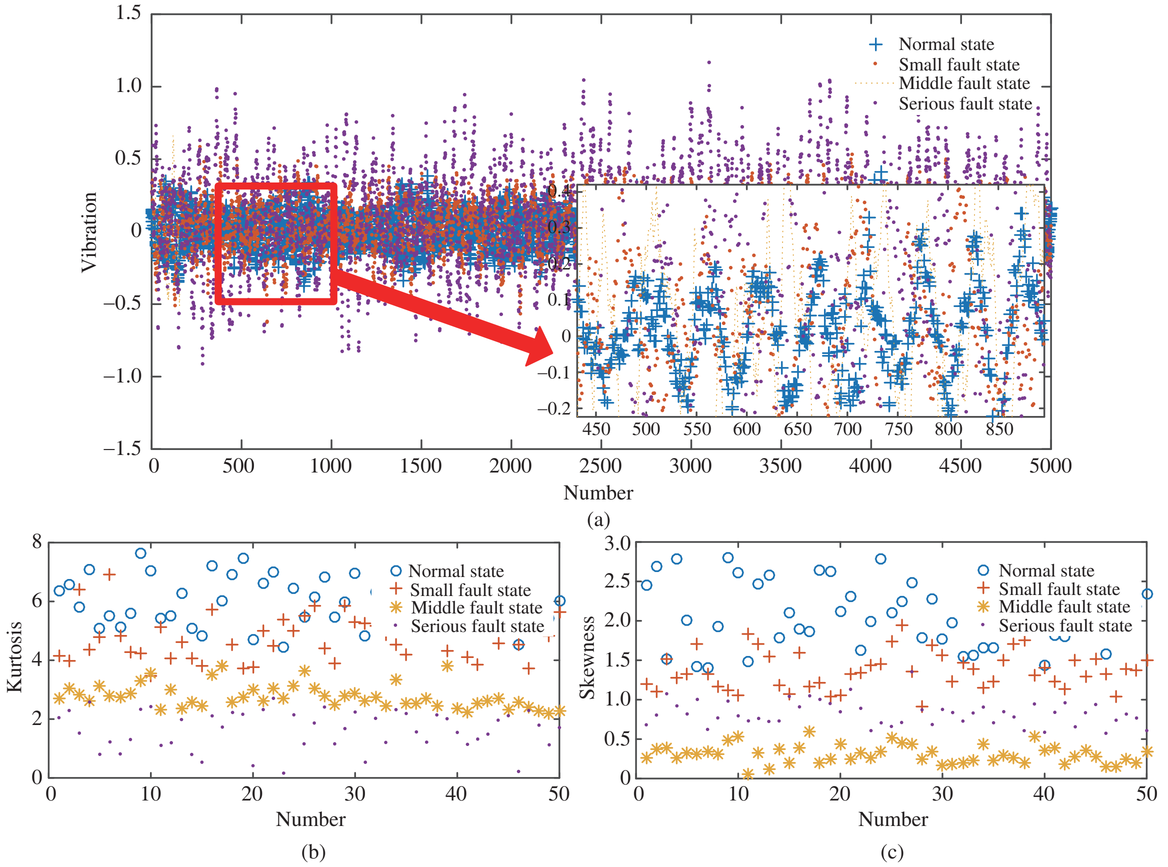

Figure 6 (Color online) The original vibration data and features. (a) The original vibration data; (b) kurtosis; (c) skewness.

Table 1 Referential values of attributes and output

Attribute	δ_i	L	M	H	VH	Output	N	S	M	SE
K	1	0.1	2.4	5.2	9.2	Health state	0	0.25	0.75	1
S	1	0	0.8	1.4	2.9					

and skewness. In engineering practice, kurtosis and skewness are described by using four levels of the semantic value as "low (L)", "middle (M)", "high (H)", and "very high (VH)". Based on the knowledge accumulated in long-term testing of the aero-engine, the referential values and the initial attribute weights are given in Table 1.

To describe the relationship between features and the health state, the initial rule should be given by analyzing the aero-engine. In the normal state, kurtosis and skewness of the vibration are located at the level "VH". Under this condition, the health state becomes worse as the level of skewness goes to be "M". When the level of skewness is "M" and the level of kurtosis is "L" or "M", the health state is in "SE" level. Generally, the health state becomes worse as the levels of kurtosis and skewness decrease. The initial rules are shown in Table 2.

5.2 Optimization and testing of interpretable BRB-based model

Due to the limitation of expert knowledge, the initial BRB should be optimized by using the observational data. Moreover, the optimization of the initial BRB should be conducted considering the initial judgment information from experts based on the analysis in Sections 2 and 3.

At first, the parameter vector $\tilde{\boldsymbol{\Omega}} = (\delta_{\tilde{i}}, \theta_{\tilde{k}}, \beta_{n\tilde{l}})$ should be determined. In this case study, the rule-activation factor is calculated by using the training set and Eqs. (14) and (15) as

$$\omega = \{\omega_1, \ldots, \omega_k\} = \{1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1\}, \tag{26}$$

which means that the 4th, 13th, and 14th rules are not activated by the limited training set, and the

Table 2 The initial rules

Number	θ_k	$K \wedge S$	Health state levels $\frac{\{N,S,M,SE\}}{\{\beta_1,\beta_2,\beta_3,\beta_4\}}$	Health state	Number	θ_k	$K \wedge S$	Health state levels $\frac{\{N,S,M,SE\}}{\{\beta_1,\beta_2,\beta_3,\beta_4\}}$	Health state
1	0.7	L ∧ L	{0.00, 0.10, 0.53, 0.37}	0.7925	9	1	H ∧ L	{0.00, 0.40, 0.60, 0.00}	0.55
2	0.8	L ∧ M	{0.00, 0.00, 0.00, 1.00}	1	10	0.7	H ∧ M	{0.00, 0.75, 0.25, 0.00}	0.375
3	0.7	L ∧ H	{0.00, 0.00, 0.30, 0.70}	0.925	11	1	H ∧ H	{0.15, 0.85, 0.00, 0.00}	0.2125
4	0.8	L ∧ VH	{0.20, 0.60, 0.20, 0.00}	0.3	12	1	H ∧ VH	{0.80, 0.20, 0.00, 0.00}	0.05
5	1	M ∧ L	{0.00, 0.00, 0.90, 0.10}	0.775	13	0.8	VH ∧ L	{0.85, 0.15, 0.00, 0.00}	0.0375
6	0.7	M ∧ M	{0.00, 0.00, 0.00, 1.00}	1	14	1	VH ∧ M	{0.40, 0.60, 0.00, 0.00}	0.15
7	0.7	M ∧ H	{0.00, 0.00, 0.75, 0.25}	0.8125	15	1	VH ∧ H	{0.90, 0.10, 0.00, 0.00}	0.025
8	1	M ∧ VH	{0.10, 0.70, 0.20, 0.00}	0.325	16	0.7	VH ∧ VH	{1.00, 0.00, 0.00, 0.00}	0

Table 3 The initial referential values of health state

The parameter	Value	The parameter	Value	The parameter	Value
Maximum generation G	100	The zoom factor F	$[0.2, 05]$	Population size λ	50
Covariance matrix $\boldsymbol{\sigma} = \kappa * \boldsymbol{I}$	$\kappa = 0.1$	The crossover rate Cr	0.2		

Table 4 The optimized rules

Number	θ_k	$K \wedge S$	Health state levels $\frac{\{N,S,M,SE\}}{\{\beta_1,\beta_2,\beta_3,\beta_4\}}$	Health state $[0, 1]$
1	0.69	L ∧ L	{0.020, 0.068, 0.505, 0.407}	0.80275
2	0.96	L ∧ M	{0.000, 0.007, 0.031, 0.962}	0.987
3	0.62	L ∧ H	{0.027, 0.081, 0.234, 0.658}	0.85375
4	0.8	L ∧ VH	{0.20, 0.60, 0.20, 0.00}	0.3
5	1.00	M ∧ L	{0.016, 0.048, 0.868, 0.068}	0.731
6	0.61	M ∧ M	{0.000, 0.000, 0.019, 0.981}	0.99525
7	0.60	M ∧ H	{0.002, 0.009, 0.727, 0.262}	0.8095
8	0.91	M ∧ VH	{0.104, 0.644, 0.141, 0.111}	0.37775
9	1.00	H ∧ L	{0.000, 0.119, 0.639, 0.242}	0.751
10	0.60	H ∧ M	{0.108, 0.770, 0.119, 0.003}	0.28475
11	1.00	H ∧ H	{0.232, 0.768, 0.000, 0.000}	0.192
12	0.98	H ∧ VH	{0.980, 0.017, 0.003, 0.000}	0.0065
13	0.8	VH ∧ L	{0.85, 0.15, 0.00, 0.00}	0.0375
14	1	VH ∧ M	{0.40, 0.60, 0.00, 0.00}	0.15
15	0.99	VH ∧ H	{0.903, 0.086, 0.008, 0.003}	0.0305
16	1.00	VH ∧ VH	{0.943, 0.045, 0.007, 0.005}	0.0215

corresponding parameters should not be optimized. The final parameter vector is presented as

$$\tilde{\boldsymbol{\Omega}} = (\delta_{\tilde{t}}, \theta_{\tilde{k}}, \beta_{n\tilde{l}}),\ \tilde{t} \in \{1, 2\};\ \tilde{k} \in \{1, \dots, 16\},\ \tilde{k} \neq 4, 13, 14;\ n = 1, 2, 3, 4. \tag{27}$$

Combined with the final parameter vector, the objective function can be determined by using Eq. (16), where the basic constraint for the belief distribution expressed as Eq. (12) is used. The parameters of P-DE-I are given in Table 3. The optimization process is implemented in MATLAB R2014a on Core(TM) i5-3337U CPU 1.80 GHz with Windows 7. As a result, the optimized BRB is shown in Table 4. The optimized attribute weights are 1 and 0.7864, respectively.

In the testing part, the testing data set is treated as the input of optimized BRB. The estimated health state is shown in Figure 7(a). The MSE between the estimated health state and the actual health state is 0.0054, which indicates that the optimized BRB can estimate the health state of the aero-engine accurately. The belief degree of the corresponding health state is shown in Figure 7(b). It can be seen that when the aero-engine is in the normal state and the small fault state, the corresponding belief degrees are not completely distinguishable (marked by the red circle). The reason may be that: (1) The original vibration data may be affected by noise. (2) The small fault state of the aero-engine may have no distinct impact on its operation.

To demonstrate the robustness of P-DE-I, the experiment is conducted 25 times. The variance of the MSEs (1.826E-07) is far smaller than the mean of the MSEs (0.00564).

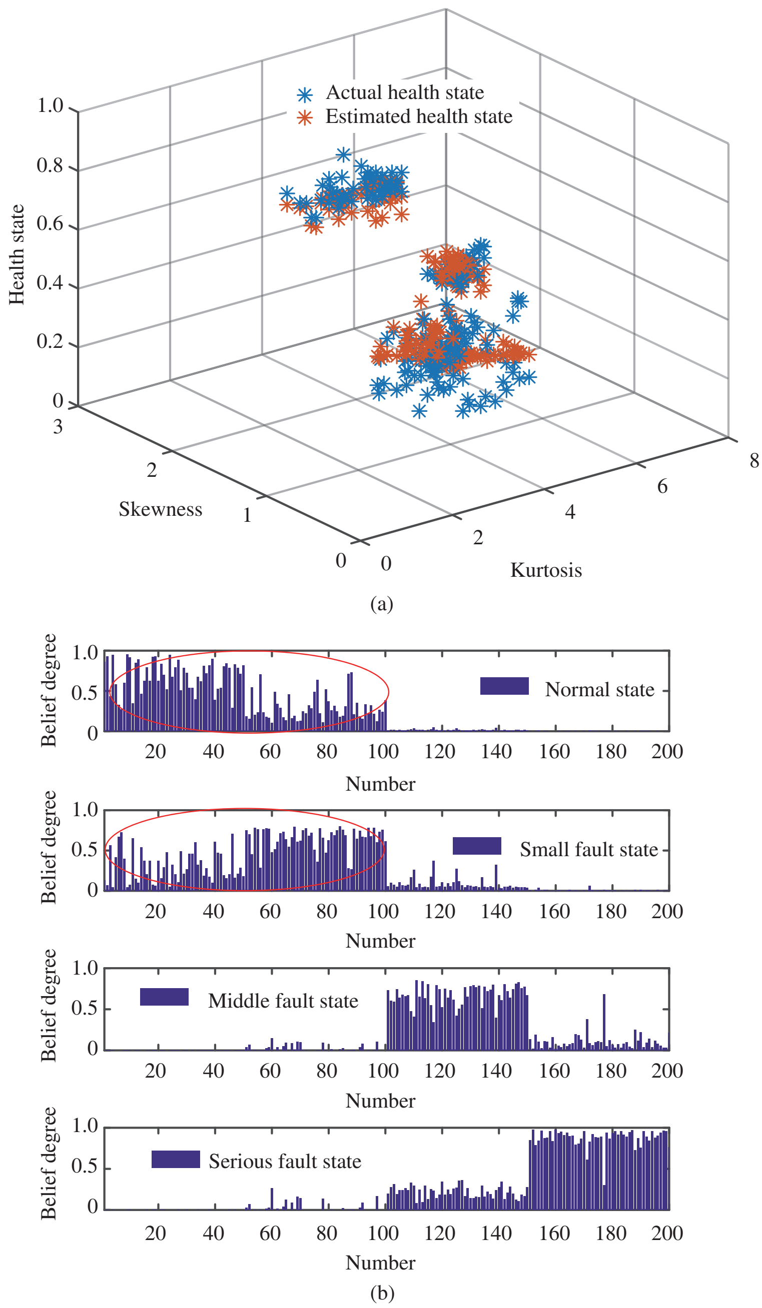

Figure 7 (Color online) (a) The estimated health state and (b) the corresponding belief degrees.

5.3 Comparative study

To verify the effectiveness of the proposed model, the comparative study is conducted between the initial BRB (denoted by BRB0), the BRB model optimized by using P-DE-I algorithm (denoted by BRB1), the BRB model optimized by using the original DE algorithm (denoted by BRB2), FRB and the extreme learning machine (ELM). The training data set and the testing data set are the same as Subsection 5.2. BRB0 is the initial model of BRB1 and BRB2, which is given in Tables 1 and 2. The FRB and ELM are used as the data-driven models.

(a) The comparison of modelling accuracy. In this comparative study, the corresponding MSEs are presented in Table 5. The errors between the model outputs and the actual value are shown in Figure 8. It can be seen that the optimized BRB models (BRB1, BRB2) achieve higher modelling accuracy than other models. Compared with FRB and ELM, the modelling accuracy of BRB1 increases by 41.30% and 15.63%. Moreover, the belief distribution from the BRB1 model shown in Figure 7(b)

Table 5 MSEs generated by five models

	BRB0	BRB1	BRB2	FRB	ELM
MSE	0.0114	0.0054	0.0051	0.0092	0.0064

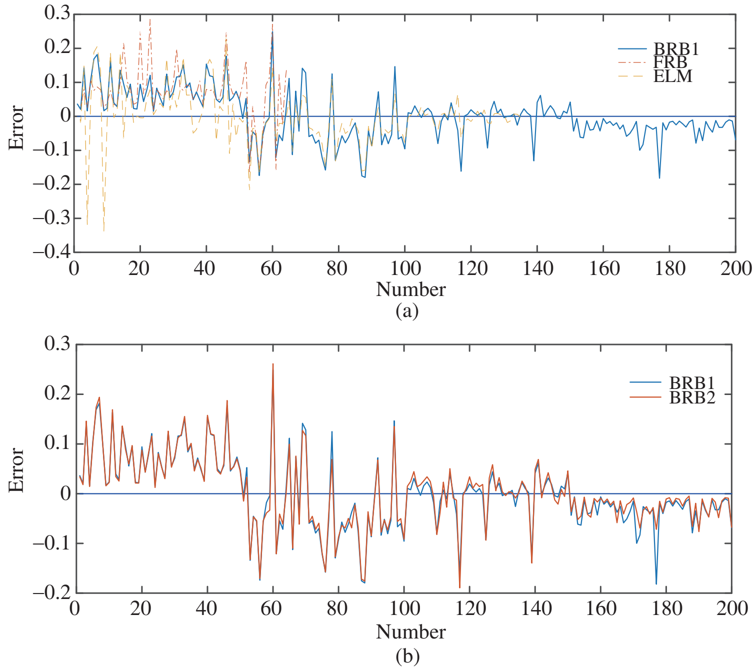

Figure 8 (Color online) The errors between model outputs (BRB1, BRB2, FRB, ELM) and the actual health state. (a) The errors of BRB1, FRB, and ELM; (b) the errors of BRB1 and BRB2.

provides a clearer description of the health state. Compared with BRB1, the modelling accuracy of BRB2 increases by 5.56%. In Figure 8(b), the outputs between these two models are similar, which indicates that BRB1 and BRB2 have similar modelling performances in this experiment.

Remark 3. Theoretically, both BRB and ELM have the universal approximation property, but their modelling performances may be different in engineering practice [5]. Thus, the above comparison between BRB1 and ELM has limitations, which should be further discussed: (1) The parameters of ELM may not be fully optimized, which means that the modelling accuracy of ELM can be improved if more observational data can be used in the optimization process. The advantage of BRB is that the initial parameters given by experts can be fully adjusted with fewer data samples. (2) The feasible region of parameters in BRB is smaller than ELM due to the constraints to guarantee model meanings. That is to say, the optimization of BRB may be more efficient than ELM, which can reduce the difficulty in optimizing high-dimensional parameters.

(b) The comparison of model interpretability. In terms of the model interpretability, the rule weights and belief distributions of BRB0, BRB1 and BRB2 are compared as shown in Figures 9 and 10. It can be seen from Figure 9 that the rule weights in BRB1 are close to those in BRB0, while most of the rule weights in BRB2 are far from those in BRB0. Taking the third rule as an example, the rule weights in BRB0 and BRB1 are 0.7 and 0.62 respectively, which indicate that the third rule is important. However, the rule weight of the third rule in BRB2 is 0.0278, which indicates that this rule is unimportant. It is completely conflicted with the initial judgment from experts. As shown in Figure 10, the belief distributions in BRB0 and BRB1 are close. Most of the belief distributions in BRB2 are different from those in BRB0. For example, the belief distribution in the eighth rule does not meet the interpretability requirement mentioned in Eq. (12).

All above discussions indicate that the parameters determined by expert knowledge are adjusted slightly and the modelling accuracy is improved. It further shows that the optimization process of BRB1 is implemented based on expert initial judgement, which is different from the traditional optimization process that only aims at a higher modelling accuracy.

(c) The comparison by using different optimization algorithms. The P-DE-I algorithm is

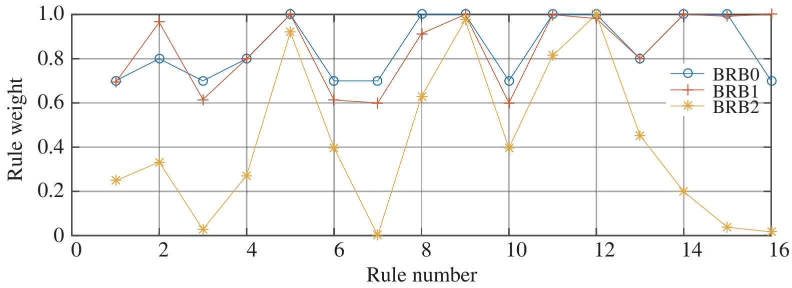

Figure 9 (Color online) The rule weights in BRB0, BRB1, and BRB2.

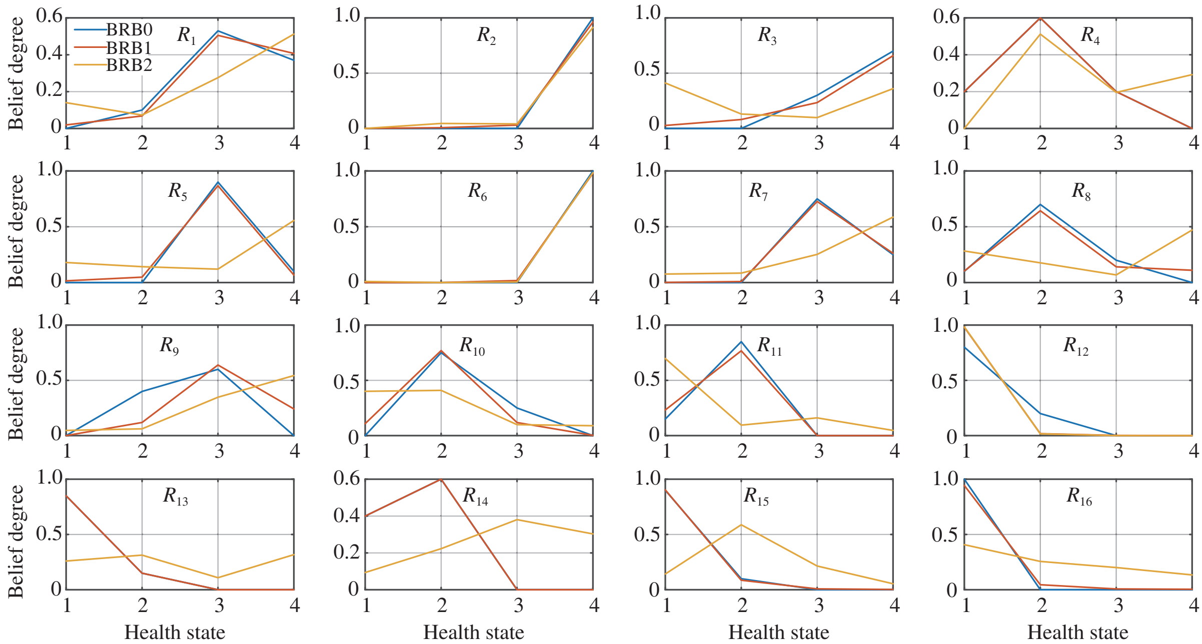

Figure 10 (Color online) The belief distributions in BRB0, BRB1, and BRB2.

Table 6 Average MSEs generated by using three optimization algorithms

Algorithm	MSE	Satisfy interpretability constraint in Eq. (12)	Be consistent with initial judgement
PSO-I	0.0049	YES	YES
GA-I	0.0057	YES	YES
SA-I	0.0052	YES	YES

introduced as an example in the above studies to realize the optimization considering interpretability. To illustrate the generality of the proposed method, the particle swarm optimization algorithm, the genetic algorithm, and the simulated annealing algorithm are modified (denoted by PSO-I, GA-I, and SA-I, respectively). In 10 repetitive experiments, the average MSEs generated by using the above three algorithms are shown in Table 6. It can be seen that the above algorithms have similar performances and can preserve the interpretability in the optimization process.

6 Conclusion

Interpretability, the advantage of a BRB-based model, is an important requirement to achieve reliable and accurate health-state assessments of complex systems. However, in current studies, the interpretability of BRB was destroyed in the optimization process. There are three typical problems: expert knowledge is not effectively used in the optimization process; optimized rules of BRB may be in conflict with real systems; and some not-activated parameters are over-optimized.

The following concepts aimed to address these three problems: For the first problem, searching intensity is defined to generate new solutions that are close to experts' initial judgment. For the second problem, an interpretability constraint is proposed to make the optimized-belief distributions consistent with real systems. For the third problem, a rule-activation factor is defined to construct the activated-parameter vector. Hence, we propose a new health-state assessment model based on the interpretable BRB, whereby a new optimization method is used to improve its modelling ability. A case study of the health-state assessment of the aero-engine was conducted to verify the effectiveness of the proposed model. The results show that the new optimization method can improve BRB modelling accuracy and preserve its interpretability.

We analyzed the interpretability of BRB in the parameter-optimization process, which has limitations. The interpretability of BRB for structure optimization (or joint optimization of the parameters and the structure) can be equally important. How to maintain the interpretability in structure optimization (or joint optimization of the parameters and the structure) should be further discussed.

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References

1 Zhao F J, Zhou Z J, Hu C H, et al. A new evidential reasoning-based method for online safety assessment of complex systems. IEEE Trans Syst Man Cybern Syst, 2018, 48: 954–966
2 Narmada S, Jacob M. Reliability analysis of a complex system with a deteriorating standby unit under common-cause failure and critical human error. Microelectron Reliab, 1996, 36: 1287–1290
3 Dao C D, Zuo M J. Optimal selective maintenance for multi-state systems in variable loading conditions. Reliab Eng Syst Saf, 2017, 166: 171–180
4 Yin X J, Wang Z L, Zhang B C, et al. A double layer BRB model for health prognostics in complex electromechanical system. IEEE Access, 2017, 5: 23833–23847
5 Chen Y W, Yang J B, Xu D L, et al. On the inference and approximation properties of belief rule based systems. Inf Sci, 2013, 234: 121–135
6 Hinton G E. Reducing the dimensionality of data with neural networks. Science, 2006, 313: 504–507
7 Mulder Y D, Wyseur B, Preneel B. Cryptanalysis of a perturbated white-box AES implementation. In: Proceedings of International Conference on Cryptology in India, 2010. 292–310
8 Yang J B, Liu J, Wang J, et al. Belief rule-base inference methodology using the evidential reasoning approach-RIMER. IEEE Trans Syst Man Cybern A, 2006, 36: 266–285
9 Zhou Z J, Hu G Y, Hu C H, et al. A survey of belief rule-base expert system. IEEE Trans Syst Man Cybern Syst, 2019. doi: 10.1109/TSMC.2019.2944893
10 Zhou Z J, Hu C H, Yang J B, et al. Online updating belief rule based system for pipeline leak detection under expert intervention. Expert Syst Appl, 2009, 36: 7700–7709
11 Chang L L, Dong W, Yang J B, et al. Hybrid belief rule base for regional railway safety assessment with data and knowledge under uncertainty. Inf Sci, 2020, 518: 376–395
12 Cheng C, Qiao X Y, Teng W X, et al. Principal component analysis and belief-rule-base aided health monitoring method for running gears of high-speed train. Sci China Inf Sci, 2020, 63: 199202
13 Zhou Z J, Feng Z C, Hu C H, et al. A hidden fault prediction model based on the belief rule base with power set and considering attribute reliability. Sci China Inf Sci, 2019, 62: 202202
14 Yang J B, Liu J, Xu D L, et al. Optimization models for training belief-rule-based systems. IEEE Trans Syst Man Cybern A, 2007, 37: 569–585
15 Chen Y W, Yang J B, Xu D L, et al. Inference analysis and adaptive training for belief rule based systems. Expert Syst Appl, 2011, 38: 12845–12860
16 Chang L L, Zhou Y, Jiang J, et al. Structure learning for belief rule base expert system: a comparative study. Knowledge-Based Syst, 2013, 39: 159–172
17 Chang L L, Zhou Z J, Chen Y W, et al. Belief rule base structure and parameter joint optimization under disjunctive assumption for nonlinear complex system modeling. IEEE Trans Syst Man Cybern Syst, 2018, 48: 1542–1554
18 Zhou Z J, Hu C H, Yang J B, et al. Online updating belief-rule-base using the RIMER approach. IEEE Trans Syst Man Cybern A, 2011, 41: 1225–1243
19 Zhou Z J, Hu C H, Yang J B, et al. A sequential learning algorithm for online constructing belief-rule-based systems. Expert Syst Appl, 2010, 37: 1790–1799
20 Li G L, Zhou Z J, Hu C H, et al. A new safety assessment model for complex system based on the conditional generalized minimum variance and the belief rule base. Saf Sci, 2017, 93: 108–120
21 Casillas J, Cordón O, Herrera F, et al. Accuracy Improvements in Linguistic Fuzzy Modeling. Berlin: Springer, 2013
22 Zadeh L A. A fuzzy-set-theoretic interpretation of linguistic hedges. J Cybernetics, 1972, 2: 4–34
23 Zhou S M, Gan J Q. Low-level interpretability and high-level interpretability: a unified view of data-driven interpretable fuzzy system modelling. Fuzzy Sets Syst, 2008, 159: 3091–3131
24 Wu W K, Yang L H, Fu Y G, et al. Parameter training approach for belief rule base using the accelerating of gradient algorithm. J Front Comput Sci Technol, 2009, 22: 5166–5170
25 Li G L, Zhou Z J, Hu C H, et al. An optimal safety assessment model for complex systems considering correlation and redundancy. Int J Approx Reason, 2019, 104: 38–56
26 Chang L L, Zhou Z J, Liao H, et al. Generic disjunctive belief-rule-base modeling, inferencing, and optimization. IEEE Trans Fuzzy Syst, 2019, 27: 1866–1880