SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

July 2021, Vol. 64 172212:1–172212:12
https://doi.org/10.1007/s11432-020-3122-6

Cooperative neural-adaptive fault-tolerant output regulation for heterogeneous nonlinear uncertain multiagent systems with disturbance

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Received 11 August 2020/Revised 7 October 2020/Accepted 30 October 2020/Published online 20 May 2021

Abstract This paper investigates the cooperative output regulation problem for heterogeneous nonlinear uncertain multiagent networked systems subject to actuator failure, bounded matched or mismatched disturbances or disturbances produced by a given linear exosystem. Accurate information about nonlinearity, actuator failure and disturbance may be completely unknown. First, a distributed finite-time observer is designed to estimate the dynamics of the exosystem on a finite-time interval over a communication digraph. Then, a neural-adaptive control protocol is proposed. It is shown that (i) closed-loop multiagent systems are asymptotically stable, with output synchronization errors that tend to zero in the absence of mismatched disturbance, and (ii) the states of the closed-loop multiagent systems and the output synchronization errors are bounded in the presence of mismatched disturbance. Finally, a simulation example is given to demonstrate the effectiveness of the proposed control strategy.

Keywords cooperative output regulation, actuator failure, matched/mismatched disturbance, distributed finite-time observer, neural-adaptive control

Citation Dong S L, Chen G R, Liu M Q, et al. Cooperative neural-adaptive fault-tolerant output regulation for heterogeneous nonlinear uncertain multiagent systems with disturbance. Sci China Inf Sci, 2021, 64(7): 172212, https://doi.org/10.1007/s11432-020-3122-6

1 Introduction

Various cooperative control problems for multiagent systems have received considerable attention from the scientific and engineering communities because of their wide range of applications such as sensor networks, spacecraft, mobile robots and unmanned aerial vehicles [1–7]. In particular, cooperative output regulation investigates heterogeneous multiagent systems in which the agents have different dynamics. The aim of cooperative output regulation is to obtain both disturbance rejection and asymptotic tracking of the exosystem via the interactions among the agents. For this task, the internal model method and distributed observer design were developed, e.g., in [8–12].

Disturbance from the exosystem can be completely eliminated by suitably using the regulator equations so that the closed-loop systems can attenuate such disturbance. However, other kinds of additive disturbance such as matched/mismatched disturbance are seldom taken into consideration in cooperative output regulation. In [13], H_∞ performance index was adopted to study the effect of energy-bounded mismatched disturbance on system performance. For Markov jump systems subject to the bounded matched disturbance, an adaptive sliding mode control strategy was used to achieve system stochastic

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stability in [14]. The present work is motivated by the need for a further study on the system performance of cooperative output regulation for multiagent systems subject to bounded matched/mismatched disturbance.

In practice, actuators may experience unexpected failure because of physical restrictions [15, 16]. If actuator failure is not accommodated well, serious consequences may follow such as system instability, performance degradation and even catastrophic accidents. Recently, many effective techniques have been proposed to handle such failure in order to increase the safety and reliability of physical systems. An information-based diagnostic method was developed to detect and isolate failure of nonlinear systems in [17]. In [18], both backstepping control and command filter methodologies were used to eliminate the impact of actuator failure. The adaptive compensation control technique was used to deal with failure in a single system [19] and in multiagent systems [20].

On the other hand, the mathematical framework of neural networks (NNs) has universal capability in approximating unknown smooth nonlinear functions. An NN-based quadratic optimization control technique was applied to robot manipulators in [21]. An adaptive neural impedance control approach was proposed to solve the tracking problem for robot manipulators subject to uncertainty and input saturation in [22]. NNs were also widely employed to solve various cooperative control problems in multiagent systems, for example, the tracking consensus problem [23], robust containment control issue [24] and leader-follower consensus problem [25]. Despite these achievements, to the best of our knowledge no work has been reported on the cooperative output regulation for heterogeneous nonlinear uncertain multiagent systems subject to actuator failure and additive disturbance. The present work attempts to fill this gap.

Specifically, in this study, an NN-based adaptive control algorithm is designed to achieve cooperative output synchronization for heterogeneous nonlinear uncertain multiagent systems over a communication digraph. The studied nonlinear systems are subject to matched disturbance with an unknown upper bound or mismatched disturbance with a given upper bound or disturbance generated by an exosystem and actuator failure. The new scheme does not rely on the accurate information of actuator failure. A distributed finite-time observer is constructed to estimate the exosystem dynamics, and then an NN-based adaptive control law is designed by using the states from the observer and the nonlinear systems. It will be shown that (i) in the absence of mismatched disturbance, the output synchronization error asymptotically tends to zero; (ii) in the presence of mismatched disturbance, all states of the closed-loop systems and the output synchronization error converge to a bounded set in terms of the upper bound of the mismatched disturbance. It will be proved that the proposed NN-based adaptive control strategy can completely eliminate the effects from matched disturbance, disturbance from the exosystem, nonlinearity and actuator failure except for mismatched disturbance; nevertheless, even for mismatched disturbance, the errors will be bounded. Finally, the effectiveness of the new control strategy will be demonstrated through a simulation example.

This study makes three contributions as the following.

(1) A more complicated cooperative output regulation problem is investigated, where matched disturbance, mismatched disturbance, disturbance from the exosystem, actuator failure and nonlinearity are coupled together in heterogeneous multiagent systems. The nonlinear term and matched disturbance are totally unknown, posing great difficulty for control stability analysis. To deal with this challenge, an NN-based adaptive control law is developed that can completely offset all of the unknown and uncertain effects and ensure the system performance.

(2) Compared with [26], a distributed finite-time observer is designed to estimate the exosystem dynamics over a general communication digraph instead of an undirected graph. Moreover, control parameters are suitably chosen to avoid the use of the global information of the digraph.

(3) Most published studies, such as [14,27,28], have used the projection technique to estimate the failure values, where the maximal upper and minimal lower bounds of failure must be known. By contrast, this work only needs the minimal lower bound to determine the desired control parameters. When the minimal lower bound is unknown, this paper recommends a solution of selecting the relevant control parameter to be sufficiently large such that failure is totally compensated.

The rest of the paper is organized as follows. Section 2 formulates the problem, and Section 3 presents the main results. Section 4 shows a simulation example, followed by a conclusion in Section 5.

2 Problem formulation

Consider a group of heterogeneous nonlinear uncertain agents with the following dynamics:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i \left[u_i^f(t) + f_i(x_i(t)) + w_i(t)\right] + C_i d_i(t) + D_i \nu(t), \\ y_i(t) &= E_i x_i(t),\ i \in \mathcal{N} = \{1, 2, \ldots, N\}, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i^f(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^l$ are the state, failure control input and output of agent i, respectively; $w_i(t)$ and $d_i(t)$ denote the bounded matched and mismatched disturbances, satisfying $\|w_i(t)\|_2 \leqslant \overline{w}_i$ and $\|d_i(t)\|_2 \leqslant \overline{d}_i$, respectively; $\overline{w}_i$ is unknown but $\overline{d}_i$ is given; $f_i(x_i(t))$ is an uncertain nonlinear smooth function; A_i, B_i, C_i, D_i and E_i are known constant matrices; and $\nu(t) \in \mathbb{R}^{n_0}$ is the reference input produced by the following exosystem:

$$\dot{\nu}(t) = S\nu(t),\ y_0(t) = F\nu(t), \tag{2}$$

where $y_0(t) \in \mathbb{R}^l$ is the reference output. The output synchronization error of agent i is defined as $\varepsilon_i(t) = y_i(t) - y_0(t)$.

In this work, a multiagent system of $N+1$ agents is composed by the exosystem (2) (labeled as node 0) and N systems (1) (labeled as node $1, \ldots, N$, respectively). Define a diagraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with node set $\mathcal{V} = \{0\} \cup \mathcal{N}$ and edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The adjacency matrix of graph $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}]$ with $i \in \mathcal{N}$ and $j \in \mathcal{V}$, where $a_{ij} > 0$ means that a communication channel from agent j to agent i exists, namely, $(j, i) \in \mathcal{E}$; otherwise, $a_{ij} = 0$.

The actuator failure model $u_i^f(t)$ of agent i is assumed to have κ_i failure modes. Each mode is denoted as $u_{ik}^f(t) = [u_{ik1}^f(t), \ldots, u_{ikm_i}^f(t)]^{\mathrm{T}}$ with $k \in \{1, \ldots, \kappa_i\}$ and

$$u_{ikv}^f(t) = \varrho_{iv}^k u_{iv}(t) + \lambda_{iv}(t),\ v \in \{1, \ldots, m_i\}, \tag{3}$$

where ϱ_{iv}^k is the unknown actuator efficiency factor of the kth failure mode in the vth actuator of agent i. Assume that $0 < \underline{\varrho}_{iv}^k \leqslant \varrho_{iv}^k \leqslant \overline{\varrho}_{iv}^k \leqslant 1$ with $\underline{\varrho}_{iv}^k$ and $\overline{\varrho}_{iv}^k$ being unknown. Let $\varrho_i^k \triangleq \mathrm{diag}\{\varrho_{i1}^k, \ldots, \varrho_{im_i}^k\}$ and $\triangle_{\varrho_i^k} \triangleq \{\varrho_i^k : \varrho_i^k = \mathrm{diag}\{\varrho_{i1}^k, \ldots, \varrho_{im_i}^k\},\ \varrho_{iv}^k \in [\underline{\varrho}_{iv}^k, \overline{\varrho}_{iv}^k]\}$ represents the sets of ϱ_{iv}^k. Moreover, let $\lambda_{iv}(t)$ denote the unknown time-varying bounded failure-derivation in the vth actuator with $\|\lambda_{iv}(t)\|_2 \leqslant \overline{\lambda}_i$, where $\overline{\lambda}_i$ is unknown. For convenience, rewrite the failure model as

$$u_i^f(t) = \varrho_i u_i(t) + \lambda_i(t), \tag{4}$$

where $\varrho_i \triangleq \mathrm{diag}\{\varrho_{i1}, \ldots, \varrho_{im_i}\} \in \{\varrho_i^1, \ldots, \varrho_i^{\kappa_i}\} \in \triangle_{\varrho_i^k}$, and $\lambda_i(t) \triangleq [\lambda_{i1}(t), \ldots, \lambda_{im_i}(t)]^{\mathrm{T}}$. In the following, for simplicity, $u_i(t)$ is rewritten as u_i and other variables are similarly denoted.

This work has two problems to solve, as follows.

Problem 1. For given systems (1) and (2) with $d_i \equiv 0$, develop a distributed control law u_i such that
(i) the closed-loop systems are asymptotically stable when $\nu \equiv 0$, and
(ii) for any initial conditions, the output synchronization is achieved, namely, $\lim_{t\to\infty} \varepsilon_i = 0,\ i \in \mathcal{N}$.

Problem 2. For given systems (1) and (2) with $d_i \not\equiv 0$, develop a distributed control law u_i such that, for any initial conditions,
(i) the states of closed-loop systems are bounded, and
(ii) the output synchronization errors are bounded.

To solve the above problems, some basic assumptions are needed.

Assumption 1. The digraph $\mathcal{G}$ has a spanning tree with node 0 as the root.

Assumption 2. The real parts of all of the eigenvalues of S in (2) are nonnegative.

Assumption 3. The pairs $(A_i,\ B_i)$ are stabilizable.

Assumption 4. There exist Π_i and Γ_i satisfying the following regulator equations:

$$\Pi_i S = A_i \Pi_i + B_i \Gamma_i + D_i, \quad E_i \Pi_i = F. \tag{5}$$

Remark 1. In [26], regulator equations involving actuator failure were utilized and a rank constraint was imposed on system matrices to ensure a feasible solution. However, this approach is rather conservative.

In this work, the existing regulator equations for heterogeneous linear multiagent systems are adopted that are proved to be effective for multiagent nonlinear systems subject to actuator failure. The solvable condition of (5) is that

$$\operatorname{rank}\begin{bmatrix} A_i-\lambda I & B_i \\ E_i & 0 \end{bmatrix} = n_i + l, \tag{6}$$

where $\lambda \in \{\lambda \in \mathbb{C} | \det(\lambda I - S) = 0\}$, as can be found from Assumption 1.4 in [29].

Lemma 1. If Assumption 1 holds, there exists a matrix Q with $Q = \operatorname{diag}\{q_1, \dots, q_N\}$ such that $QL + L^{\mathrm{T}}Q > 0$, where $L = [l_{ij}]_{i,j=1}^N$ with $l_{ii} = a_{i0} + \sum_{j=1}^N a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$.

Definition 1 ([30])**.** Let V: $\mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz-continuous. The Clarke generalized gradient of V at x is

$$\partial V(x) \triangleq \operatorname{co}\left\{ \lim_{i\to\infty} \nabla V(x_i) | x_i \to x, x_i \notin \Omega_v \right\}, \tag{7}$$

where co is the convex hull, and Ω_v is the set of measure zero where gradient $\nabla V(x)$ is not defined.

3 Main results

First, a distributed finite-time observer is constructed to estimate the exosystem dynamics. Then, based on the designed observer, a neural-adaptive control law u_i is built to solve Problems 1 and 2, respectively.

3.1 Distributed finite-time observer design

The following distributed finite-time observer is designed:

$$\dot{\zeta}_i = S\zeta_i - c\sum_{j=0}^N a_{ij}(\zeta_i - \zeta_j) - \gamma \operatorname{sign}\left(\sum_{j=0}^N a_{ij}(\zeta_i - \zeta_j)\right), \tag{8}$$

where $c > 0$ and $\gamma > 0$ are to be chosen, and $\zeta_0 = \nu$. The function sign$(\cdot)$ is defined as

$$\operatorname{sign}(h) = \begin{cases} 1, & \text{if } h > 0, \\ 0, & \text{if } h = 0, \\ -1, & \text{if } h < 0. \end{cases}$$

Theorem 1. Under Assumption 1, observer (8) with $c > \max\{\frac{\|Q\otimes S\|_F}{\varpi}, \frac{\sqrt{n_0 N}\|Q\otimes S\|_F}{2\varpi}\}$, $0 < \varpi \leqslant \lambda_{\min}(\frac{QL+L^{\mathrm{T}}Q}{2})$ and $\gamma > 0$ can estimate the state of the exosystem (2) in finite time.

Proof. Let $e_i = \zeta_i - \nu$. It follows from (2) and (8) that

$$\dot{e} = (I \otimes S - cL \otimes I)e - \gamma \operatorname{sign}((L\otimes I)e), \tag{9}$$

where $e = [e_1^{\mathrm{T}}, \dots, e_N^{\mathrm{T}}]^{\mathrm{T}}$. Let $\tilde{e} = (L \otimes I)e$. We obtain

$$\dot{\tilde{e}} = (I \otimes S - cL \otimes I)\tilde{e} - \gamma (L\otimes I)\operatorname{sign}(\tilde{e}), \tag{10}$$

where $\tilde{e} = [\tilde{e}_1^{\mathrm{T}}, \dots, \tilde{e}_N^{\mathrm{T}}]^{\mathrm{T}}$.

Select the following non-smooth Lyapunov function:

$$V = \sum_{i=1}^N q_i \left(\frac{c}{2}|\tilde{e}_i|^2 + \gamma|\tilde{e}_i|\right), \tag{11}$$

where q_i is defined in Lemma 1.

Computing the Clarke generalized gradient of V yields

$$\partial V = \begin{bmatrix} q_1(c\tilde{e}_1 + \gamma \mathrm{SGN}(\tilde{e}_1)) \\ \vdots \\ q_N(c\tilde{e}_N + \gamma \mathrm{SGN}(\tilde{e}_N)) \end{bmatrix} = (Q\otimes I)(c\tilde{e} + \gamma \mathrm{SGN}(\tilde{e})), \tag{12}$$

where $Q = \mathrm{diag}\{q_1, \ldots, q_N\}$. $\mathrm{SGN}(\cdot)$ is a multivalued function, defined by

$$\mathrm{SGN}(h) = \begin{cases} 1, & \text{if } h > 0, \\ [-1, 1], & \text{if } h = 0, \\ -1, & \text{if } h < 0. \end{cases}$$

The set-valued Lie derivative of V with respect to $\tilde{e}$ is defined as $L_{\mathcal{F}}V(\tilde{e}) = \{a \in \mathbb{R} : \exists\ \alpha \in \mathcal{K}_{[f]}(\tilde{e}, t)$ with $\beta^{\mathrm{T}}\alpha = a,\ \forall \beta \in \partial V\}$ [31–33], where $\mathcal{K}_{[f]}(\tilde{e}, t) = (I \otimes S - cL \otimes I)\tilde{e} - \gamma(L \otimes I)\mathrm{SGN}(\tilde{e})$. The differential inclusion of (10) is $\dot{\tilde{e}} \in \mathcal{K}_{[f]}(\tilde{e}, t)$. Moreover, $\alpha \in \mathcal{K}_{[f]}(\tilde{e}, t)$ means that there exists $\phi = \mathrm{SGN}(\tilde{e})$ such that $\alpha = (I \otimes S - cL \otimes I)\tilde{e} - \gamma(L \otimes I)\phi$. For $\beta = (Q \otimes I)(c\tilde{e} + \gamma\phi) \in \partial V$, it follows that

$$\begin{aligned} a &= \beta^{\mathrm{T}}\alpha \\ &= (c\tilde{e} + \gamma\phi)^{\mathrm{T}}(Q \otimes I)[(I \otimes S - cL \otimes I)\tilde{e} - \gamma(L \otimes I)\phi] \\ &= (c\tilde{e} + \gamma\phi)^{\mathrm{T}}(Q \otimes S)\tilde{e} - 0.5(c\tilde{e} + \gamma\phi)^{\mathrm{T}}((QL + L^{\mathrm{T}}Q) \otimes I)(c\tilde{e} + \gamma\phi) \\ &< (c\tilde{e} + \gamma\phi)^{\mathrm{T}}(Q \otimes S)\tilde{e} - \varpi(c\tilde{e} + \gamma\phi)^{\mathrm{T}}(c\tilde{e} + \gamma\phi) \\ &< c\|Q \otimes S\|_F\|\tilde{e}\|_2^2 + \gamma\sqrt{n_0N}\|Q \otimes S\|_F\|\tilde{e}\|_2 - \varpi c^2\|\tilde{e}\|_2^2 - 2\varpi\gamma c\|\tilde{e}\|_1 - \varpi\gamma^2\phi^{\mathrm{T}}\phi \\ &< c\|Q \otimes S\|_F\|\tilde{e}\|_2^2 + \gamma\sqrt{n_0N}\|Q \otimes S\|_F\|\tilde{e}\|_1 - \varpi c^2\|\tilde{e}\|_2^2 - 2\varpi\gamma c\|\tilde{e}\|_1 - \varpi\gamma^2\phi^{\mathrm{T}}\phi \\ &< c(\|Q \otimes S\|_F - \varpi c)\|\tilde{e}\|_2^2 + \gamma(\sqrt{n_0N}\|Q \otimes S\|_F - 2\varpi c)\|\tilde{e}\|_1 - \varpi\gamma^2\phi^{\mathrm{T}}\phi, \end{aligned} \tag{13}$$

where $0 < \varpi \leqslant \lambda_{\min}(\frac{QL+L^{\mathrm{T}}Q}{2})$, $\phi^{\mathrm{T}}\tilde{e} = \|\tilde{e}\|_1$ and $\|\phi\|_2 \leqslant \sqrt{n_0N}$. Thus, for

$$c > \max\left\{\frac{\|Q \otimes S\|_F}{\varpi}, \frac{\sqrt{n_0N}\|Q \otimes S\|_F}{2\varpi}\right\}, \tag{14}$$

it is obtained that $a < -\varpi\gamma^2\phi^{\mathrm{T}}\phi$. For any $\tilde{e} \neq 0$, $\phi^{\mathrm{T}}\phi > 1$ and $a < -\varpi\gamma^2$ hold. This further yields that $\frac{\mathrm{d}}{\mathrm{d}t}V < -\varpi\gamma^2$ for almost every t and there exists t^* with $t^* < \frac{V(0)}{\varpi\gamma^2}$ such that $\lim_{t\to t^*} V = 0$. Thus, both $\tilde{e}$ and e will approach zero in finite time.

Remark 2. In [34], it was shown that we can ignore the case that $\tilde{e} = 0$ is true only for some isolated time points of measure zero. On the other hand, from $t^* < \frac{V(0)}{\varpi\gamma^2}$, it is found that the time required to obtain $\tilde{e} = 0$ decreases with increasing γ. Moreover, by choosing c to be sufficiently large, the use of global information of the interaction graph can be avoided.

3.2 Neural-adaptive control law design

Let $\eta_i = x_i - \Pi_i\zeta_i$. Combining (1), (5) and (8) yields

$$\begin{aligned} \dot{\eta}_i &= A_i\eta_i + B_i\left(u_i^f + f_i(x_i) + w_i\right) + C_id_i - B_i\Gamma_i\zeta_i - D_ie_i + c\Pi_i\tilde{e}_i + \gamma\Pi_i\mathrm{sign}(\tilde{e}_i), \\ \varepsilon_i &= E_i\eta_i + Fe_i, \end{aligned} \tag{15}$$

where $\tilde{e}_i = \sum_{j=0}^{N} a_{ij}(\zeta_i - \zeta_j)$.

Because the NN has a universal approximation capability [35, 36], it is adopted to approximate the smooth function $f_i(x_i(t))$ on a compact set in the form of

$$f_i(x_i(t)) = H_i^{\mathrm{T}}\varphi_i(x_i) + \delta_i, \tag{16}$$

where $\varphi_i(x_i)$: $\mathbb{R}^{n_i} \to \mathbb{R}^{s_i}$ is a chosen vector-valued basis function and $\varphi_i(x_i) = \sigma(\mathcal{V}_i^{\mathrm{T}}\bar{x}_i)$ with $\bar{x}_i = [1\ x_i^{\mathrm{T}}]^{\mathrm{T}}$; s_i is the number of hidden-layer neurons; matrix $\mathcal{V}_i$ contains the first-layer weights and thresholds that can be selected randomly; $\sigma(\cdot)$ is the standard sigmoid function with $\sigma(x) = \frac{1}{1+\mathrm{e}^{-x}}$; $H_i \in \mathbb{R}^{s_i \times m_i}$ is the unknown ideal second-layer weight matrix; and δ_i is the bounded approximation error with $\|\delta_i\|_2 \leqslant \overline{\delta}_i$, where $\overline{\delta}_i$ is usually unknown.

Then, the following neural-adaptive control law is constructed:

$$u_i = -\frac{a_i + b_i + \mu}{b_i + \mu}\psi_iB_i^{\mathrm{T}}P_i\eta_i - \hat{H}_i^{\mathrm{T}}\varphi_i(x_i) - \theta_i\mathrm{sign}(B_i^{\mathrm{T}}P_i\eta_i), \tag{17}$$

where $a_i = \|\Gamma_i\|_F^2\|\zeta_i\|_2^2$ and $b_i = \|B_i^{\mathrm{T}}P_i\eta_i\|_2\|\Gamma_i\|_F\|\zeta_i\|_2$; μ is a given exponential function with $\mu = \mu_1\mathrm{e}^{-\mu_2 t}$, $\mu_1 > 0$ and $\mu_2 > 0$; matrix $P_i > 0$ and constant ψ_i are to be determined; and $\hat{H}_i$ is the estimation of H_i that satisifes

$$\begin{aligned}\dot{\hat{H}}_i &= r_i\varphi_i(x_i)\eta_i^{\mathrm{T}}P_iB_i - r_ig_i(\hat{H}_i - \bar{H}_i),\\ \dot{\bar{H}}_i &= h_ig_i(\hat{H}_i - \bar{H}_i),\end{aligned} \tag{18}$$

where $r_i > 0$, $h_i > 0$, and $g_i > 0$ are given. The proposed update law (18) can effectively inhibit the undesired high-frequency oscillations that may occur during operation [37]. The controller also updates θ_i as follows:

$$\begin{aligned}\dot{\theta}_i &= \|B_i^{\mathrm{T}}P_i\eta_i\|_2 - \bar{m}_i(\theta_i - \hat{\theta}_i),\\ \dot{\hat{\theta}}_i &= \bar{m}_i(\theta_i - \hat{\theta}_i),\end{aligned} \tag{19}$$

where $\bar{m}_i > 0$ is given.

Remark 3. NNs have obtained great success in pattern recognition, classification, digital signal processing and many other applications. In this work, an NN is used to design an output regulation control law (17) for system (1) with unknown nonlinearity because of its universal approximation capability. We note that the approximation error δ_i generally decreases with increasing number of hidden-layer neurons s_i [36], which will lead to too many adaptive parameters in (18) and give rise to an additional computational burden.

Based on (4), substituting the control law (17) into (15), the following closed-loop system is obtained:

$$\begin{aligned}\dot{\eta}_i = &\left(A_i - \frac{a_i + b_i + \mu}{b_i + \mu}\psi_iB_i\varrho_iB_i^{\mathrm{T}}P_i\right)\eta_i + C_id_i - B_i\Gamma_i\zeta_i - \theta_iB_i\varrho_i\mathrm{sign}(B_i^{\mathrm{T}}P_i\eta_i)\\ &+ B_i(\tilde{H}_i^{\mathrm{T}}\varphi_i(x_i) + \lambda_i + \delta_i + w_i) - D_ie_i + c\Pi_i\tilde{e}_i + \gamma\Pi_i\mathrm{sign}(\tilde{e}_i),\\ \varepsilon_i = &E_i\eta_i + Fe_i,\end{aligned} \tag{20}$$

where $\tilde{H}_i = H_i - \hat{H}_i\varrho_i$.

Now, we are ready to solve Problems 1 and 2 posed in Section 2.

Theorem 2. Under Assumptions 1–4, Problem 1 is solved by using observer (8) and controller (17) with $\psi_i > \frac{1}{\rho_i}$, if there exist matrices $P_i > 0$ and scalars $\phi_i > 0$ satisfying

$$P_iA_i + A_i^{\mathrm{T}}P_i - 2P_iB_iB_i^{\mathrm{T}}P_i < -\phi_iI, \tag{21}$$

where $\rho_i = \min\{\underline{\varrho}_{iv}^k\}$ for all $k \in \{1,\ldots,\kappa_i\}$ and $v \in \{1,\ldots,m_i\}$.

Proof. Consider the following Lyapunov function:

$$V_i = \eta_i^{\mathrm{T}}P_i\eta_i + \rho_i(\theta - \theta_i)^2 + \rho_i(\theta - \hat{\theta}_i)^2 + \frac{1}{r_i}\mathrm{tr}(\tilde{H}_i\varrho_i^{-1}\tilde{H}_i^{\mathrm{T}}) + \frac{1}{h_i}\mathrm{tr}(\breve{H}_i\varrho_i^{-1}\breve{H}_i^{\mathrm{T}}), \tag{22}$$

where $\breve{H}_i = H_i - \bar{H}_i\varrho_i$, and $\theta > 0$ is to be chosen. It follows that

$$\dot{V}_i = 2\eta_i^{\mathrm{T}}P_i\dot{\eta}_i - 2\rho_i(\theta - \theta_i)\dot{\theta}_i - 2\rho_i(\theta - \hat{\theta}_i)\dot{\hat{\theta}}_i - \frac{2}{r_i}\mathrm{tr}(\dot{\hat{H}}_i\tilde{H}_i^{\mathrm{T}}) - \frac{2}{h_i}\mathrm{tr}(\dot{\bar{H}}_i\breve{H}_i^{\mathrm{T}}). \tag{23}$$

On the other hand, we have

$$\begin{aligned}2\eta_i^{\mathrm{T}}P_i\dot{\eta}_i = &\eta_i^{\mathrm{T}}(P_iA_i + A_i^{\mathrm{T}}P_i - 2\psi_iP_iB_i\varrho_iB_i^{\mathrm{T}}P_i)\eta_i - 2\eta_i^{\mathrm{T}}P_iB_i\Gamma_i\zeta_i(t)\\ &- \frac{2a_i\psi_i}{b_i + \mu}\eta_i^{\mathrm{T}}P_iB_i\varrho_iB_i^{\mathrm{T}}P_i\eta_i - 2\theta_i\eta_i^{\mathrm{T}}P_iB_i\varrho_i\mathrm{sign}(B_i^{\mathrm{T}}P_i\eta_i)\\ &+ 2\eta_i^{\mathrm{T}}P_iB_i(\lambda_i + \delta_i + w_i + \tilde{H}_i^{\mathrm{T}}\varphi_i(x_i)) + 2\eta_i^{\mathrm{T}}P_i(-D_ie_i + c\Pi_i\tilde{e}_i\\ &+ \gamma\Pi_i\mathrm{sign}(\tilde{e}_i)) + 2\eta_i^{\mathrm{T}}P_iC_id_i,\end{aligned} \tag{24}$$

$$2\rho_i(\theta - \theta_i)\dot{\theta}_i + 2\rho_i(\theta - \hat{\theta}_i)\dot{\hat{\theta}}_i = -2\rho_i(\theta_i - \theta)\|B_i^{\mathrm{T}}P_i\eta_i\|_2 + 2\rho_i\bar{m}_i(\theta_i - \hat{\theta}_i)^2, \tag{25}$$

and

$$\frac{2\mathrm{tr}(\dot{\hat{H}}_i\tilde{H}_i^{\mathrm{T}})}{r_i}+\frac{2\mathrm{tr}(\dot{\bar{H}}_i\breve{H}_i^{\mathrm{T}})}{h_i}=2g_i\mathrm{tr}((\hat{H}_i-\bar{H}_i)\varrho_i(\hat{H}_i-\bar{H}_i)^{\mathrm{T}})+2\eta_i^{\mathrm{T}}P_iB_i\tilde{H}_i^{\mathrm{T}}\varphi_i(x_i). \tag{26}$$

Considering a_i and b_i in (17), we obtain

$$\begin{aligned}&-\frac{2a_i\psi_i}{b_i+\mu}\eta_i^{\mathrm{T}}P_iB_i\varrho_iB_i^{\mathrm{T}}P_i\eta_i-2\eta_i^{\mathrm{T}}P_iB_i\Gamma_i\zeta_i(t)\\&<2\|B_i^{\mathrm{T}}P_i\eta_i\|_2\|\Gamma_i\|_F\|\zeta_i(t)\|_2-\frac{2a_i\psi_i\rho_i}{b_i+\mu}\|B_i^{\mathrm{T}}P_i\eta_i\|_2^2\\&<2b_i-\frac{2b_i^2}{b_i+\mu}\\&<2\mu,\end{aligned}\tag{27}$$

where $\rho_i=\min\{\underline{\varrho}_{iv}^k\}$ for all $k\in\{1,\dots,\kappa_i\}$ and $v\in\{1,\dots,m_i\}$. A sufficiently large ψ_i such that $\psi_i>\frac{1}{\rho_i}$ can be selected, yielding the second inequality.

From $\eta_i^{\mathrm{T}}P_iB_i\varrho_i\mathrm{sign}(B_i^{\mathrm{T}}P_i\eta_i)$ and $\rho_i=\min\{\underline{\varrho}_{iv}^k\}$ with $\varrho_i>0$, we obtain

$$\begin{aligned}\eta_i^{\mathrm{T}}P_iB_i\varrho_i\mathrm{sign}(B_i^{\mathrm{T}}P_i\eta_i)&=\eta_i^{\mathrm{T}}P_iB_i\varrho_i\mathrm{sign}(\varrho_iB_i^{\mathrm{T}}P_i\eta_i)\\&=\|\varrho_iB_i^{\mathrm{T}}P_i\eta_i\|_1\\&\geqslant\rho_i\|B_i^{\mathrm{T}}P_i\eta_i\|_1\\&\geqslant\rho_i\|B_i^{\mathrm{T}}P_i\eta_i\|_2.\end{aligned}\tag{28}$$

Since $\|\lambda_i\|_2<\overline{\lambda}_i$, $\|\delta_i\|_2<\bar{\delta}_i$ and $\|w_i\|_2<\overline{w}_i$, we obtain

$$\eta_i^{\mathrm{T}}P_iB_i(\lambda_i+\delta_i+w_i)\leqslant(\overline{\lambda}_i+\bar{\delta}_i+\overline{w}_i)\|B_i^{\mathrm{T}}P_i\eta_i\|_2. \tag{29}$$

Because $\rho_i=\min\{\underline{\varrho}_{iv}^k\}$ and $\psi_i>\frac{1}{\rho_i}$, it follows from (21) that $P_iA_i+A_i^{\mathrm{T}}P_i-2\psi_iP_iB_i\varrho_iB_i^{\mathrm{T}}P_i<-\phi_iI$. Then, combining (24)–(29) yields

$$\begin{aligned}\dot{V}_i<&-\phi_i\|\eta_i\|_2^2+2\mu+2(\overline{\lambda}_i+\bar{\delta}_i+\overline{w}_i-\theta\rho_i)\|B^{\mathrm{T}}P\eta_i\|_2\\&+2\eta_i^{\mathrm{T}}P_i(-D_ie_i+c\Pi_i\tilde{e}_i+\gamma\Pi_i\mathrm{sign}(\tilde{e}_i))+2\eta_i^{\mathrm{T}}P_iC_id_i\\&-2\rho_i\bar{m}_i(\theta_i-\hat{\theta}_i)^2-2g_i\mathrm{tr}((\hat{H}_i-\bar{H}_i)\varrho_i(\hat{H}_i-\bar{H}_i)^{\mathrm{T}}).\end{aligned}\tag{30}$$

Choosing a large enough θ such that $\overline{\lambda}_i+\bar{\delta}_i+\overline{w}_i-\theta\rho_i<0$, we further obtain

$$\begin{aligned}\dot{V}_i<&-\phi_i\|\eta_i\|_2^2+2\mu+2\eta_i^{\mathrm{T}}P_i(-D_ie_i+c\Pi_i\tilde{e}_i\\&+\gamma\Pi_i\mathrm{sign}(\tilde{e}_i))+2\eta_i^{\mathrm{T}}P_iC_id_i.\end{aligned}\tag{31}$$

When $d_i\equiv0$, it follows that

$$\dot{V}_i<-\phi_i\|\eta_i\|_2^2+2\mu+2\eta_i^{\mathrm{T}}P_i(-D_ie_i+c\Pi_i\tilde{e}_i+\gamma\Pi_i\mathrm{sign}(\tilde{e}_i)). \tag{32}$$

Because e_i and $\tilde{e}_i$ tend to zero after finite time t^*, we obtain

$$V_i(\infty)<V_i(t^*)+2\bar{\mu}-\int_{t^*}^{\infty}\phi_i\|\eta_i(\tau)\|_2^2\mathrm{d}\tau, \tag{33}$$

where $\bar{\mu}=\frac{\mu_1}{\mu_2}=\int_0^{\infty}\mu_1\mathrm{e}^{-\mu_2\tau}\mathrm{d}\tau>\int_{t^*}^{\infty}\mu_1\mathrm{e}^{-\mu_2\tau}\mathrm{d}\tau$. Thus, it follows that

$$\int_{t^*}^{\infty}\|\eta_i(\tau)\|_2^2\mathrm{d}\tau<\frac{V(t^*)}{\phi_i}+\frac{2\bar{\mu}}{\phi_i}. \tag{34}$$

When $\nu\equiv0$, $\zeta_i\equiv0$. Thus, $\int_{t^*}^{\infty}\|x_i(\tau)\|_2^2\mathrm{d}\tau<\frac{V(t^*)}{\phi_i}+\frac{2\bar{\mu}}{\phi_i}$, implying the asymptotical stability of system (20). When $\nu\not\equiv0$, $\lim_{t\to\infty}\eta_i=0$, and thus $\lim_{t\to\infty}\varepsilon_i=\lim_{t\to\infty}E_i\eta_i+Fe_i=0$.

Remark 4. The linear matrix inequality can be used to solve the Riccatic inequality (21) by using the following equivalent transformation: it follows from (21) that $A_iP_i^{-1}+P_i^{-1}A_i^{\rm T}-2B_iB_i^{\rm T}+\phi_iP_i^{-1}P_i^{-1}<0$. By the Schur complement, it is further obtained that

$$\begin{bmatrix} A_iP_i^{-1}+P_i^{-1}A_i^{\rm T}-2B_iB_i^{\rm T} & P_i^{-1} \\ * & -\phi_i^{-1}I \end{bmatrix}<0, \tag{35}$$

which is equivalent to inequality (21).

Theorem 3. Under Assumptions 1–4, Problem 2 is solved by using observer (8) and controller (17) with $\psi_i>\frac{1}{\rho_i}$, if there exist matrices $P_i>0$ and scalars $\phi_i>0$ satisfying (21). Moreover, (i) the states of the closed-loop system (20) are bounded and

$$\lim_{t\to\infty}\|\eta_i\|_2\leqslant\frac{2\sqrt{\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)}}{\phi_i}\overline{d}_i,$$

and (ii) the output synchronization error is bounded, namely,

$$\lim_{t\to\infty}\|\varepsilon_i\|_2<\frac{2\sqrt{\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)\lambda_{\max}(E_i^{\rm T}E_i)}}{\phi_i}\overline{d}_i.$$

Proof. Use the same Lyapunov function V_i as in Theorem 2. Then, by a similar analysis with $\|d_i\|_2\leqslant\overline{d}_i$, it is obtained that

$$\dot{V}_i<-\phi_i\|\eta_i\|_2^2+2\mu+2\eta_i^{\rm T}P_i(-D_ie_i+c\Pi_i\tilde{e}_i+\gamma\Pi_i{\rm sign}(\tilde{e}_i))+2\eta_i^{\rm T}P_iC_id_i. \tag{36}$$

Since $2\eta_i^{\rm T}P_iC_id_i<\frac{\phi_i}{2}\|\eta_i\|_2^2+\frac{2}{\phi_i}\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)\|\overline{d}_i\|_2^2$, after time t^*, it is obtained that

$$\dot{V}_i<-\frac{\phi_i}{2}\|\eta_i\|_2^2+\frac{2\tilde{\varpi}_i}{\phi_i}\overline{d}_i^2+2\mu, \tag{37}$$

where $\tilde{\varpi}_i=\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)$.

According to Theorem 2.7 in [38], it can be concluded that system (20) is robustly input-to-state stable with inputs d_i and μ, and the system states are bounded owing to the bounded property of d_i for any initial conditions. When $\|\eta_i\|_2^2>\frac{4\tilde{\varpi}_i}{\phi_i^2}\overline{d}_i^2+\frac{4\mu}{\phi_i}$, $\dot{V}_i<0$ holds. This implies that η_i will eventually enter into the set $\Psi\triangleq\{\eta_i:\|\eta_i\|_2^2\leqslant\frac{4\tilde{\varpi}_i}{\phi_i^2}\overline{d}_i^2+\frac{4\mu_1}{\phi_i}\}$ because $\|d_i\|_2\leqslant\overline{d}_i$ and $\mu=\mu_1{\rm e}^{-\mu_2t}\leqslant\mu_1$ for $t>0$. As $t\to\infty$, $\lim_{t\to\infty}\|\eta_i\|_2^2\leqslant\frac{4\tilde{\varpi}_i}{\phi_i^2}\overline{d}_i^2$ because $\lim_{t\to\infty}\mu=\lim_{t\to\infty}\mu_1{\rm e}^{-\mu_2t}=0$. Thus, it follows that

$$\lim_{t\to\infty}\|\varepsilon_i\|_2<\lim_{t\to\infty}\sqrt{\lambda_{\max}(E_i^{\rm T}E_i)}\|\eta_i\|_2+\|Fe_i\|_2<\frac{2\sqrt{\tilde{\varpi}_i\lambda_{\max}(E_i^{\rm T}E_i)}}{\phi_i}\overline{d}_i.$$

Remark 5. The constant c in observer (8) can be chosen to be sufficiently large to avoid the use of global information of the communication graph $\mathcal{G}$. Similarly, the analysis is also applicable to constant ψ_i in controller (17) and ψ_i can be chosen to be larger enough to compensate the effect from actuator failure when their minimal lower bound is unknown. This is quite different from the published studies such as [14, 26–28]. Exact and complete information of actuator failure must be known for the method reported in [26] in order to solve the regulator equations when dealing with cooperative output regulation for homogenous linear multiagent systems. The projection approach was adopted in [14,27,28] to estimate the actuator failure, where both lower and upper bounds must be known in advance. The present paper relaxes most of these restrictions.

Remark 6. It can be seen from Theorem 3 that the upper bounds of both $\lim_{t\to\infty}\|\eta_i\|_2$ and $\lim_{t\to\infty}\|\varepsilon_i\|_2$ are related to $\frac{\sqrt{\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)}}{\phi_i}$. The following result provides the minimal upper bound of $\frac{\sqrt{\lambda_{\max}(C_i^{\rm T}P_iP_iC_i)}}{\phi_i}$, denoted as $\sqrt{\lambda_{\max}(C_i^{\rm T}C_i)}\bar{\phi}_i^*$.

Theorem 4. The minimal upper bound $\sqrt{\lambda_{\max}(C_i^{\mathrm{T}}C_i)}\bar{\phi}_i^*$ can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min\ \bar{\phi}_i \\ &\text{s.t. } A_iQ_i + Q_iA_i^{\mathrm{T}} - 2B_iB_i^{\mathrm{T}} + \bar{\phi}_i^{-1}Q_i < 0, \\ &\quad\ \ Q_i = P_i^{-1} > 0. \end{aligned} \tag{38}$$

And $\lim_{t\to\infty}\|\eta_i\|_2 < 2\sqrt{\lambda_{\max}(C_i^{\mathrm{T}}C_i)}\bar{\phi}_i^*\bar{d}_i$ and $\lim_{t\to\infty}\|\varepsilon_i\|_2 < 2\sqrt{\lambda_{\max}(C_i^{\mathrm{T}}C_i)\lambda_{\max}(E_i^{\mathrm{T}}E_i)}\bar{\phi}_i^*\bar{d}_i$.

Proof. Because $P_i > 0$, it follows that

$$\frac{\sqrt{\lambda_{\max}(C_i^{\mathrm{T}}P_iP_iC_i)}}{\phi_i} < \sqrt{\lambda_{\max}(C_i^{\mathrm{T}}C_i)}\bar{\phi}_i, \tag{39}$$

where $\bar{\phi}_i = \frac{\lambda_{\max}(P_i)}{\phi_i}$. Pre- and post-multiplying P_i^{-1} to (21) yields

$$A_iP_i^{-1} + P_i^{-1}A_i^{\mathrm{T}} - 2B_iB_i^{\mathrm{T}} + \phi_iP_i^{-1}P_i^{-1} < 0. \tag{40}$$

Since $P_i^{-1} > \frac{1}{\lambda_{\max}(P_i)}I$, it is obtained that $\phi_iP_i^{-1} > \frac{\phi_i}{\lambda_{\max}(P_i)}I = \bar{\phi}_i^{-1}I$. Then, it follows from (40) that

$$A_iP_i^{-1} + P_i^{-1}A_i^{\mathrm{T}} - 2B_iB_i^{\mathrm{T}} + \bar{\phi}_i^{-1}P_i^{-1} < 0. \tag{41}$$

Thus, the minimal value $\bar{\phi}_i^*$ can be obtained by solving the optimization problem (38).

Remark 7. We note that the nonlinear coupling between $\bar{\phi}_i$ and Q_i in (38) will cause some difficulty in computing the optimal value $\bar{\phi}_i^*$. Once $Q_i = P_i^{-1}$ is determined in advance, $\bar{\phi}_i^*$ can be solved much more easily; however this is suboptimal.

4 Illustrative example

Consider a multiagent system of five agents, shown in Figure 1. Their system parameters are given as follows:

$$\begin{aligned} &S = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad A_i = \begin{bmatrix} 1 & 1 \\ o_i & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \pi_i \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ &D_i = \begin{bmatrix} -0.9 & -0.9 \\ 1 & 0.1 \end{bmatrix}, \quad E_i = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad o_i = -1 + i, \quad \pi_i = i, \quad w_i = \cos(x_{i1} + x_{i2}), \\ &f_i(x_i) = x_{i1}\sin(x_{i1}) + x_{i2}\cos(x_{i2}), \quad d_1 = d_3 = d_4 = 0, \quad d_2 = 3\sin(t), \quad i \in \{1,2,3,4\}. \end{aligned}$$

Two possible failure modes of each agent i are considered as follows. (i) Normal mode 1: the actuator of agent i operates normally with $\varrho_{i1}^1 = \varrho_{i2}^1 = 1$. (ii) Failure mode 2: the actuator of agent i loses its operation with bounds characterized by $\overline{\varrho}_{i1}^2 = 0.8 + 0.05i$ and $\underline{\varrho}_{i1}^2 = 0.6 + 0.05i$. Thus, $\rho_i = \min\{\underline{\varrho}_{iv}^k\} = 0.6 + 0.05i$ and ψ_i in the controller (17) is then selected as $\psi_i = 2$. Set λ_i in (4) as $\lambda_i = 0.01i + 0.02i\sin(t)$. It follows from Assumption 4 that

$$\Pi_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} -1 & -\frac{1}{i} \end{bmatrix}.$$

From inequality (21), we obtain

$$P_1 = \begin{bmatrix} 51 & 20 \\ 20 & 9 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 15 & 5.25 \\ 5.25 & 2.25 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 10 & 2.9 \\ 2.9 & 1.1 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 0.6 \end{bmatrix}.$$

By using the obtained P_2, the minimal values of $\bar{\phi}_2$ in Theorem 4 can be computed, obtaining $\bar{\phi}_2^* = 0.5625$.

The initial states of systems (1), (2) and (8) are set as $x_1(0) = [12\ \ -12]^{\mathrm{T}}$, $x_2(0) = [5\ \ -5]^{\mathrm{T}}$, $x_3(0) = [0\ \ 0]^{\mathrm{T}}$, $x_4(0) = [-8\ \ 8]^{\mathrm{T}}$, $\nu(0) = [-40\ \ 10]^{\mathrm{T}}$, $\zeta_1(0) = [2\ \ -8]^{\mathrm{T}}$, $\zeta_2(0) = [-20\ \ 15]^{\mathrm{T}}$, $\zeta_3(0) = [-6\ \ 17]^{\mathrm{T}}$ and $\zeta_4(0) = [18\ \ -19]^{\mathrm{T}}$. The initial values of $\hat{H}_i$, $\bar{H}_i$, θ_i and $\hat{\theta}_i$ are chosen to be zero. We take

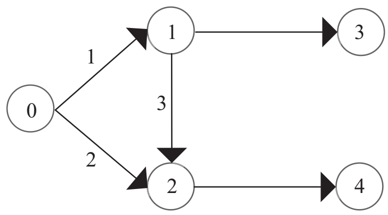

Figure 1 Communication topology.

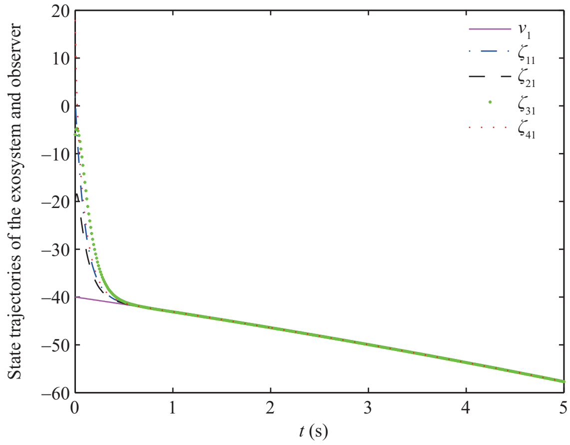

Figure 2 (Color online) States of the exosystem ν_1 and observer ζ_{i1}.

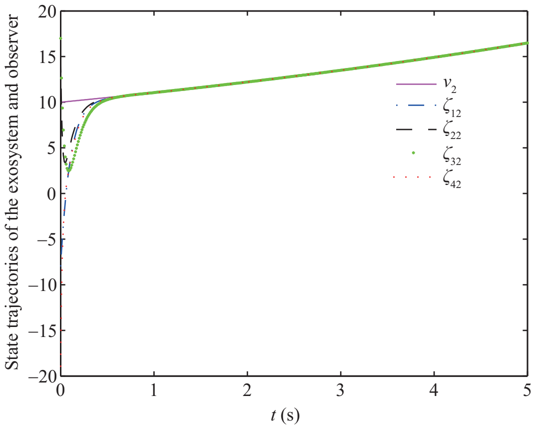

Figure 3 (Color online) States of the exosystem ν_2 and observer ζ_{i2}.

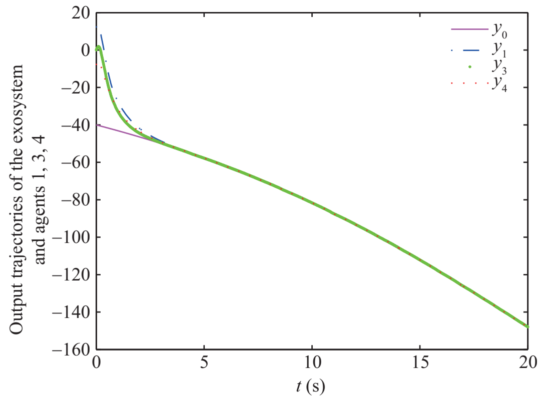

Figure 4 (Color online) Output of the exosystem (2) and agents 1, 3, 4.

$h_i = g_i = r_i = \bar{m}_i = 1$ and $\mu = 0.002\mathrm{e}^{-8t}$. The $\mathcal{V}_i$ in (16) is taken as $\mathcal{V}_i^{\mathrm{T}} = \left[\begin{smallmatrix} -2 & -2 & -3 \\ 4 & -3 & -4 \end{smallmatrix}\right]$. Select $c = 10$ and $\gamma = 0.1$ for observer (8).

Figures 2 and 3 show that observer (8) can effectively estimate the states of system (2). Figures 4–6 show that (i) agents 1, 3 and 4 that are not subject to mismatched disturbance can successfully track the output of the exosystem, and (ii) the output synchronization is not achieved for agent 2 which is affected by the mismatched disturbance, but the synchronization error ε_2 is always bounded.

5 Conclusion

This paper has investigated the cooperative output regulation problem for heterogeneous nonlinear uncertain multiagent systems. The considered systems are subject to actuator failure, matched disturbance, mismatched disturbance, and/or disturbance from an exosystem. By utilizing the non-smooth Lyapunov function technique, a distributed finite-time observer has been constructed to estimate the exosystem states over a communication digraph. Then, an NN-adaptive fault-tolerant control strategy has been developed using the states from the nonlinear systems and the finite-time observer without the use of global information. A simulation example has been presented to demonstrate the effectiveness of the proposed control strategy. A possible direction for future work is to investigate the applicability of the event-triggered output control laws to similar multiagent systems with switching communication

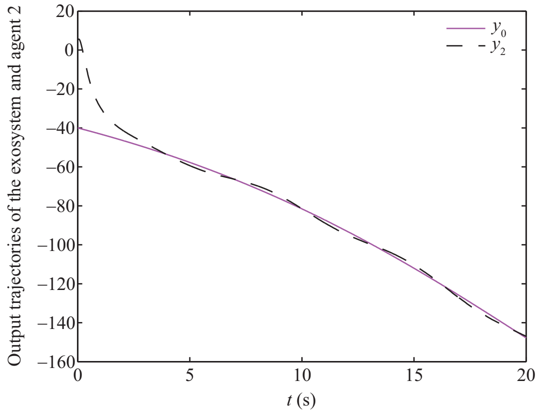

Figure 5 (Color online) Output of the exosystem (2) and agent 2.

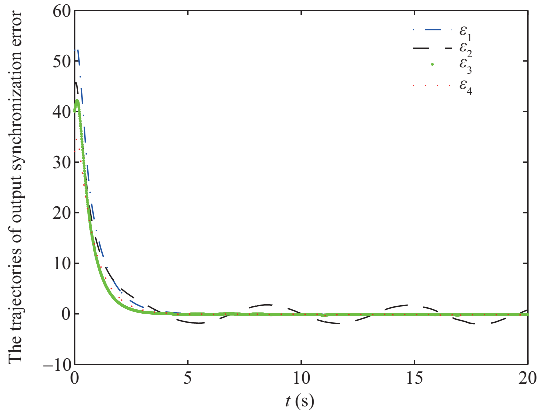

Figure 6 (Color online) Output synchronization error ε_i.

graphs [39–45].

Acknowledgements This work was supported by National Key R&D Program of China (Grant No. 2018YFB1702802), Science Fund for Creative Research Groups of the National Natural Science Foundation of China (Grant No. 61621002), NSFC-Zhejiang Joint Fund for the Integration of Industrialization and Informatization (Grant No. U1709203), Zhejiang Provincial Natural Science Foundation of China (Grant No. LZ19F030002), and Open Research Project of the State Key Laboratory of Industrial Control Technology, Zhejiang University, China (Grant No. ICT20068).

References

1 Ren W, Beard R W. Distributed Consensus in Multi-vehicle Cooperative Control. London: Springer, 2008
2 Lin Z L. Control design in the presence of actuator saturation: from individual systems to multi-agent systems. Sci China Inf Sci, 2019, 62: 026201
3 Tian L, Ji Z J, Hou T, et al. Bipartite consensus of edge dynamics on coopetition multi-agent systems. Sci China Inf Sci, 2019, 62: 229201
4 Wen G X, Chen C L P, Dou H, et al. Formation control with obstacle avoidance of second-order multi-agent systems under directed communication topology. Sci China Inf Sci, 2019, 62: 192205
5 Rehan M, Ahn C K, Chadli M. Consensus of one-sided Lipschitz multi-agents under input saturation. IEEE Trans Circ Syst II, 2020, 67: 745–749
6 Rehman A U, Rehan M, Iqbal N, et al. Toward the LPV approach for adaptive distributed consensus of Lipschitz multi-agents. IEEE Trans Circ Syst II, 2019, 66: 91–95
7 Xiao W B, Cao L, Li H Y, et al. Observer-based adaptive consensus control for nonlinear multi-agent systems with time-delay. Sci China Inf Sci, 2020, 63: 132202
8 Francis B A, Wonham W M. The internal model principle of control theory. Automatica, 1976, 12: 457–465
9 Chen J Y, Li Z H, Ding Z T. Adaptive output regulation of uncertain nonlinear systems with unknown control directions. Sci China Inf Sci, 2019, 62: 089205
10 Su Y F, Huang J. Cooperative output regulation of linear multi-agent systems. IEEE Trans Automat Contr, 2012, 57: 1062–1066
11 Zhang Y, Su Y F. Cooperative output regulation for linear uncertain MIMO multi-agent systems by output feedback. Sci China Inf Sci, 2018, 61: 092206
12 Xu Y, Fang M, Pan Y J, et al. Event-triggered output synchronization for nonhomogeneous agent systems with periodic denial-of-service attacks. Int J Robust Nonlinear Control, 2020. doi: 10.1002/rnc.5223
13 Dong S L, Ren W, Wu Z G, et al. H_∞ output consensus for Markov jump multiagent systems with uncertainties. IEEE Trans Cybern, 2020, 50: 2264–2273
14 Li H Y, Shi P, Yao D Y. Adaptive sliding mode control of Markov jump nonlinear systems with actuator faults. IEEE Trans Autom Control, 2017, 62: 1933–1939
15 Li H Y, Wu Y, Chen M. Adaptive fault-tolerant tracking control for discrete-time multiagent systems via reinforcement learning algorithm. IEEE Trans Cybern, 2021, 51: 1163–1174
16 Dong S L, Liu M Q, Wu Z G, et al. Observer-based sliding mode control for Markov jump systems with actuator failures and asynchronous modes. IEEE Trans Circ Syst II-Express Briefs, 2020. doi: 10.1109/TCSII.2020.3030703
17 Zhang X D, Parisini T, Polycarpou M M. Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach. IEEE Trans Automat Contr, 2004, 49: 1259–1274
18 Li Y X. Finite time command filtered adaptive fault tolerant control for a class of uncertain nonlinear systems. Automatica, 2019, 106: 117–123
19 Wang W, Wen C Y. Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance. Automatica, 2010, 46: 2082–2091
20 Ma H J, Yang G H. Adaptive fault tolerant control of cooperative heterogeneous systems with actuator faults and unreliable interconnections. IEEE Trans Automat Contr, 2016, 61: 3240–3255
21 Kim Y H, Lewis F L. Optimal design of CMAC neural-network controller for robot manipulators. IEEE Trans Syst Man Cybernet Part C (Appl Rev), 2000, 30: 22–31
22 He W, Dong Y T, Sun C Y. Adaptive neural impedance control of a robotic manipulator with input saturation. IEEE Trans Syst Man Cybernet Syst, 2016, 46: 334–344

23 Wang H, Yu W W, Ding Z T, et al. Tracking consensus of general nonlinear multiagent systems with external disturbances under directed networks. IEEE Trans Automat Contr, 2019, 64: 4772–4779
24 Wen G H, Wang P J, Huang T W, et al. Robust neuro-adaptive containment of multileader multiagent systems with uncertain dynamics. IEEE Trans Syst Man Cybernet Syst, 2019, 49: 406–417
25 Peng Z H, Wang D, Zhang H W, et al. Distributed neural network control for adaptive synchronization of uncertain dynamical multiagent systems. IEEE Trans Neural Netw Learn Syst, 2014, 25: 1508–1519
26 Deng C, Yang G H. Distributed adaptive fault-tolerant control approach to cooperative output regulation for linear multi-agent systems. Automatica, 2019, 103: 62–68
27 Liu M, Ho D W C, Shi P. Adaptive fault-tolerant compensation control for Markovian jump systems with mismatched external disturbance. Automatica, 2015, 58: 5–14
28 Zuo Z Q, Ho D W C, Wang Y J. Fault tolerant control for singular systems with actuator saturation and nonlinear perturbation. Automatica, 2010, 46: 569–576
29 Huang J. Nonlinear Output Regulation: Theory and Application. Philadelphia: Society for Industrial and Applied Mathematics, 2004
30 Shevitz D, Paden B. Lyapunov stability theory of nonsmooth systems. IEEE Trans Autom Control, 1994, 39: 1910–1914
31 Bacciotti A, Ceragioli F. Stability and stabilization of discontinuous systems and nonsmooth Lyapunov functions. ESAIM Control Optim Calc, 1999, 4: 361–376
32 Cortés J, Bullo F. Coordination and geometric optimization via distributed dynamical systems. SIAM J Control Optim, 2005, 44: 1543–1574
33 Cao Y, Ren W, Casbeer D W, et al. Finite-time connectivity-preserving consensus of networked nonlinear agents with unknown Lipschitz terms. IEEE Trans Automat Contr, 2016, 61: 1700–1705
34 Franceschelli M, Pisano A, Giua A, et al. Finite-time consensus with disturbance rejection by discontinuous local interactions in directed graphs. IEEE Trans Automat Contr, 2015, 60: 1133–1138
35 Sarangapani J. Neural Network Control of Nonlinear Discrete-Time Systems. Boca Raton: CRC Press, 2006
36 Lewis F L, Dawson D M, Abdallah C T. Robot Manipulator Control: Theory and Practice. Boca Raton: CRC Press, 2003
37 Yucelen T, Haddad W M. Low-frequency learning and fast adaptation in model reference adaptive control. IEEE Trans Automat Contr, 2013, 58: 1080–1085
38 Chen Z Y, Huang J. Stabilization and Regulation of Nonlinear Systems: A Robust and Adaptive Approach. Berlin: Springer, 2015
39 Lu Z H, Zhang L, Wang L. Controllability analysis of multi-agent systems with switching topology over finite fields. Sci China Inf Sci, 2019, 62: 012201
40 Liu J W, Huang J. Leader-following consensus of linear discrete-time multi-agent systems subject to jointly connected switching networks. Sci China Inf Sci, 2018, 61: 112208
41 Guan Y Q, Wang L. Structural controllability of multi-agent systems with absolute protocol under fixed and switching topologies. Sci China Inf Sci, 2017, 60: 092203
42 Chen Z Y, Han Q-L, Wu Z-G, et al. Special focus on advanced techniques for event-triggered control and estimation. Sci China Inf Sci, 2020, 63: 150200
43 Chen Z Y, Han Q-L, Yan Y M, et al. How often should one update control and estimation: review of networked triggering techniques. Sci China Inf Sci, 2020, 63: 150201
44 Ma H, Li H Y, Lu R Q, et al. Adaptive event-triggered control for a class of nonlinear systems with periodic disturbances. Sci China Inf Sci, 2020, 63: 150212
45 Wang Q, Wu Z G. Robust output feedback control for input-saturated systems based on a sliding mode observer. Circ Syst Signal Process, 2021, 40: 2267–2281